Transport properties of the QCD medium

Jacopo Ghiglieri, SUBATECH, Nantes

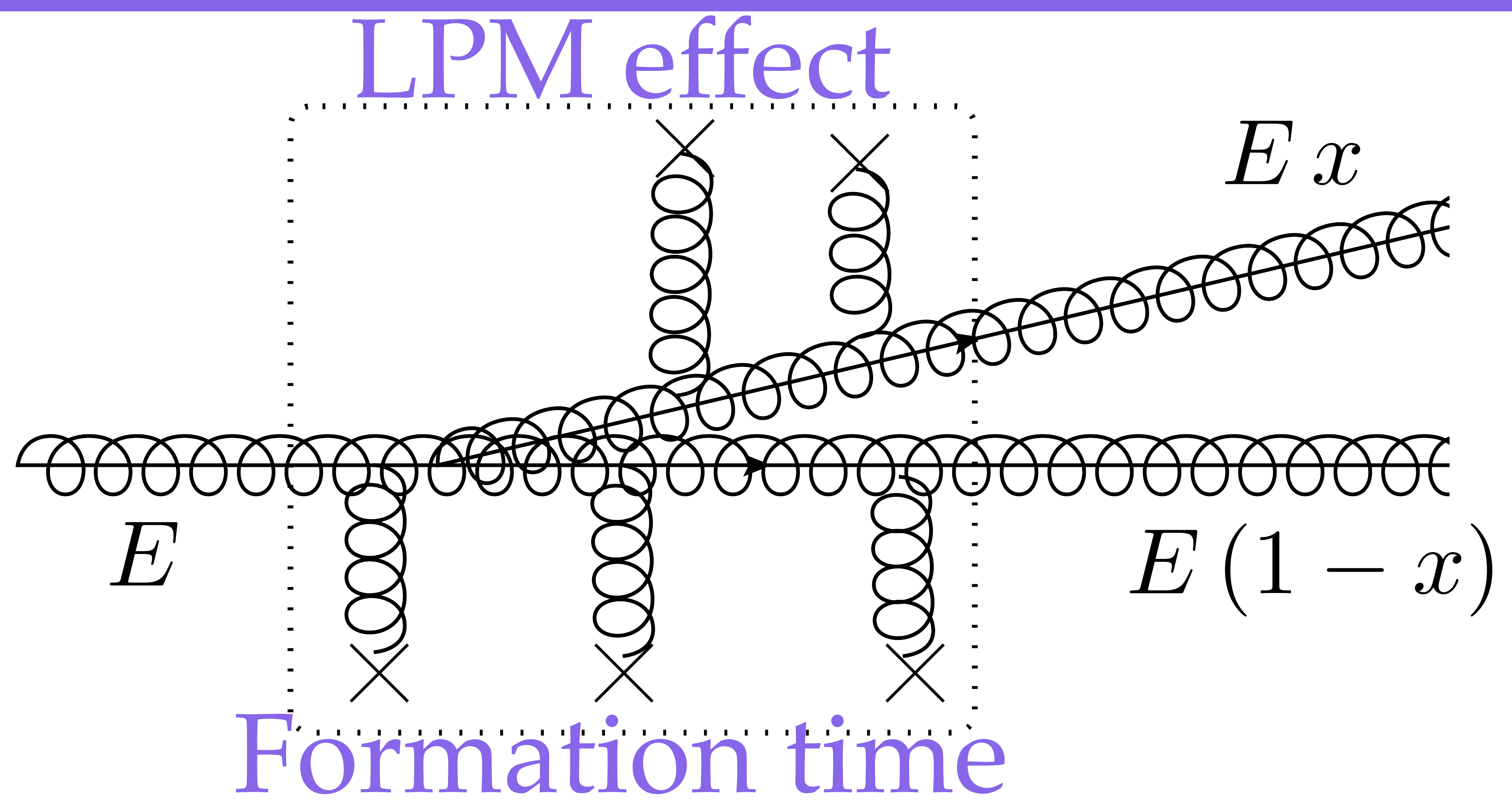
Quark Matter 2022, Kraków, April 8 2022

In this talk

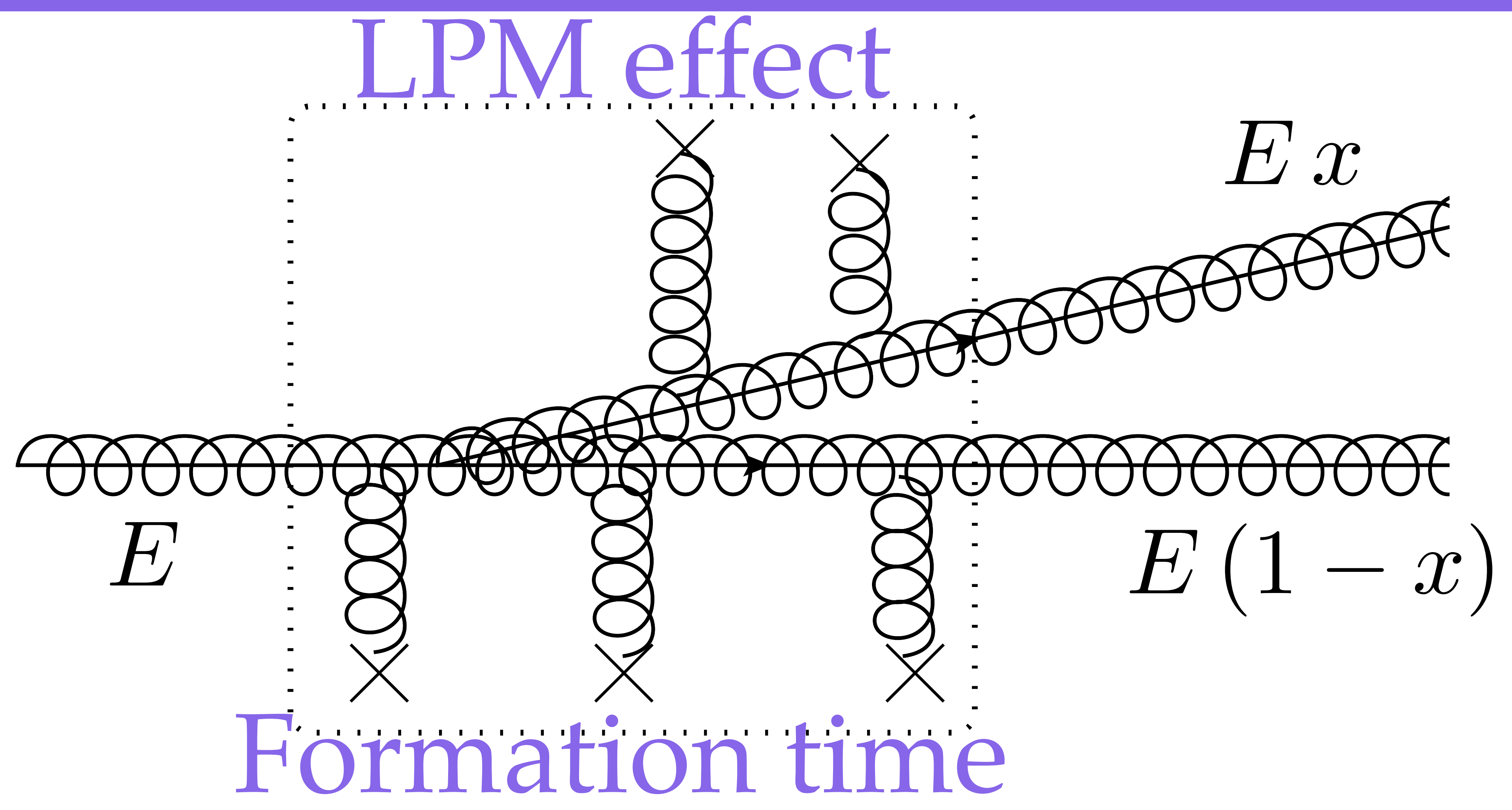
- Recent developments in the microscopic description of kinetic and transport properties of the quark gluon plasma
  - Medium-induced radiation
  - Transverse momentum broadening
  - The effective kinetic theory, transport&thermalisation
- Main driver: better understanding&control of theory and its uncertainties

Many interesting developments and results, limited sample presented here. I refer to the original contributions

Medium-induced radiation



Medium-induced radiation



- Key ingredient
  - in the description of jet modification, see **J. Brewer's talk**
  - in thermalisation&transport: effective number-violating 1↔2 process, efficient chemical equilibration and energy transport, *bottom-up thermalisation* Baier Mueller Schiff Son (2001)

Medium-induced radiation

- Probability I: vacuum DGLAP x emission vertices x transverse diffusion

$$\frac{dI}{dx} = \frac{\alpha_s P_{1\to 2}(x)}{[x(1-x)E]^2} \mathrm{Re} \int_{t_1<t_2} dt_1 dt_2 \boldsymbol{\nabla}_{\boldsymbol{b}_2} \cdot \boldsymbol{\nabla}_{\boldsymbol{b}_1} \left[\langle \boldsymbol{b}_2, t_2 | \boldsymbol{b}_1, t_1 \rangle \big|_{\boldsymbol{b}_2=\boldsymbol{b}_1=0} - \mathrm{vac.} \right]$$

Baier Dokshitzer Mueller Peigne Schiff, Zakharov (1995-97)

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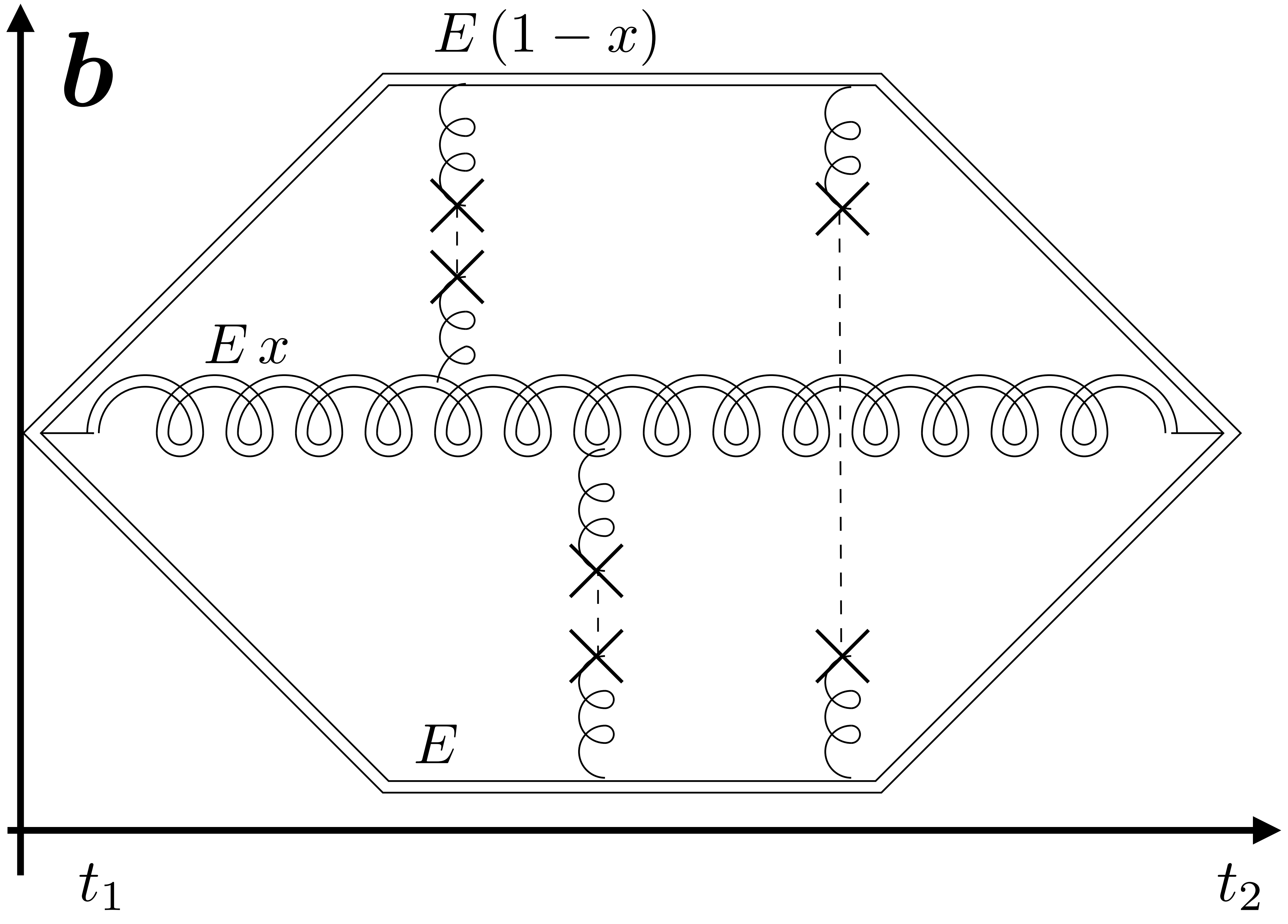
- Transverse diffusion under this Hamiltonian

$$\mathcal{H} = -\frac{\nabla_{\boldsymbol{b}}^2}{2x(1-x)E} + \sum_i \frac{m^2}{2E_i} - i\mathcal{C}(\boldsymbol{b}, x\boldsymbol{b}, (1-x)\boldsymbol{b})$$

Real part: phase accumulation (with in-medium masses)

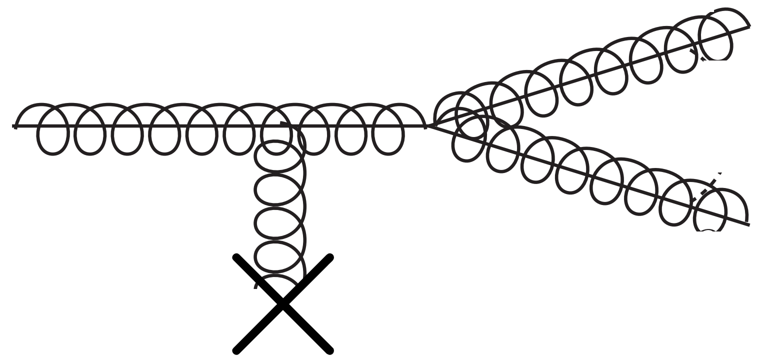
Imaginary part: Wilson lines encoding *scattering kernel* with the medium



Baier Dokshitzer Mueller Peigne Schiff, Zakharov (1995-97)

Medium-induced radiation

- In practice, most approaches resorted to limiting regimes
  - Opacity expansion, for *thin media*
    Gyulassy Levai Vitev (2000)

$N = 1$ 

  - Harmonic oscillator approximation, for **thick media**, introduce $\hat{q}$
    Diffusion under *multiple soft scatterings*

$$\mathcal{C}(\boldsymbol{b}, x\boldsymbol{b}, (1-x)\boldsymbol{b}) \approx \frac{\hat{q}}{4}\left[b^2 + (xb)^2 + ((1-x)b)^2\right]$$

  - Infinite, time-independent medium Arnold Moore Yaffe (2002)

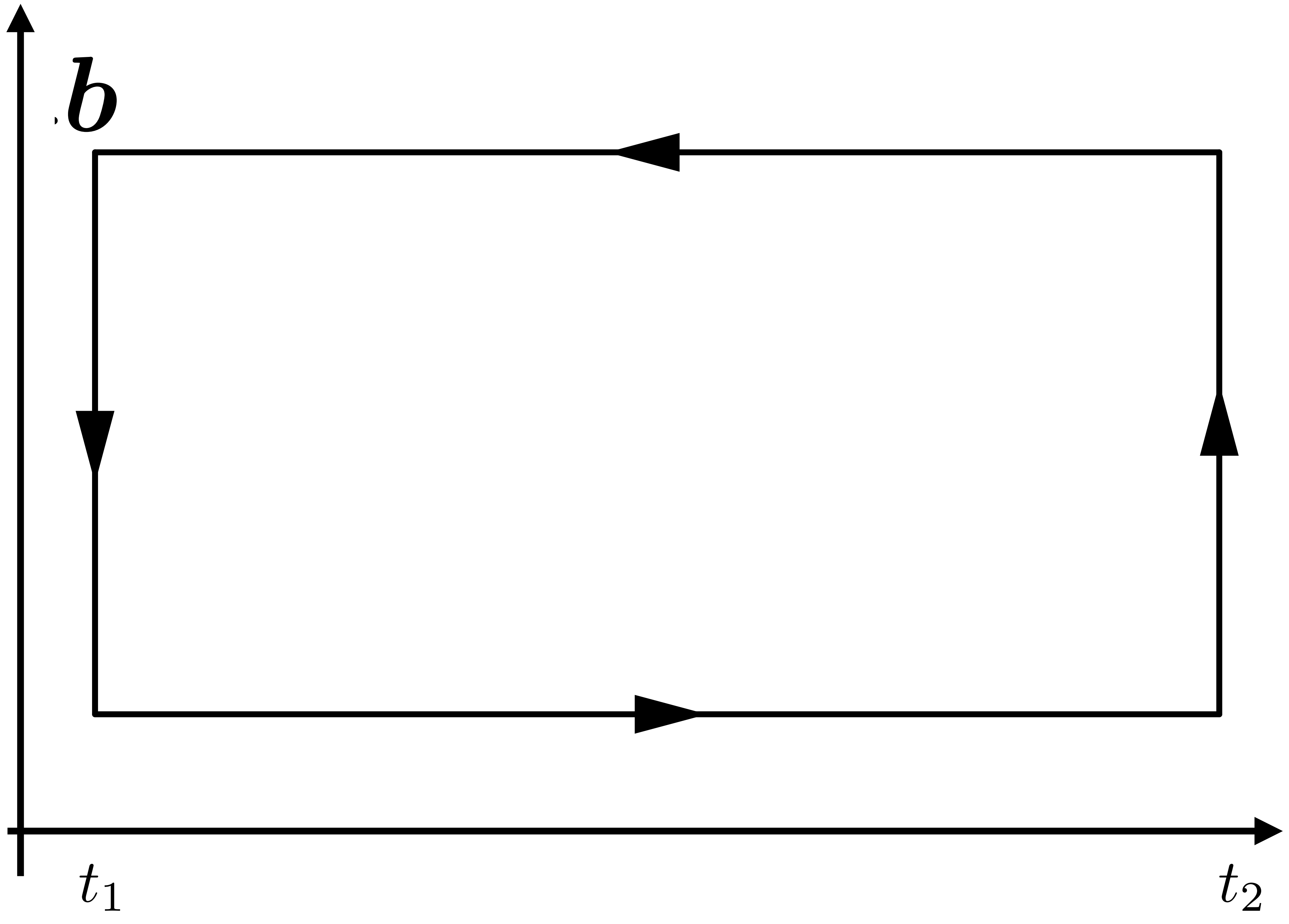
Medium-induced radiation: new developments

- Consider for simplicity the broadening of a single parton:

  $\mathcal{C}(\boldsymbol{b}, \sout{x\boldsymbol{b}, (1-x)}\boldsymbol{b})$

- Broadening probability

$$\mathcal{P}(k_\perp) = \int_{\boldsymbol{b}} e^{-i\boldsymbol{k}_\perp \cdot \boldsymbol{b}} \exp\big[-\mathcal{C}(\boldsymbol{b})L \big]$$



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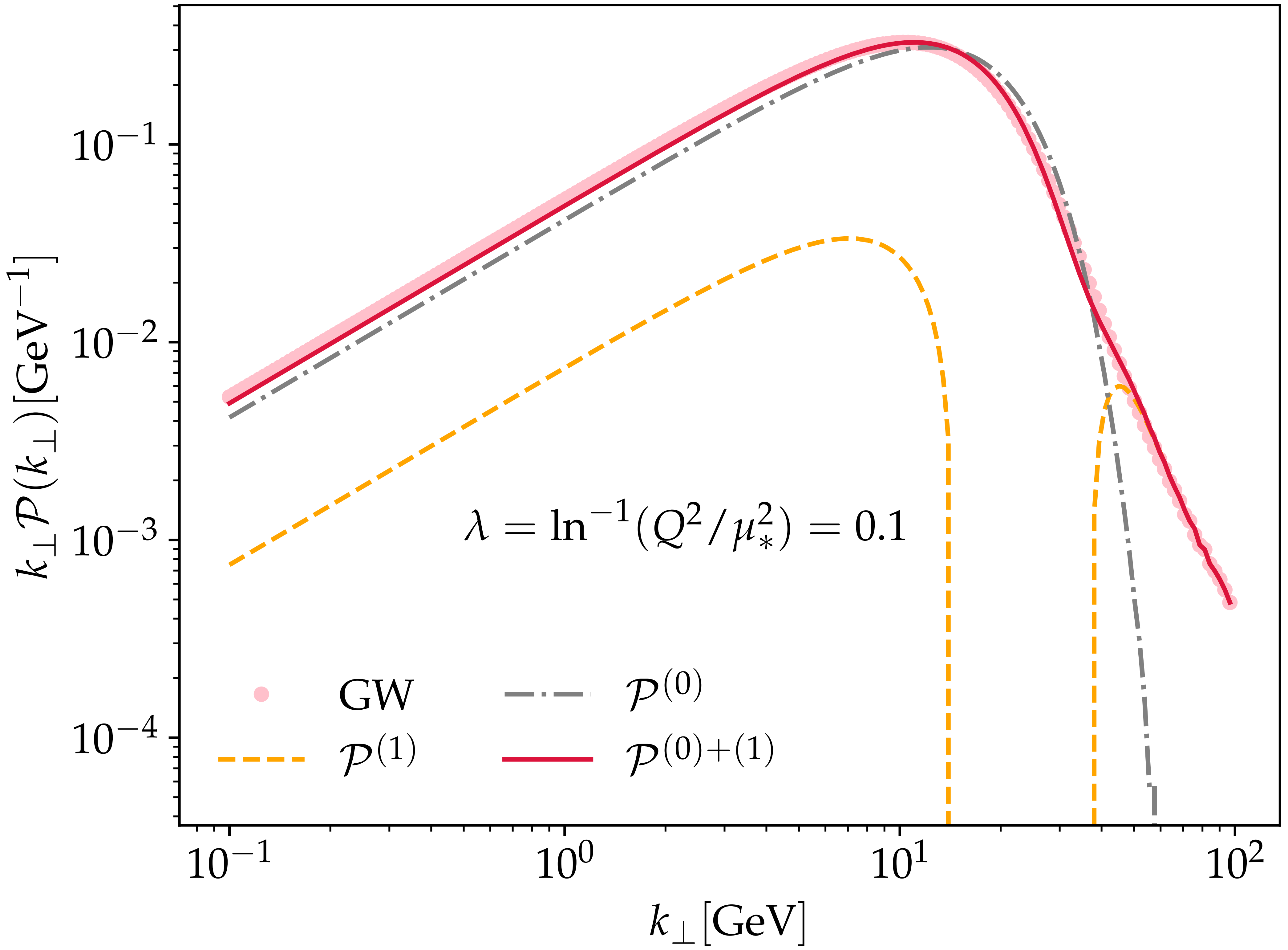

Barata Mehtar-Tani Soto-Ontoso Tywoniuk **PRD104** (2021)

Posters by **Barata**, **Takacs**, **Tywoniuk** Wednesday

Medium-induced radiation: new developments

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$$\mathcal{C}(\boldsymbol{b}, x\boldsymbol{b}, (1-x)\boldsymbol{b}) \approx \frac{\hat{q}}{4}\left[b^2 + (xb)^2 + ((1-x)b)^2\right] \equiv \mathcal{C}_{\rm HO}$$

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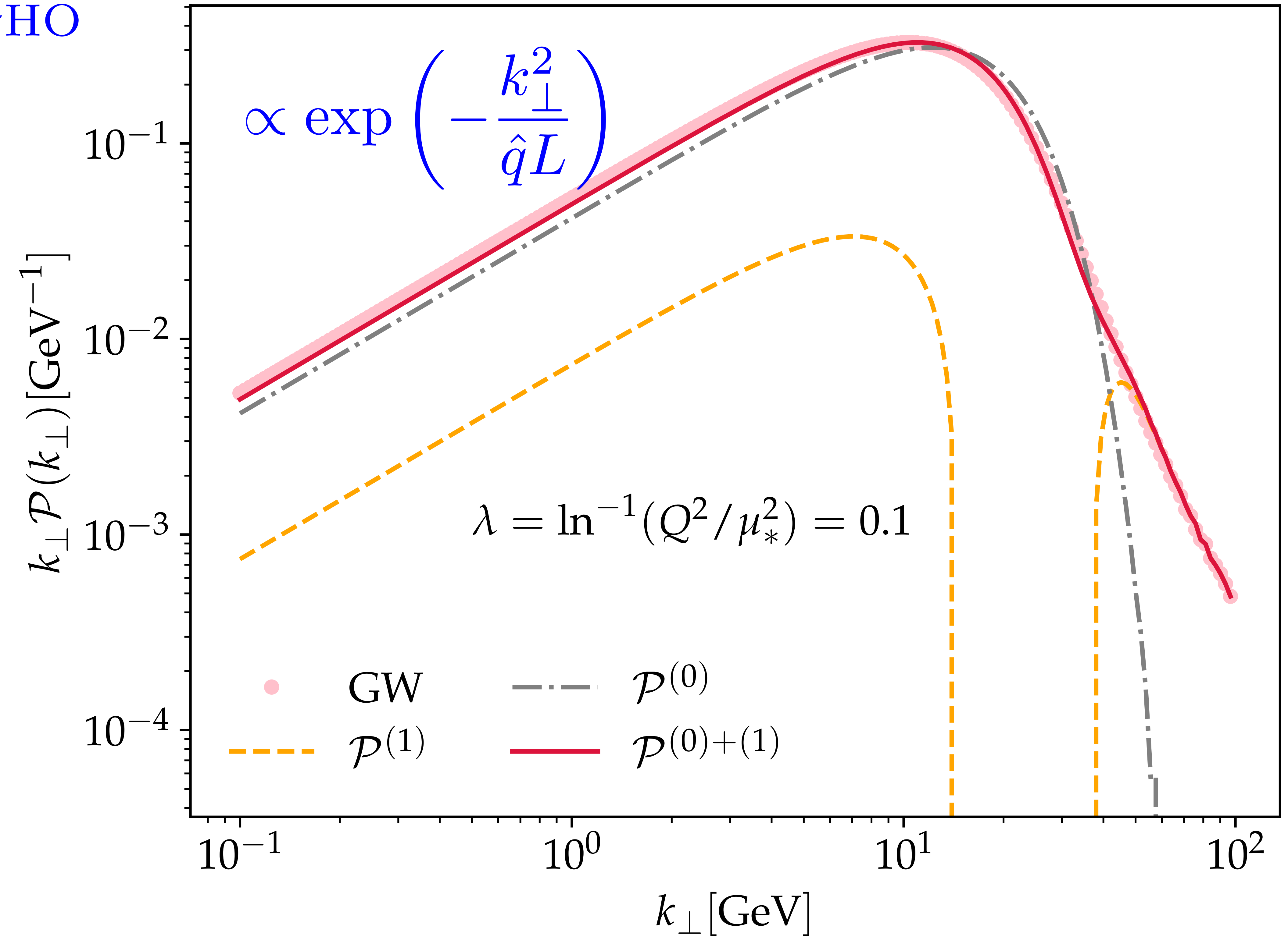
- IR Gaussian from multiple soft scatterings

$$\mathcal{P}(k_\perp)_{\rm HO} \propto \exp\left(-\frac{k_\perp^2}{\hat{q}L}\right)$$



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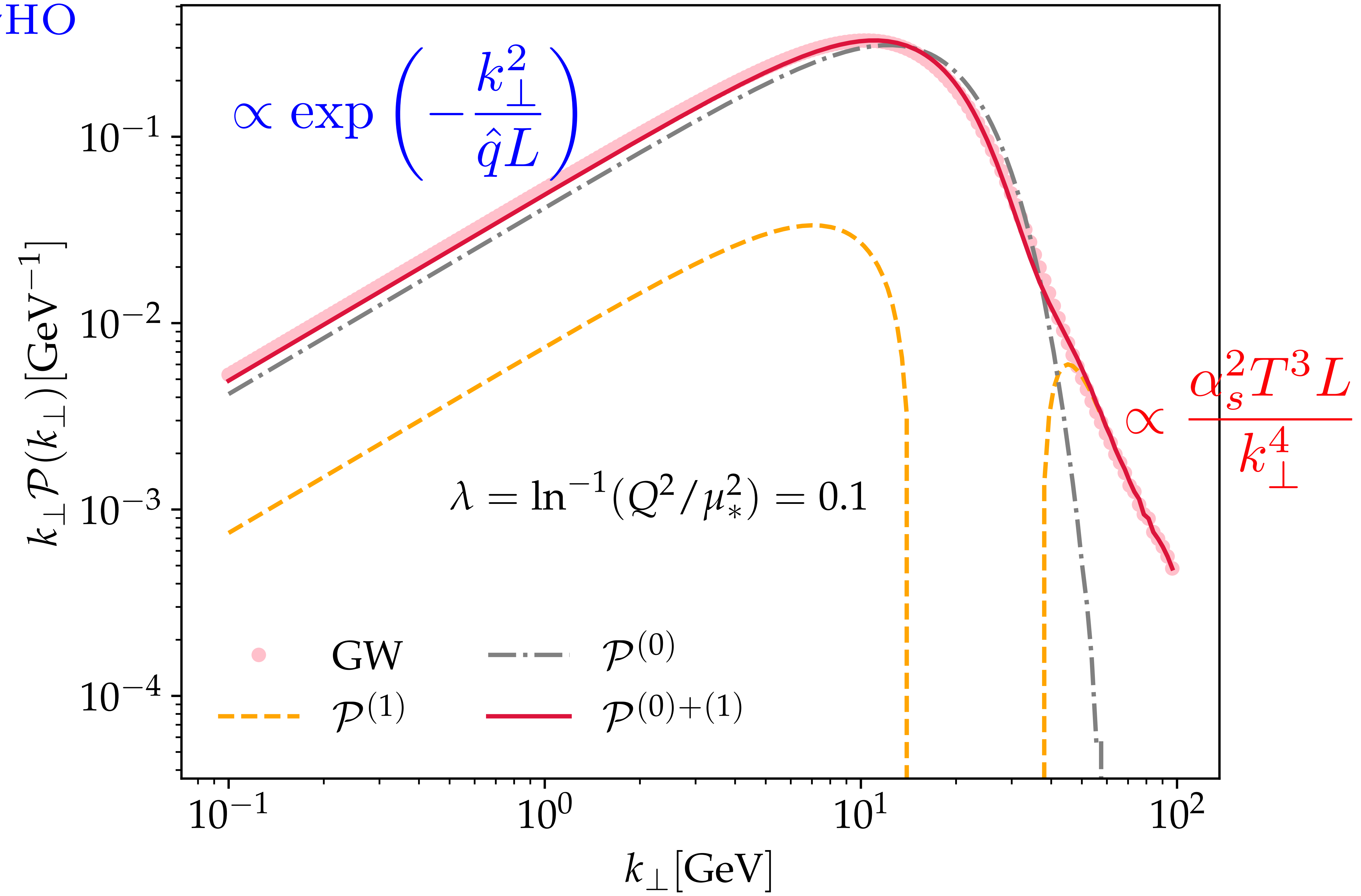
- IR Gaussian from multiple soft scatterings

$$\mathcal{P}(k_\perp)_{\rm HO} \propto \exp\left(-\frac{k_\perp^2}{\hat{q}L}\right)$$

- **asymptotic freedom** $\Rightarrow$ it has to make way to the rare large momentum scatterings

$$\mathcal{P}(k_\perp)_{\rm Coulomb} \propto \frac{\alpha_s^2 T^3 L}{k_\perp^4}$$



Barata Mehtar-Tani Soto-Ontoso Tywoniuk **PRD104** (2021)

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Medium-induced radiation: new developments

- **Improved** harmonic oscillator approximation (IHO or IOE):

$$\mathcal{C}(\boldsymbol{b}, x\boldsymbol{b}, (1-x)\boldsymbol{b}) \approx \frac{\hat{q}}{4}\left[b^2 + (xb)^2 + ((1-x)b)^2\right] \equiv \mathcal{C}_{\rm HO}$$

$$\mathcal{C} = \underbrace{\mathcal{C}_{\rm HO}}_{\propto b^2} + \left[\ \underbrace{\mathcal{C}-\mathcal{C}_{\rm HO}}_{\propto b^2 \ln(b),\ldots}\ \right]$$

Treat the non-quadratic part of the kernel as a perturbation, properly incorporating the Coulomb logarithm: includes the rarer harder *"Molière"* scatterings

Inclusion of Molière scattering in hybrid framework: **talk** by **Hulcher** Tue 18:30

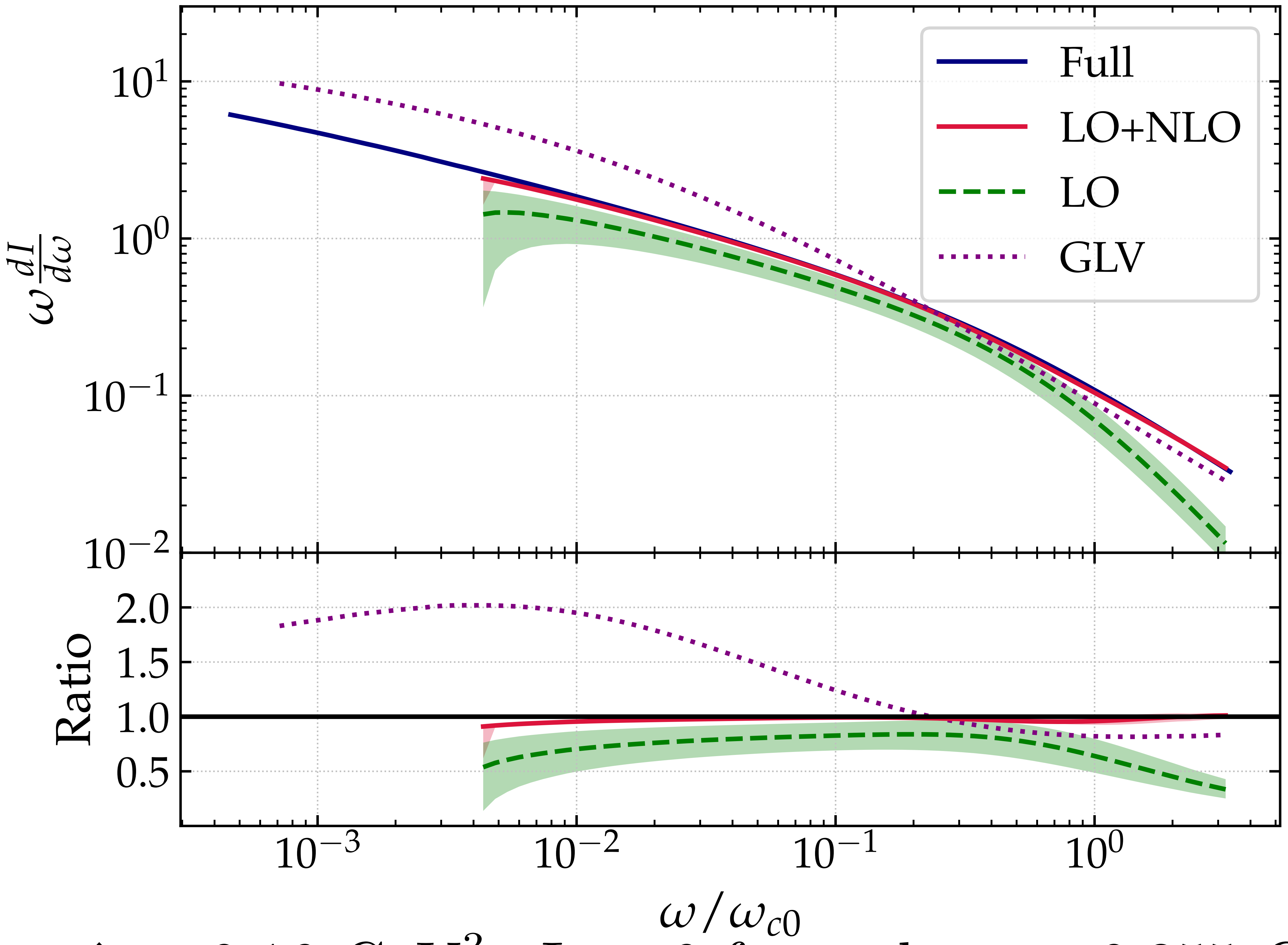
Barata Mehtar-Tani Soto-Ontoso Tywoniuk **JHEP2109** (2021)

Posters by **Barata**, **Takacs**, **Tywoniuk** Wednesday

Medium-induced radiation: new developments

- Numerical determination of the Green's function of the full Hamiltonian

Spectrum



Numerical

IOE

HO

GLV N=1

$\hat{q}_0 = 0.16\ \text{GeV}^3,\ L = 6\ \text{fm and } \mu_* = 0.355\ \text{GeV}$.

Andres Apolinario Dominguez **JHEP2007** (2020) Andres Dominguez Gonzalez-Martinez **JHEP2103** (2021)

Applications to time-dependent media in the **talk** by **Andres**, Wed 16:40

See also Caron-Huot Gale **PRC82** (2010)

Medium-induced radiation: the scattering kernel

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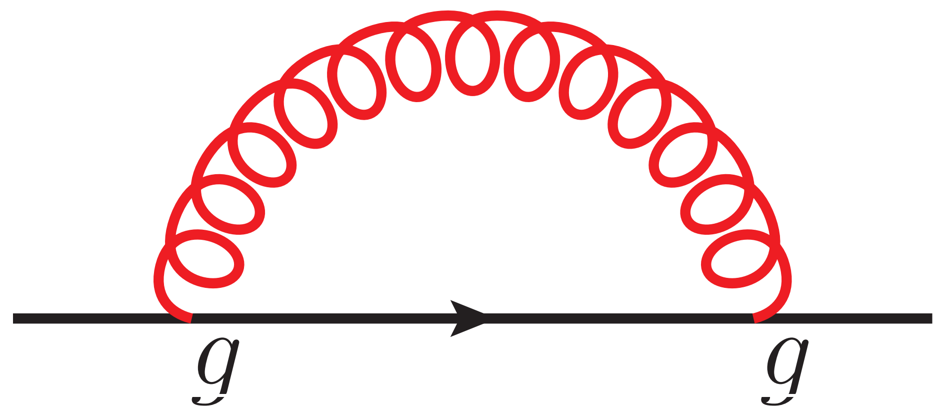
- Classical (soft gluon) corrections to the scattering/broadening kernel can be problematic for perturbation theory, Linde problem

$n_B(p) \sim T/p \gtrsim 1/g$



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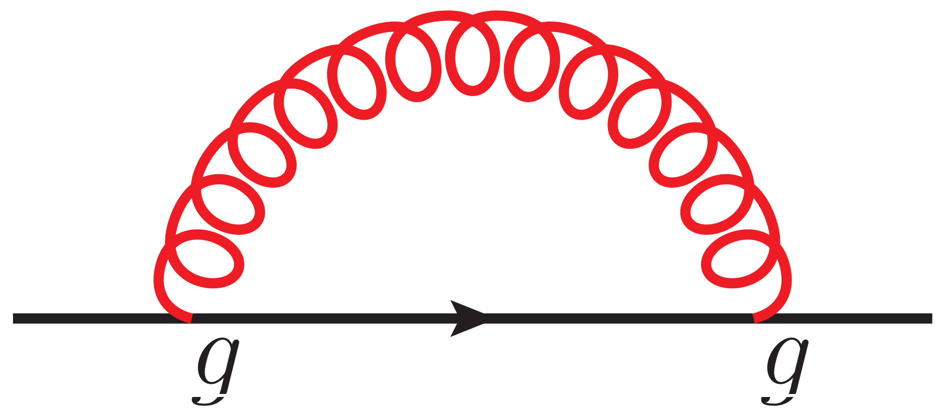

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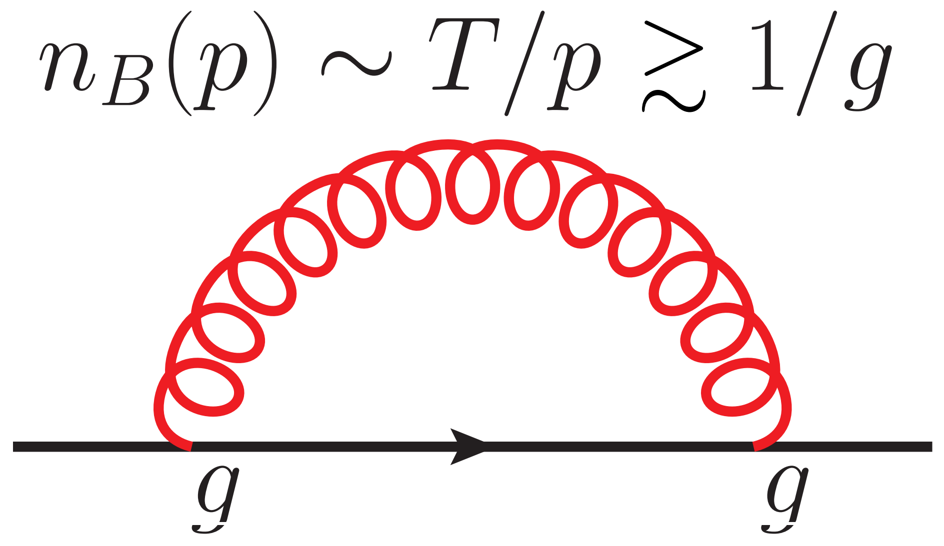
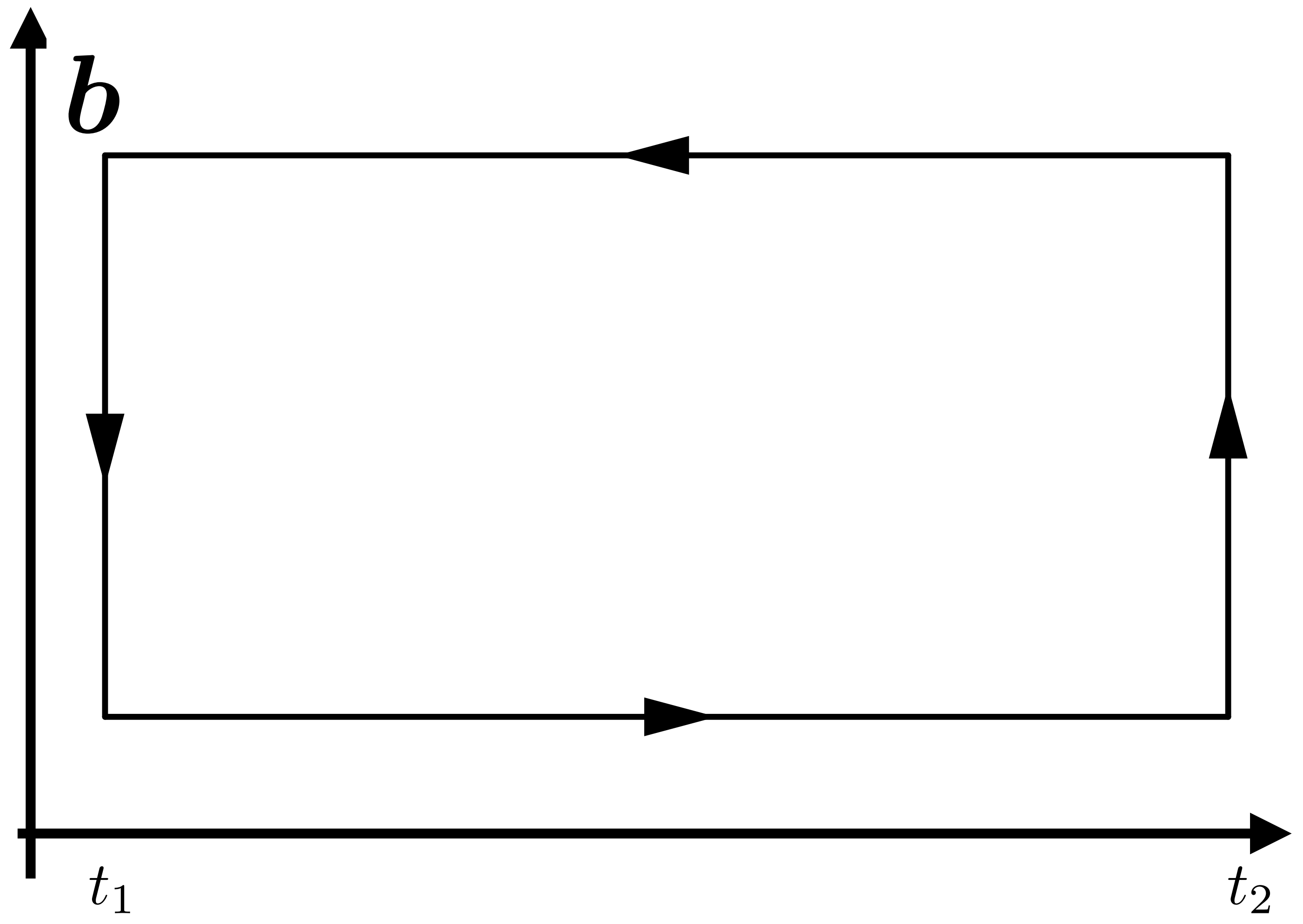


- Breakthrough: soft classical modes at space-like separations become **Euclidean** and **time-independent** Caron-Huot **PRD79** (2008)

- Horrible HTL perturbative calculation or extremely challenging 4D lattice on the light-cone become **3D** Electrostatic QCD (EQCD).
  New strategy: lattice for $\boldsymbol{b} \gtrsim 1/gT$, pQCD for $\boldsymbol{b} \lesssim 1/gT$



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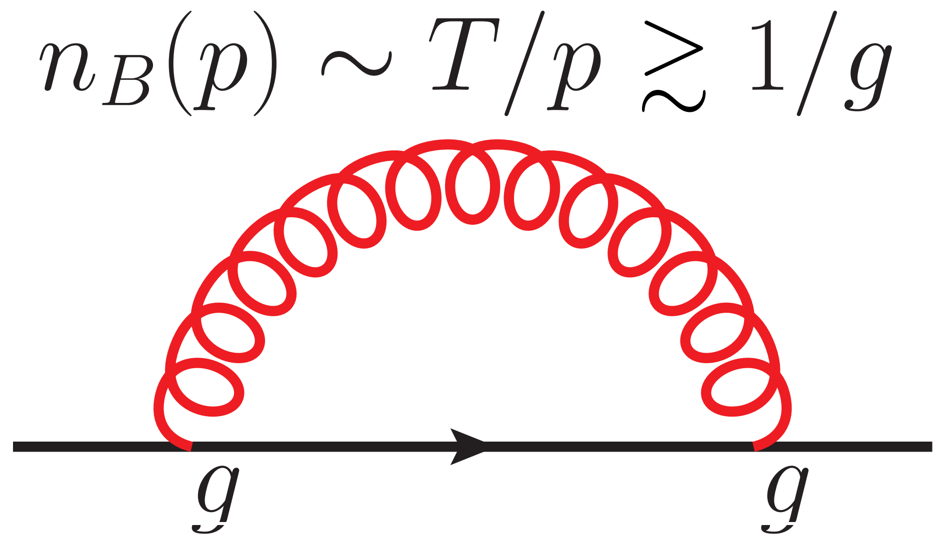

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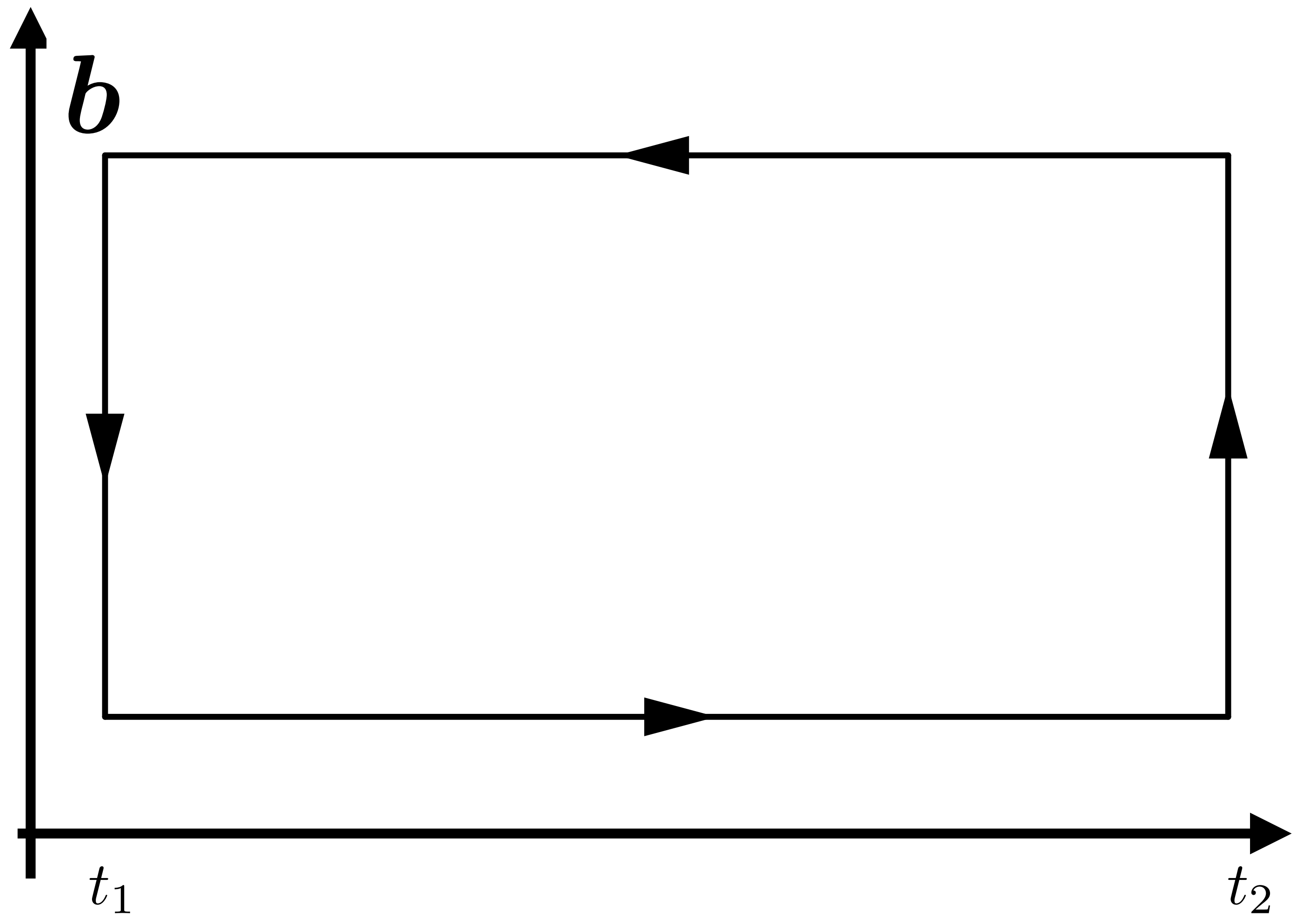
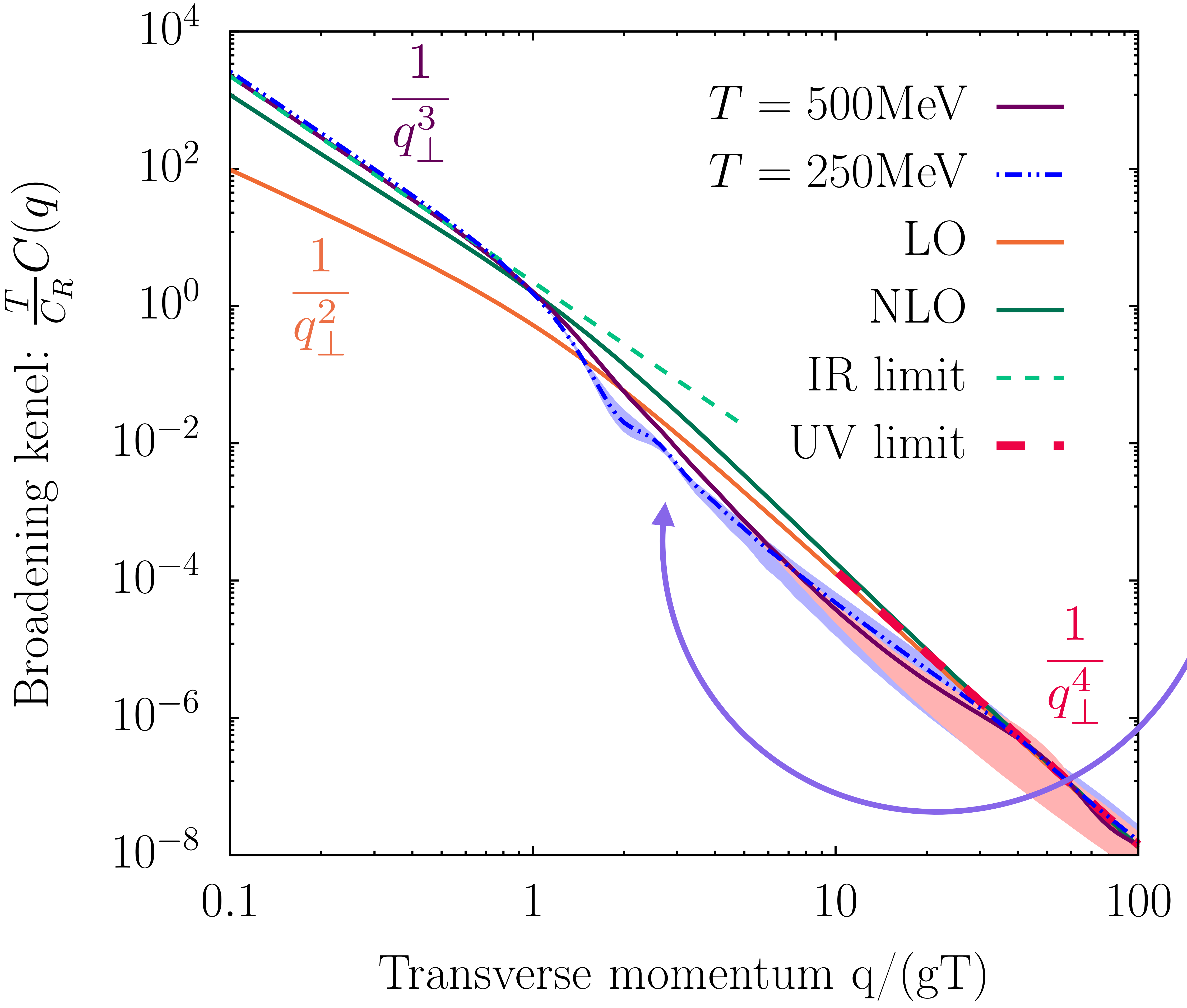


- Recently: continuum-extrapolated EQCD lattice data for the scattering kernel and merging with pQCD
  Moore Schlusser **PRD101** (2020) Moore Schlichting Schlusser Soudi **JHEP2110** (2021)

The scattering kernel



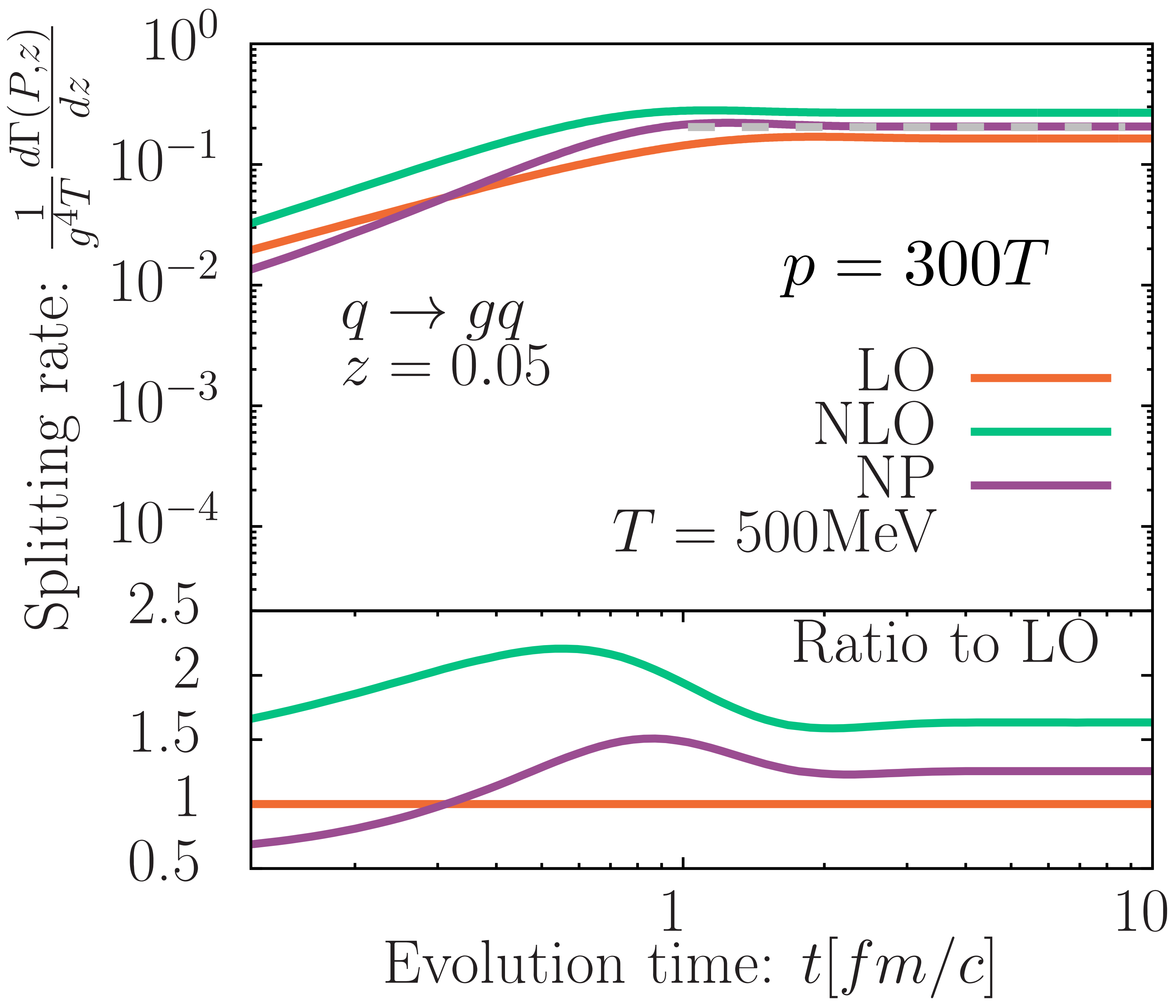
- LO and NLO perturbative EQCD: Aurenche Gelis Zaraket (2002) Caron-Huot (2008)
  LO UV ($q \gtrsim gT$) pQCD and matching: Arnold Xiao (2008) JG Kim (2018)
- Significant deviations from pQCD
- Non-perturbative magnetic "screening" means q^{-3} instead of Coulomb/Molière q^{-4}
- qhat second moment of this quantity

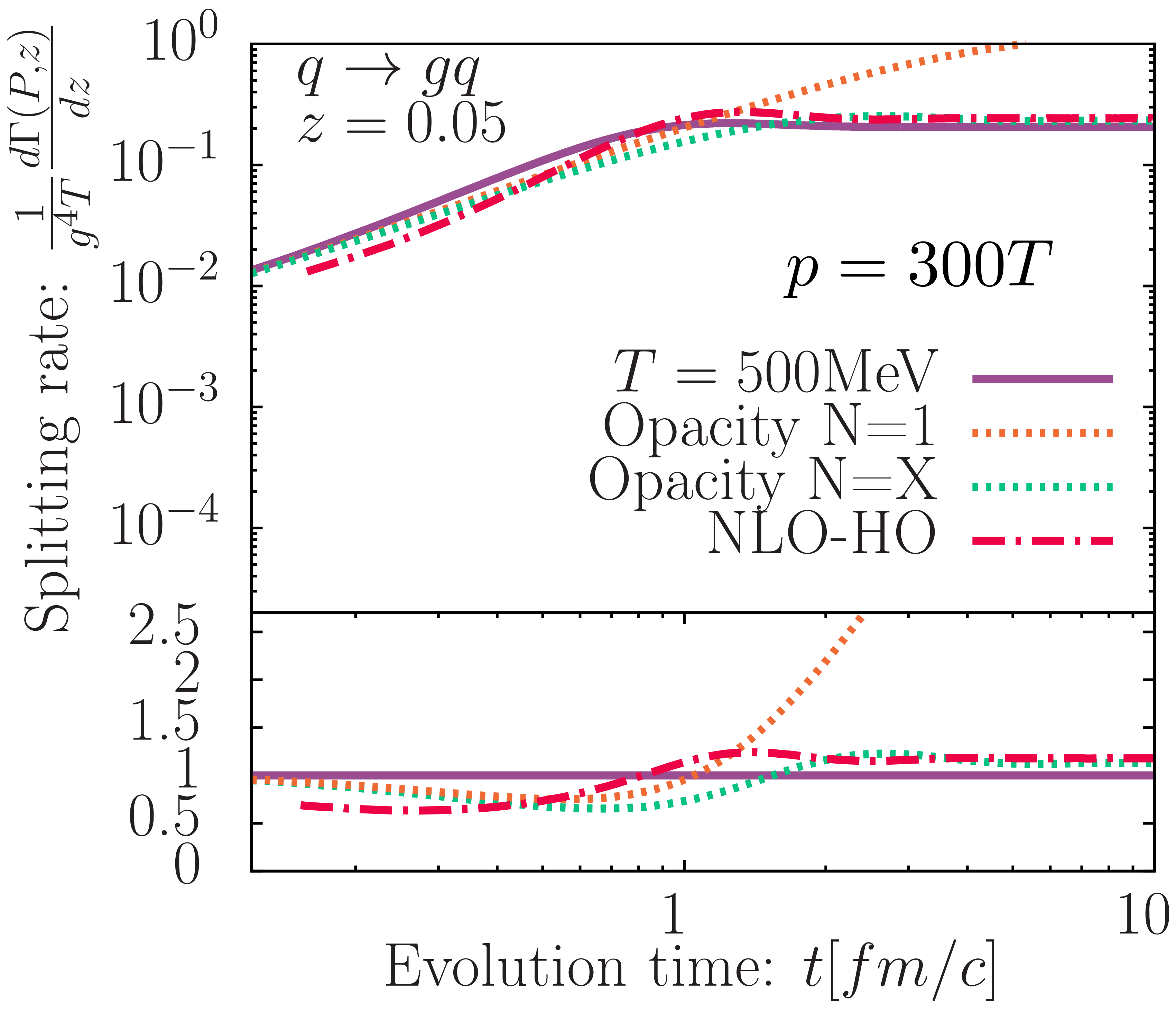
$$\hat{q} \propto \int d^2q_\perp \, q_\perp^2 \, \mathcal{C}(q_\perp)$$

Schlichting Soudi **2111.13731**, **talk** by **Soudi** Thu 12:50

Medium-induced radiation from the EQCD kernel

- (Numerical) splitting rate with the non-perturbative broadening kernel

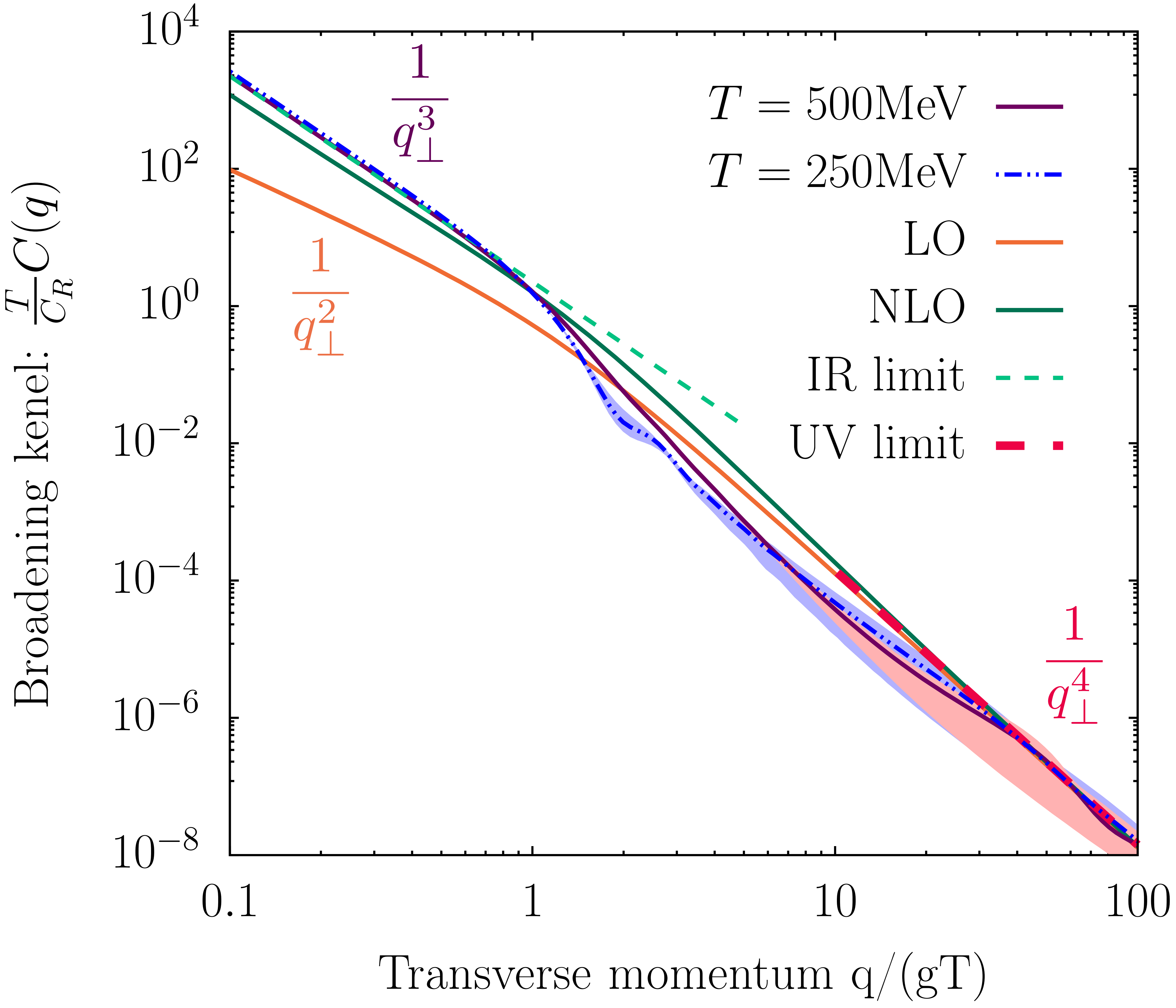




- Differences from the broadening kernel more important than differences from the more sophisticated approximations to the LPM equation

Schlichting Soudi **2111.13731**, **talk** by **Soudi** Thu 12:50

The scattering kernel

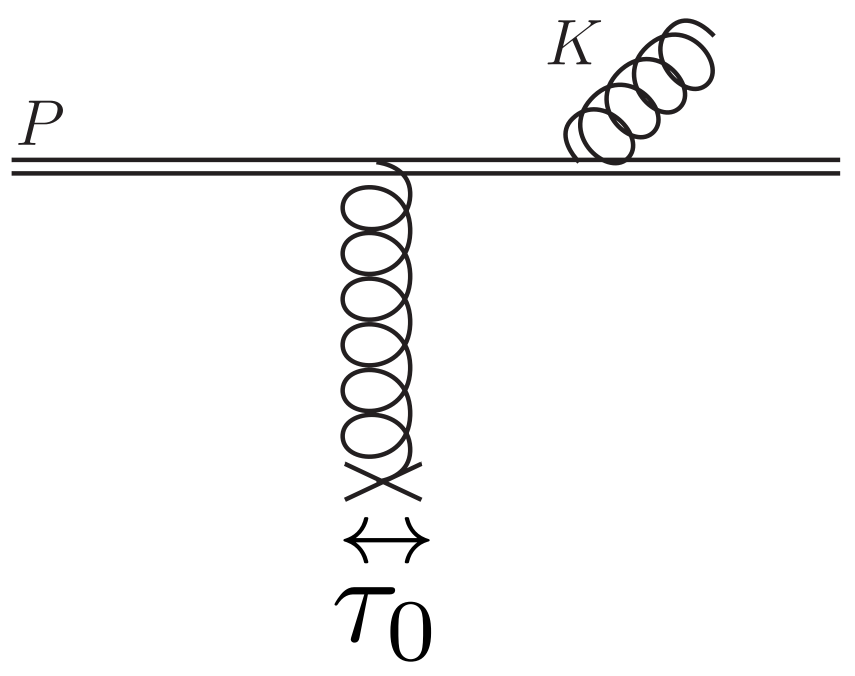


- Similar lattice EQCD+pQCD programme in progress for the in-medium jet mass
  Talk by **Schicho**, Wed 9:20
- Only classical corrections here, what happens with **quantum corrections** for $q \gtrsim gT$?

Schlichting Soudi **2111.13731**, **talk** by **Soudi** Thu 12:50

The scattering kernel: quantum corrections

- Radiative corrections to momentum broadening are enhanced by soft and collinear logarithms in the single scattering regime ⇒ double logarithm



$$\delta\hat{q} = \frac{\alpha_s N_c}{\pi}\hat{q}_0 \int_{\text{single}} \frac{d\omega}{\omega}\frac{dk_\perp^2}{k_\perp^2} = \frac{\alpha_s N_c}{\pi}\hat{q}_0 \ln^2\left(\frac{L}{\tau_0}\right)$$

Liou Mueller Wu (2013) Blaizot Dominguez Iancu Mehtar-Tani (2013)

Caucal Mehtar-Tani **2109.12041 2203.09407**

Poster by **Mehtar-Tani** later today

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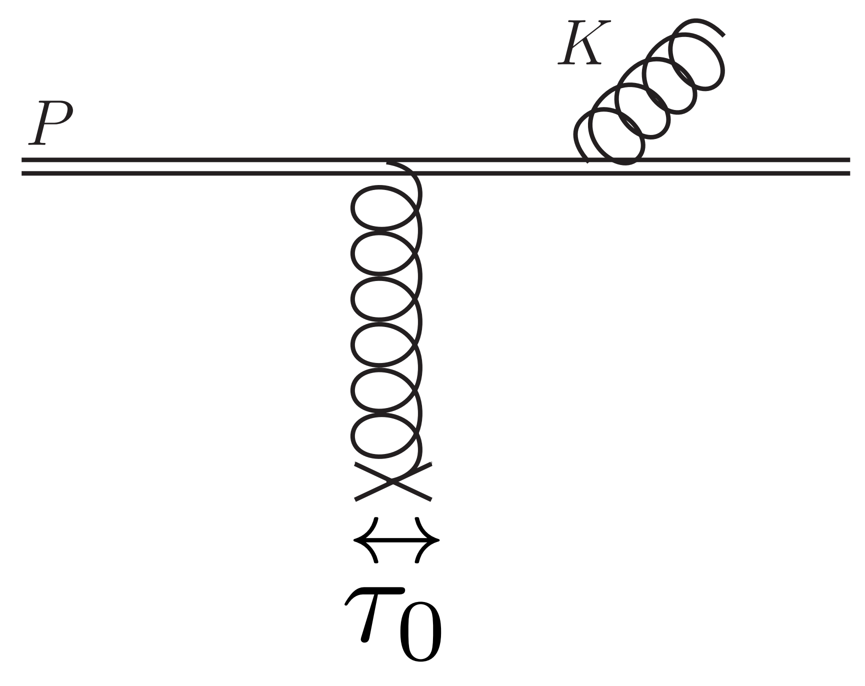

Liou Mueller Wu (2013) Blaizot Dominguez Iancu Mehtar-Tani (2013)

- This log² renormalises the LO qhat. *Resum* these logs

$$\hat{q}(\tau, \boldsymbol{k}_\perp^2) = \hat{q}^{(0)}(\tau_0, \boldsymbol{k}_\perp^2) + \int_{\tau_0}^{\tau} \frac{\mathrm{d}\tau'}{\tau'} \int_{Q_s^2(\tau')}^{\boldsymbol{k}_\perp^2} \frac{\mathrm{d}\boldsymbol{k}_\perp'^2}{\boldsymbol{k}_\perp'^2}\, \bar{\alpha}_s(\boldsymbol{k}_\perp'^2)\, \hat{q}(\tau', \boldsymbol{k}_\perp'^2)$$

$$Q_s^2(\tau) = \hat{q}(\tau, Q_s^2(\tau))\tau\,,$$

- Single hard scattering $k_\perp'^2 \gg \hat{q}\tau$
- UV cutoff
- Shortest duration τ_0

by solving the above numerically and semi-analytically

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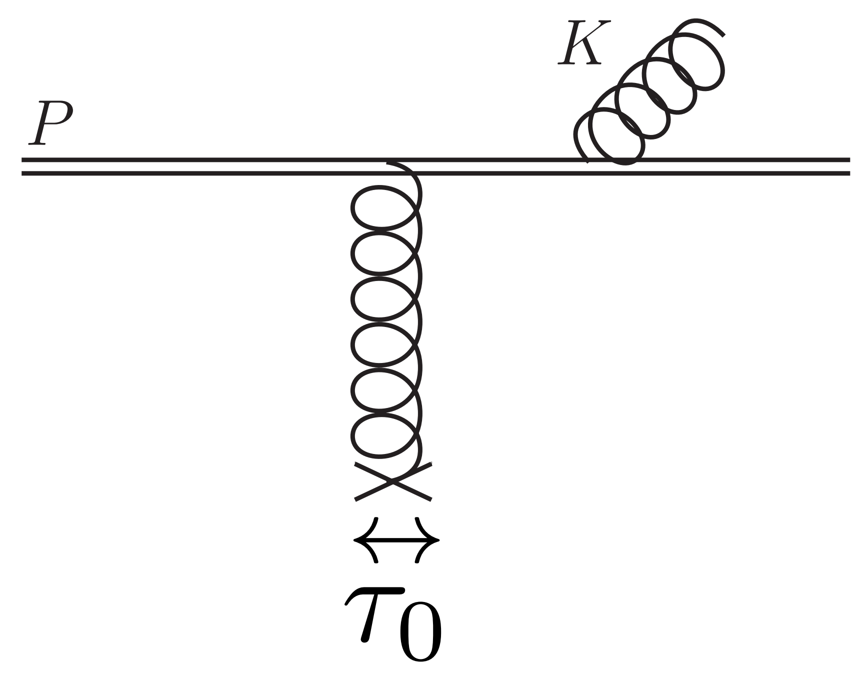

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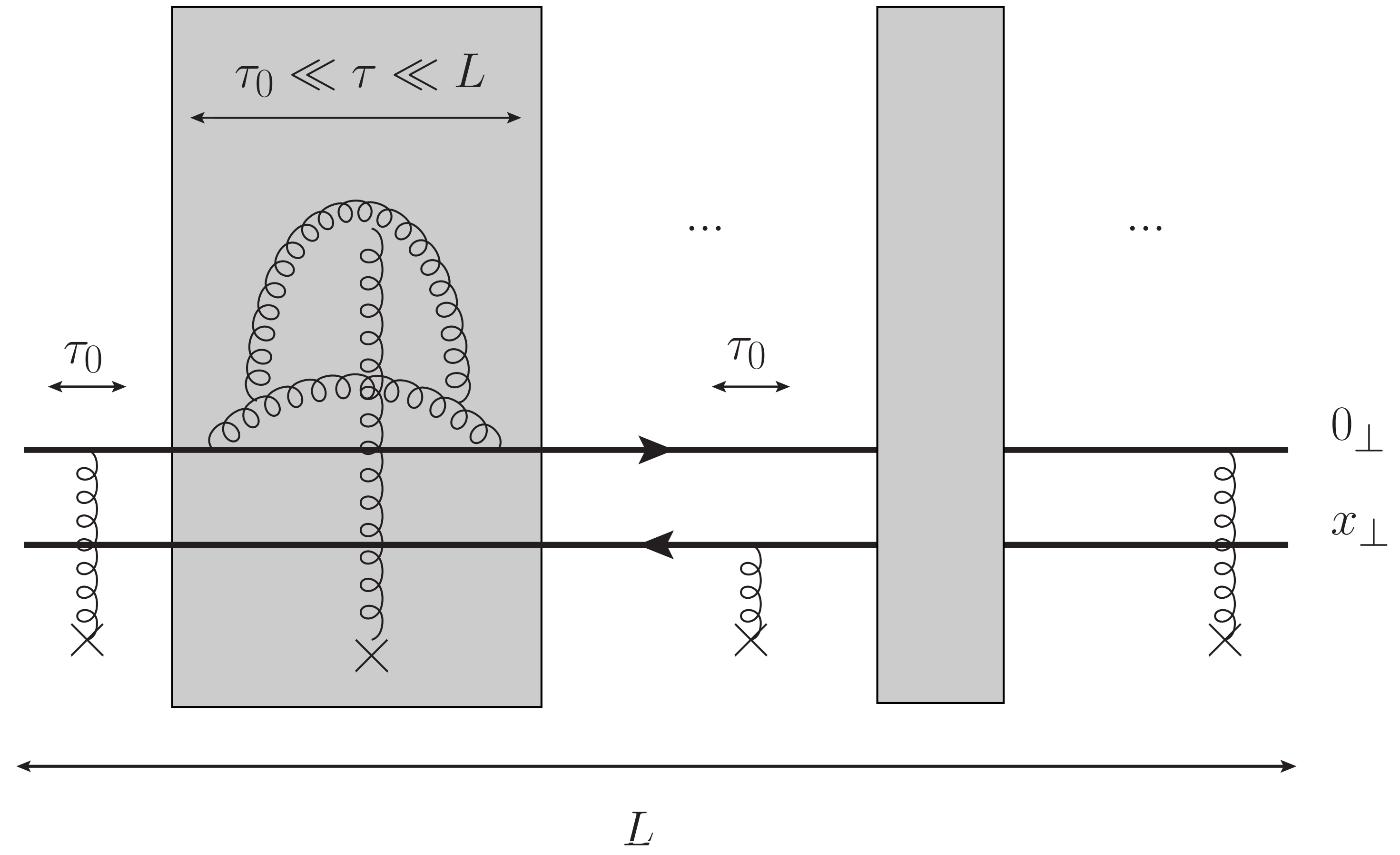

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Poster by **Mehtar-Tani** later today

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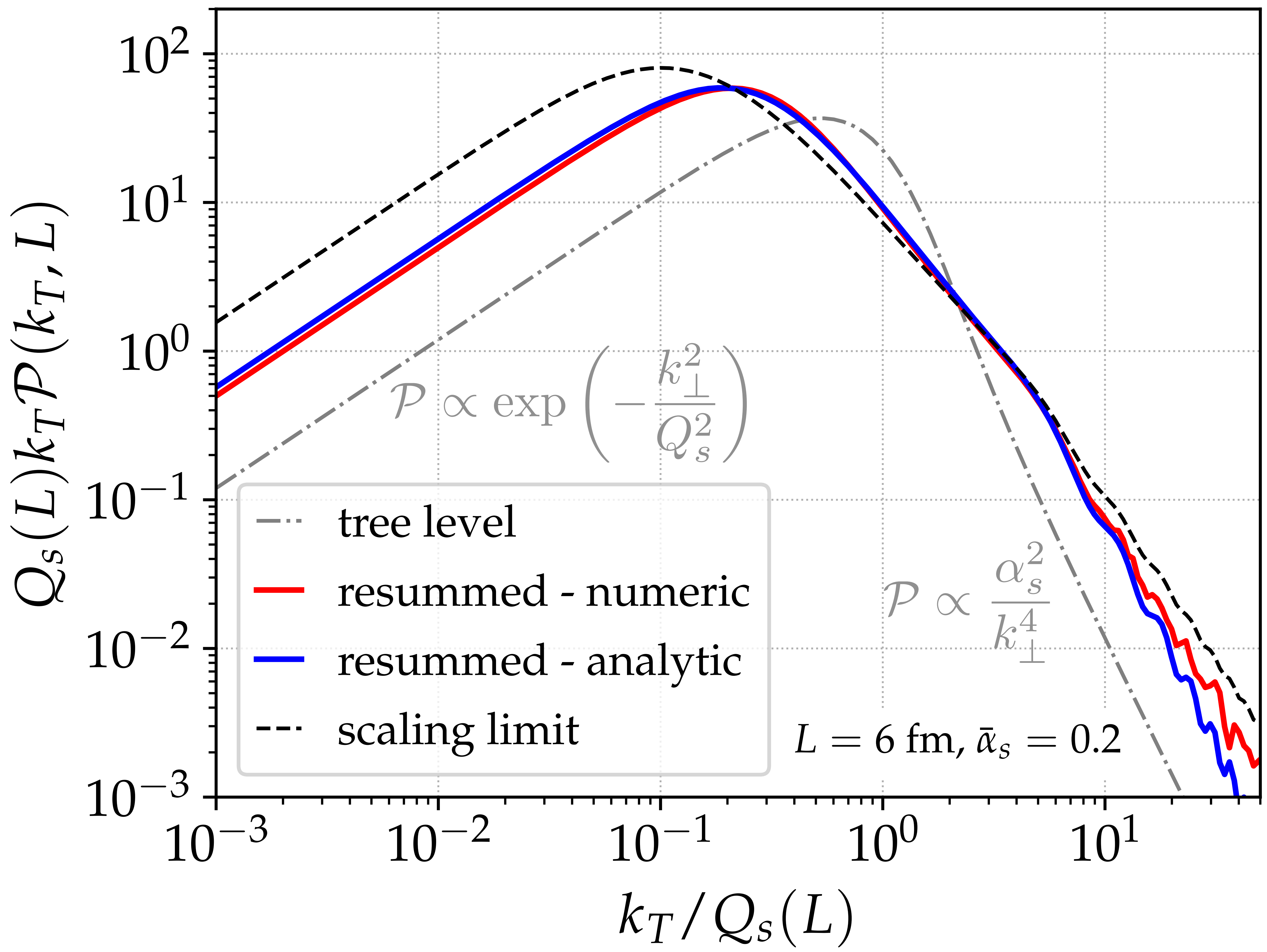


- Non-local nature of quantum radiative corrections makes Coulomb/Molière tail less steep

$$\mathcal{P} \propto k_\perp^{-4+2\bar{\alpha}_s} \qquad \bar{\alpha}_s \equiv \frac{\alpha_s N_c}{\pi}$$

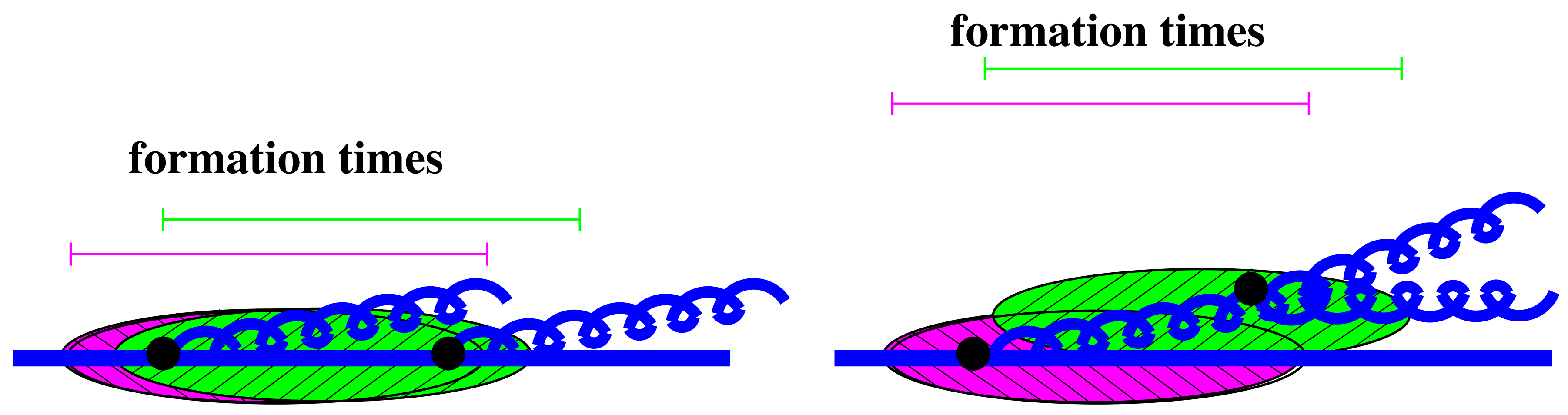
- Increased probability of large-momentum scatterings from non-local quantum corrections

Caucal Mehtar-Tani **2109.12041 2203.09407**

Poster by **Mehtar-Tani** later today

Medium-induced radiation: quantum corrections

- Universality of double logs: they also arise in the case of a **double splitting** with overlapping formation time in the soft limit
  Blaizot Mehtar-Tani, Iancu, Wu (2014)



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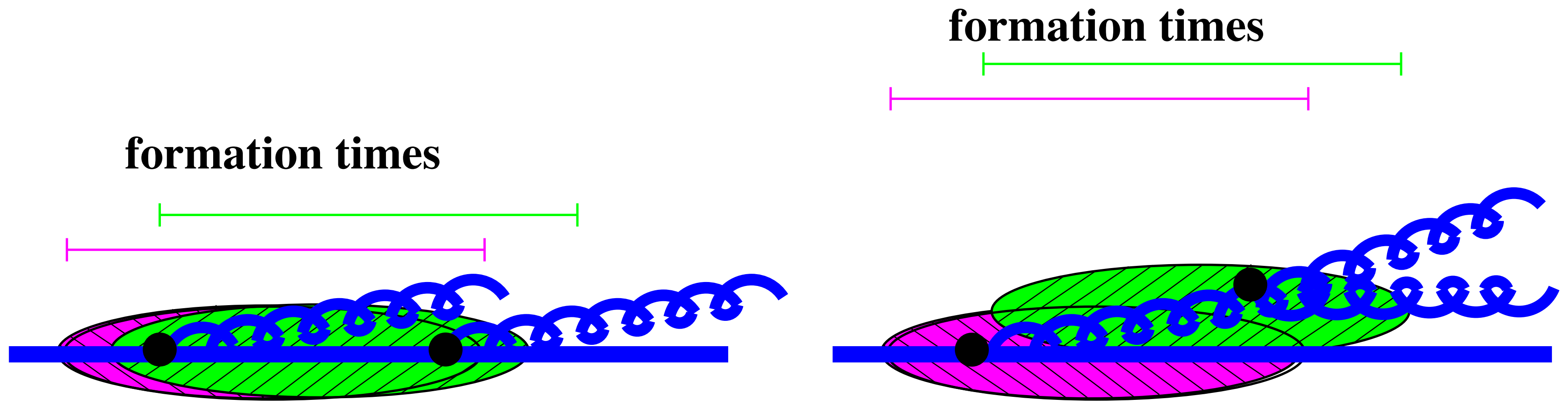

- Ongoing effort to determine all corrections from overlapping formation times (*real* and *virtual*) within the harmonic oscillator approximation. Important to understand if assumed Markovian nature of medium-induced kernel holds for the cascades
  Arnold Iqbal Chang Gorda Rase Elgeadwy (2015-2022)

Arnold Iqbal Gorda **2112.05161** Arnold **JHEP2203** (2021)
Poster by **Iqbal** later today

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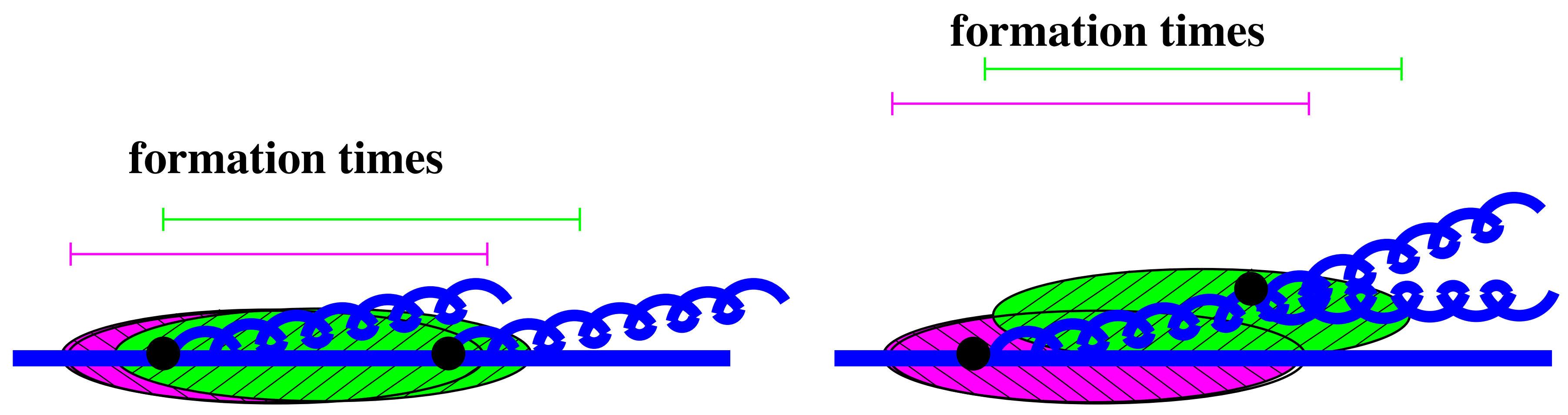

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- Latest news: universality holds not only for the double logs, but (with caveats) also for the accompanying single logs. Good news for their *resummation*!

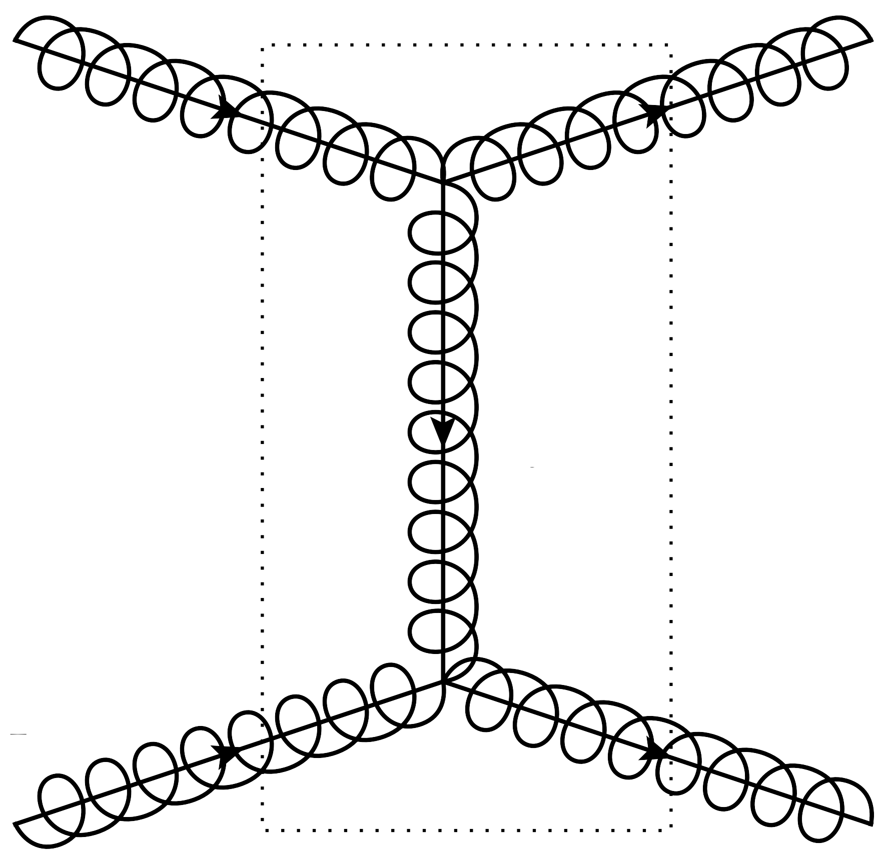
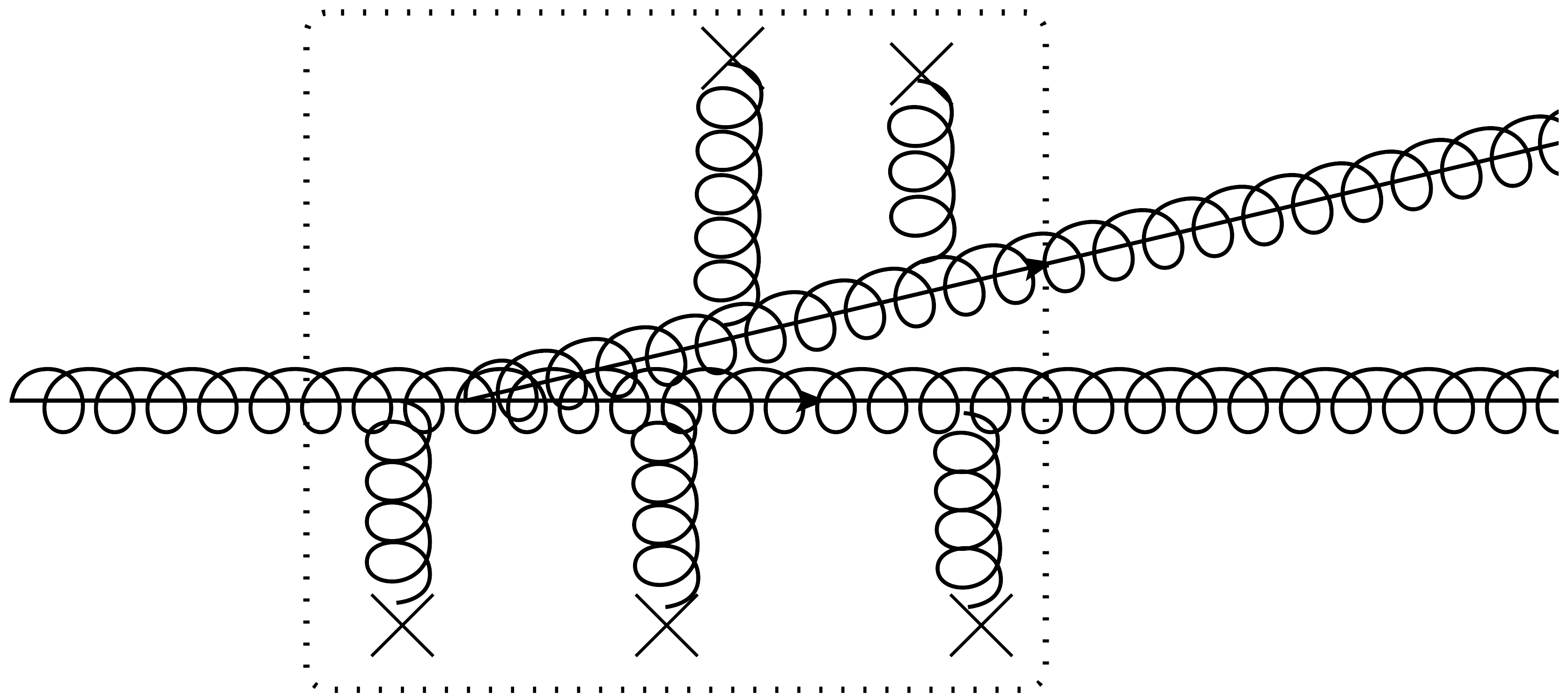
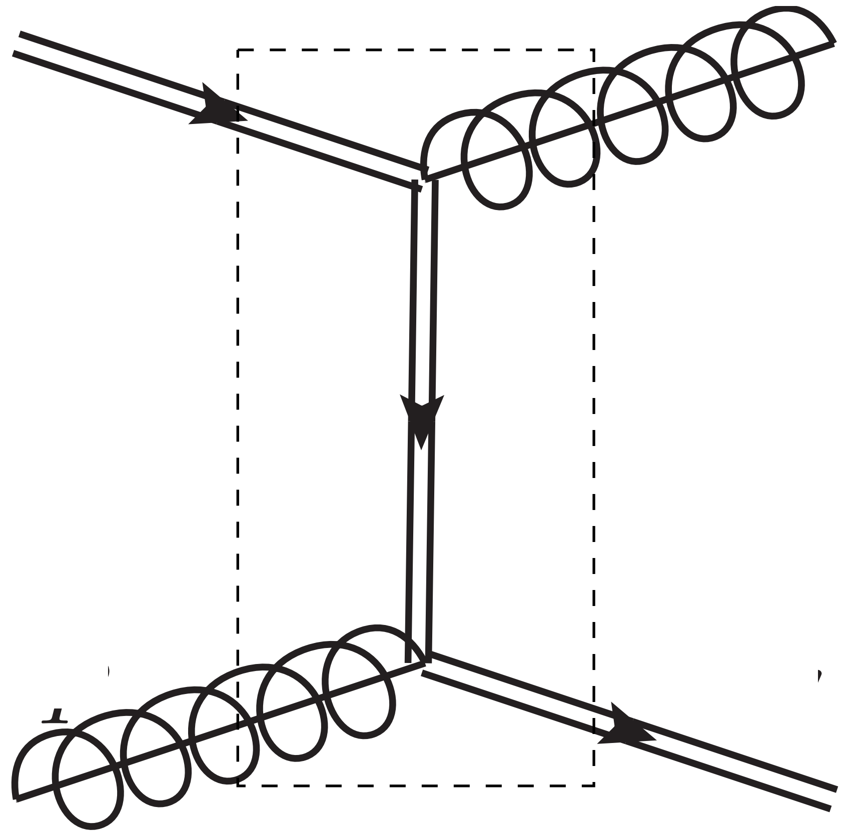
Arnold Iqbal Gorda **2112.05161** Arnold **JHEP2203** (2021)
Poster by **Iqbal** later today

The kinetic theory approach

The kinetic theory approach

- Transverse momentum broadening and radiation are **key ingredients** in the *effective kinetic theory* of QCD, together with **drag, longitudinal momentum broadening** and **conversions** Arnold Moore Yaffe (2003)

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

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Applications to jet physics:
AMY kinetic theory for jet thermalisation: Schlichting Soudi **JHEP2107** (2021), **poster** by **Soudi** Wednesday
Factorised energy loss transport approach Dai Paquet Teaney Bass **PRC105 (2022), poster** by **Dai** Wednesday
Ke Wang **JHEP2105** (2021), **talk** by **Ke** Thursday 17:30

The kinetic theory approach

- Transverse momentum broadening and radiation are **key ingredients** in the *effective kinetic theory* of QCD, together with **drag, longitudinal momentum broadening** and **conversions** Arnold Moore Yaffe (2003)

- How do these developments affect the kinetic description?

The kinetic theory approach: transport coefficients

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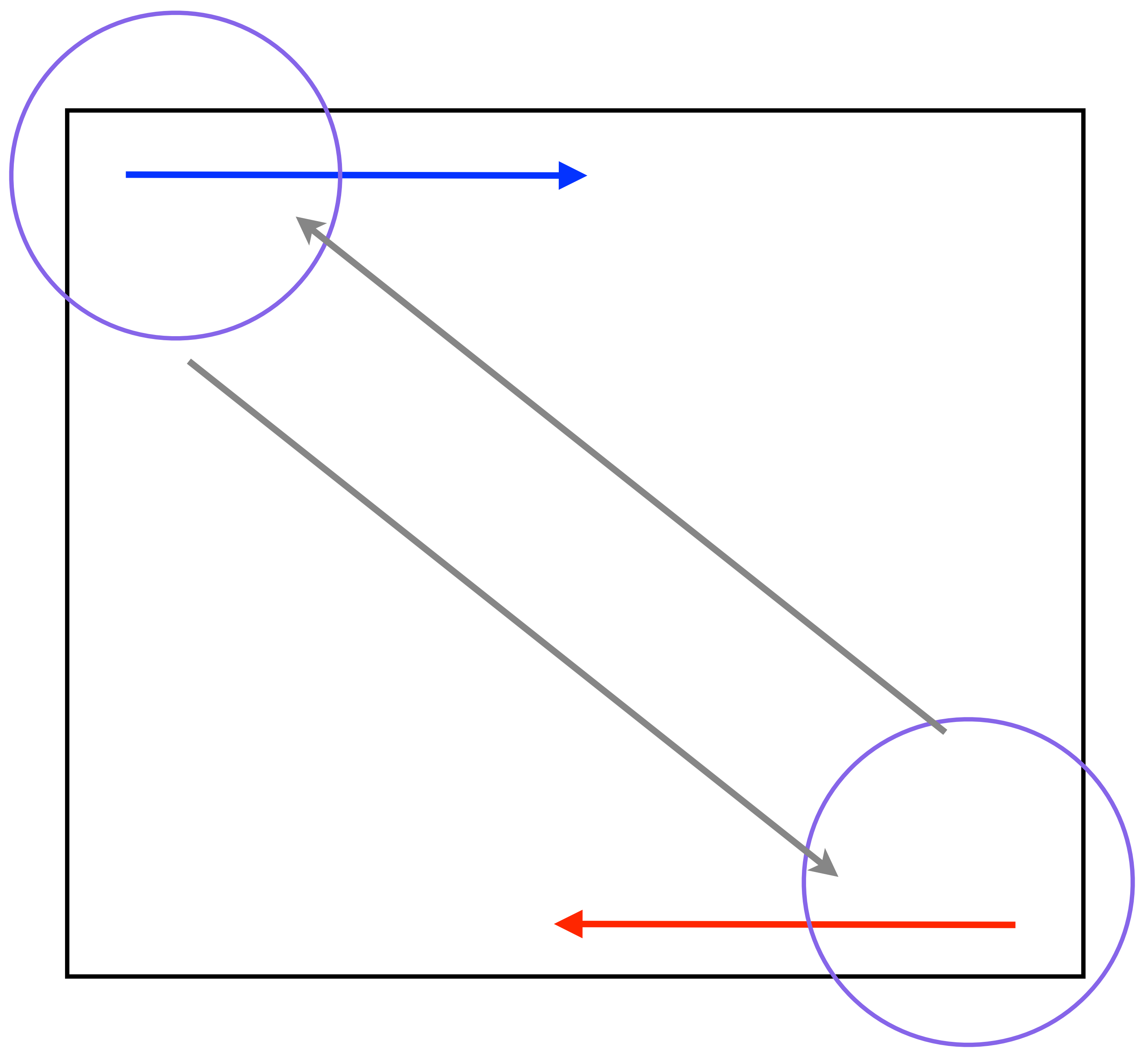
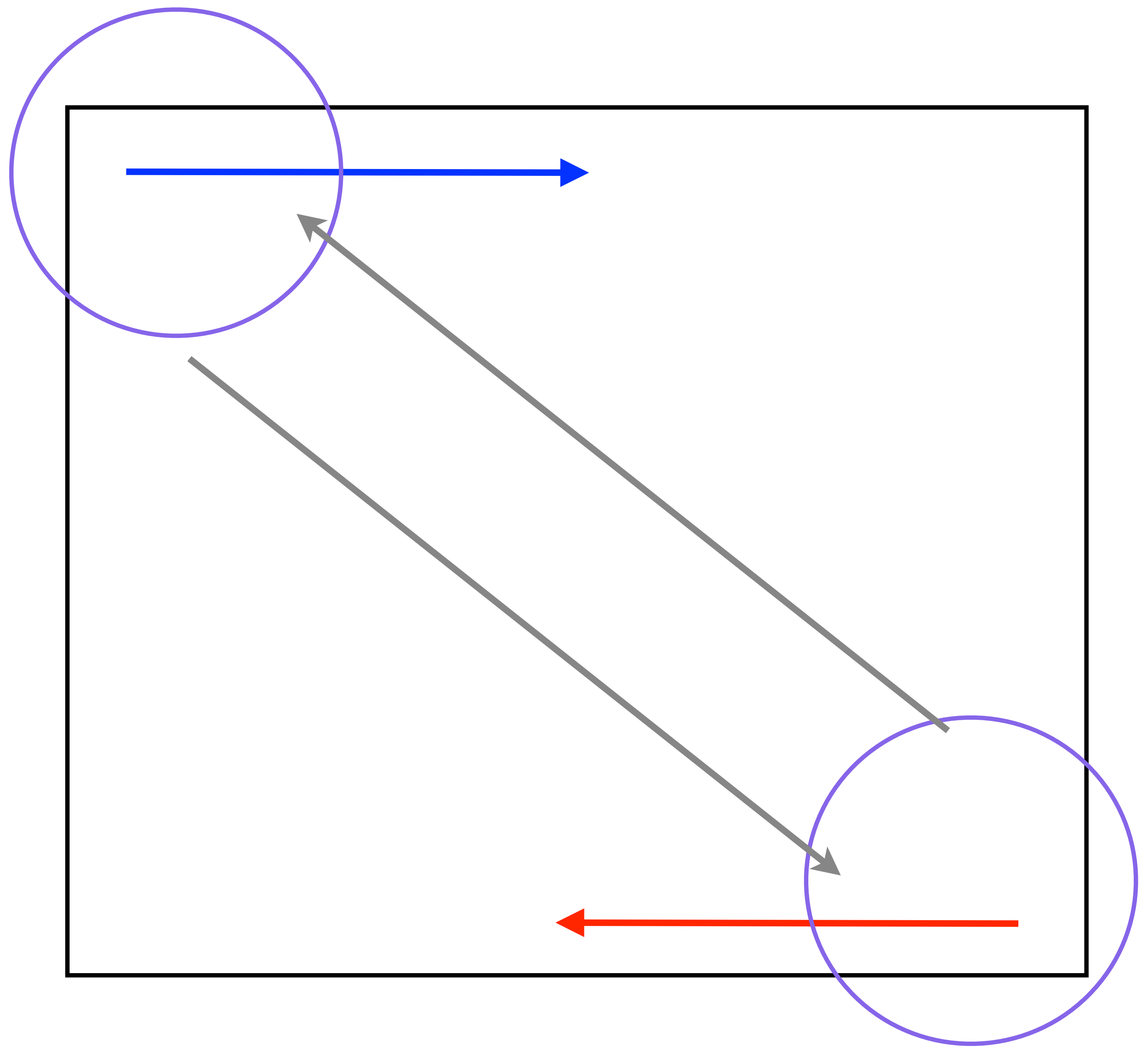
- Shear viscosity: efficiency of isotropisation is key



The kinetic theory approach: transport coefficients

- Shear viscosity: efficiency of isotropisation is key
- Direct isotropizing effect of transverse momentum broadening thus more important than its indirect effect as a driver of medium-induced radiation



The kinetic theory approach: transport coefficients

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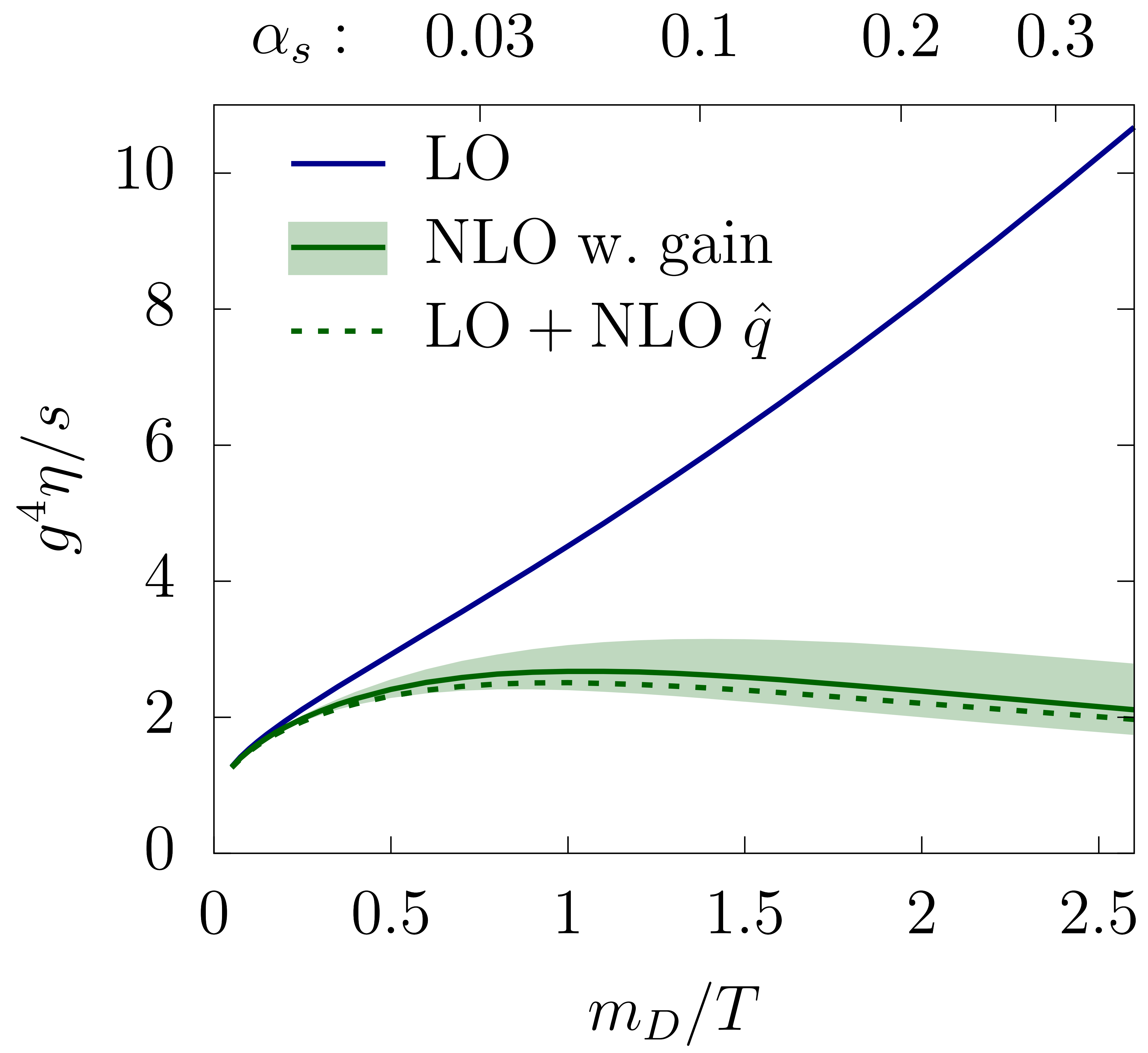

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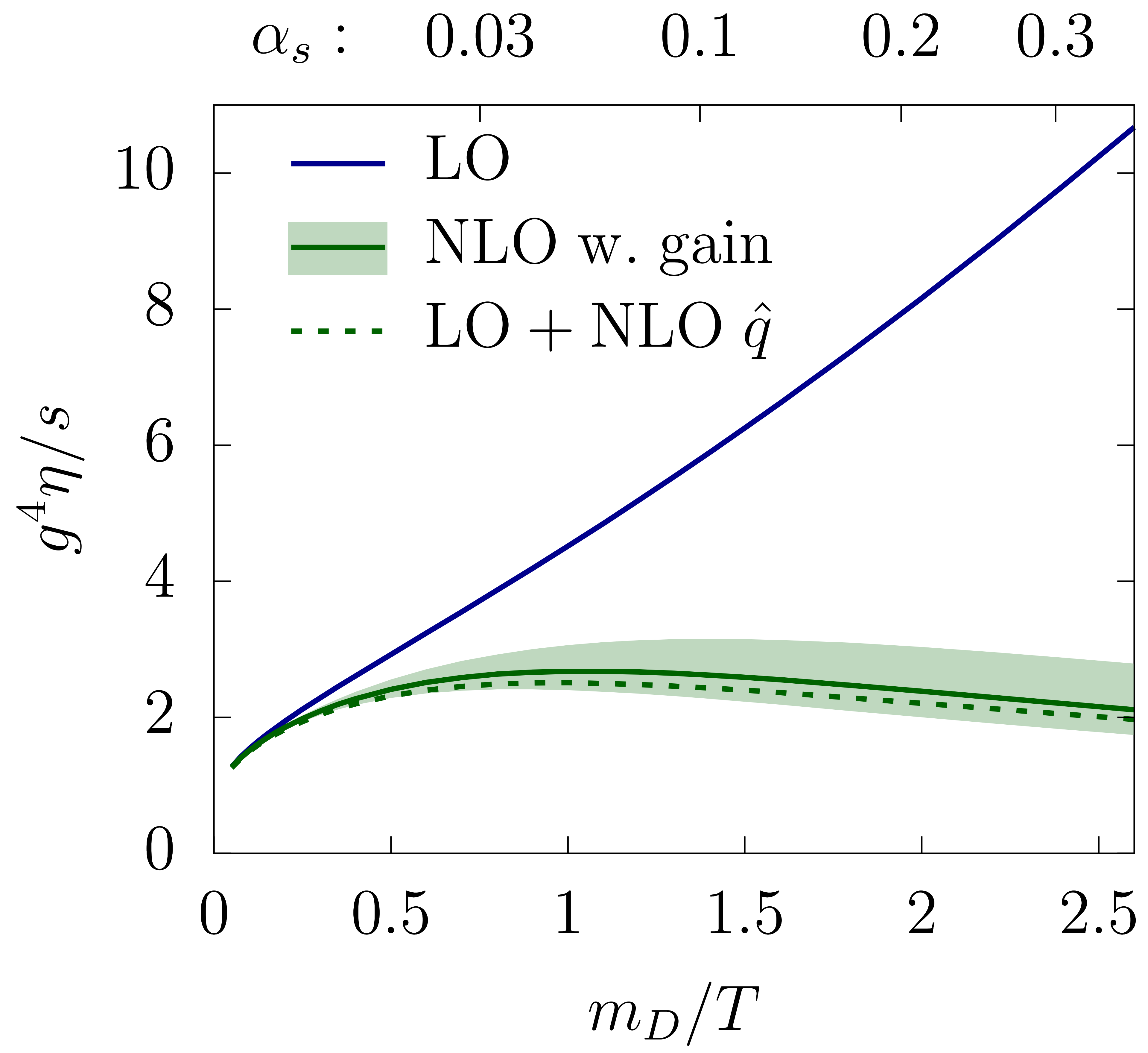
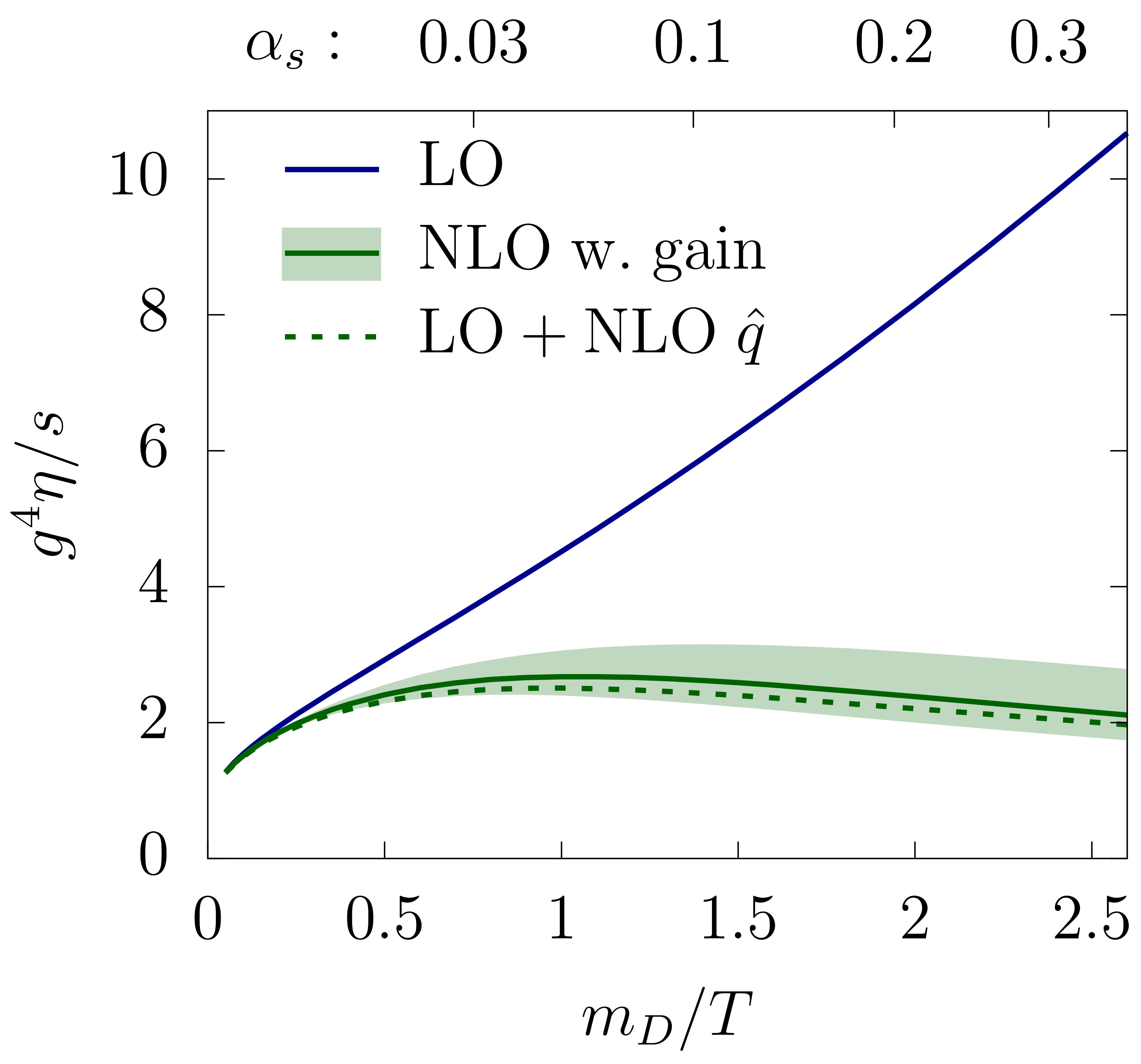


The kinetic theory approach: transport coefficients

- NLO large and completely dominated by NLO broadening
- Important observation: are we severely underestimating broadening at LO (excess screening shown before) and thus overestimating $\eta \sim 1/\hat{q}$? Müller **PRD104** (2021)
- Get as much non-perturbative input as possible!

For a different way of merging pQCD and (4D) lattice see **D. Bala**'s **poster** Wednesday for the photon rate, **L. Altenkort**'s **talk** Wed 12:10 for heavy-quark diffusion



The kinetic theory approach: thermalisation

- Many applications of kinetic theory to thermalisation

AMY at finite chemical potential and beam-energy dependence
Schlichting Du **PRL127**, **PRD104** (2021) **Poster** by **X. Du** Wednesday

Critical exponents in bottom-up thermalisation
Brewer Scheihing-Hitschfeld Yin **2203.02427**, Mikheev Mazeliauskas Berges **2203.02299**, **poster** by **Scheihing-Hitschfeld**

Attractors in kinetic theory **talks** by **Plumari** and **Almalool** Tuesday ~18

  - …

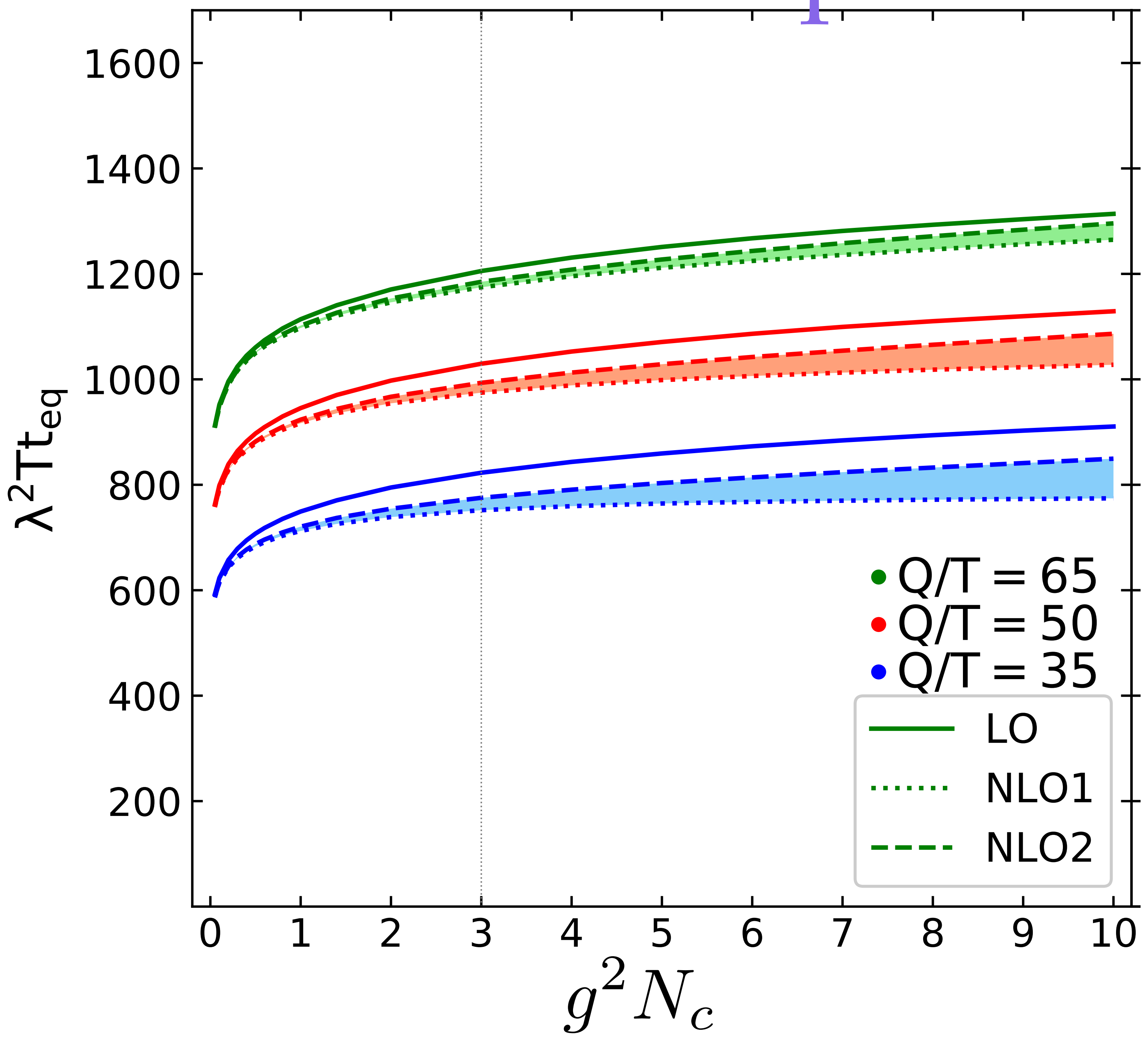
- We can worry about similarly problematic perturbative expansions for applications of kinetic theory to thermalisation. Can we try to estimate the **systematics of typical extrapolations** to α_s=0.3 (g=2)?

The kinetic theory approach: thermalisation

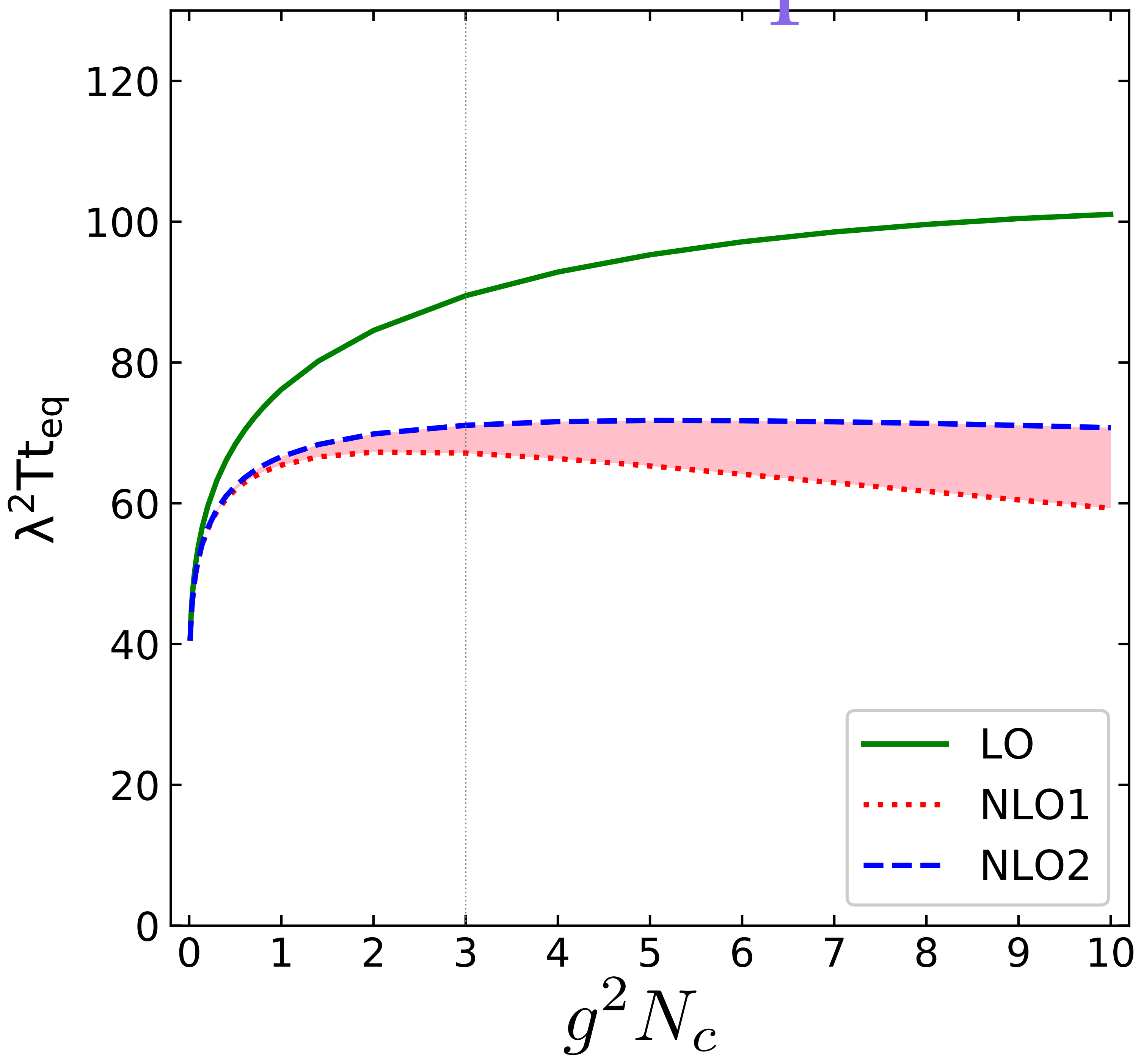
- Recently, NLO corrections to isotropic thermalisation for overoccupied and underoccupied initial conditions Fu JG Iqbal Kurkela **PRD105** (2022), **talk** by **Fu** Wed 9:20

Equilibration time

Underoccupied



Overoccupied



The kinetic theory approach: thermalisation

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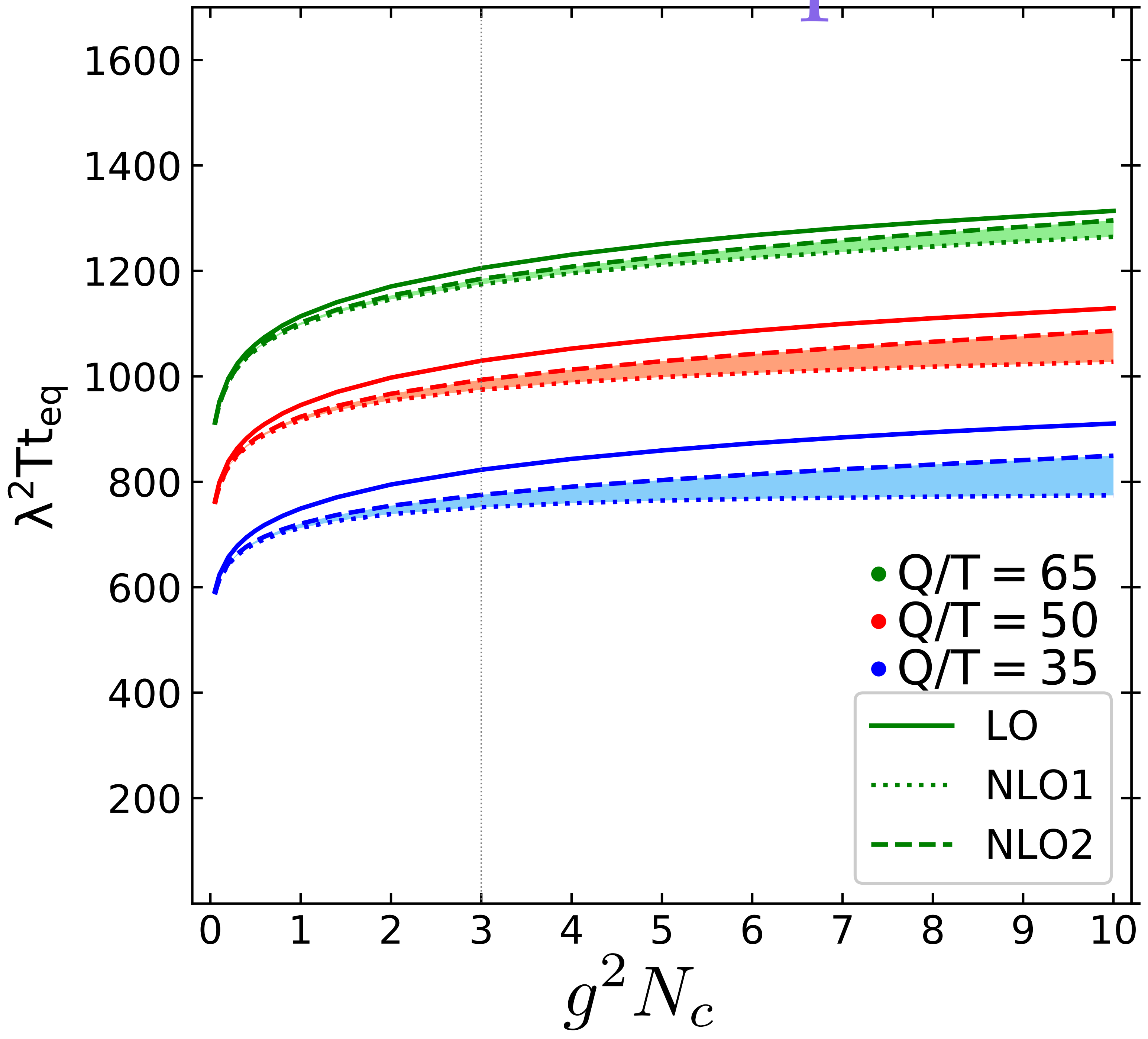
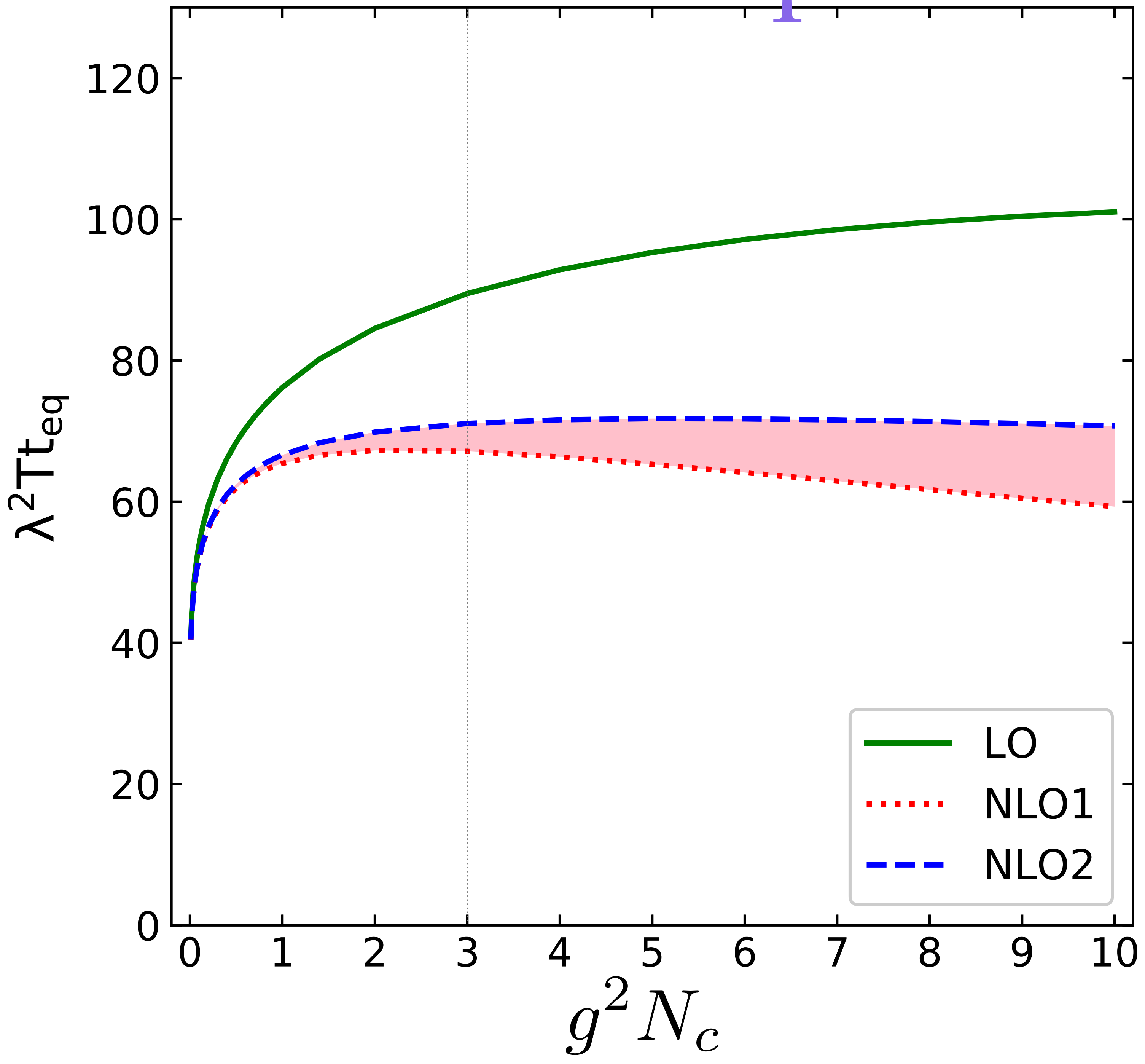


Overoccupied



- Two different NLO schemes which resum differently higher-order effects: proxy for NNLO

The kinetic theory approach: thermalisation

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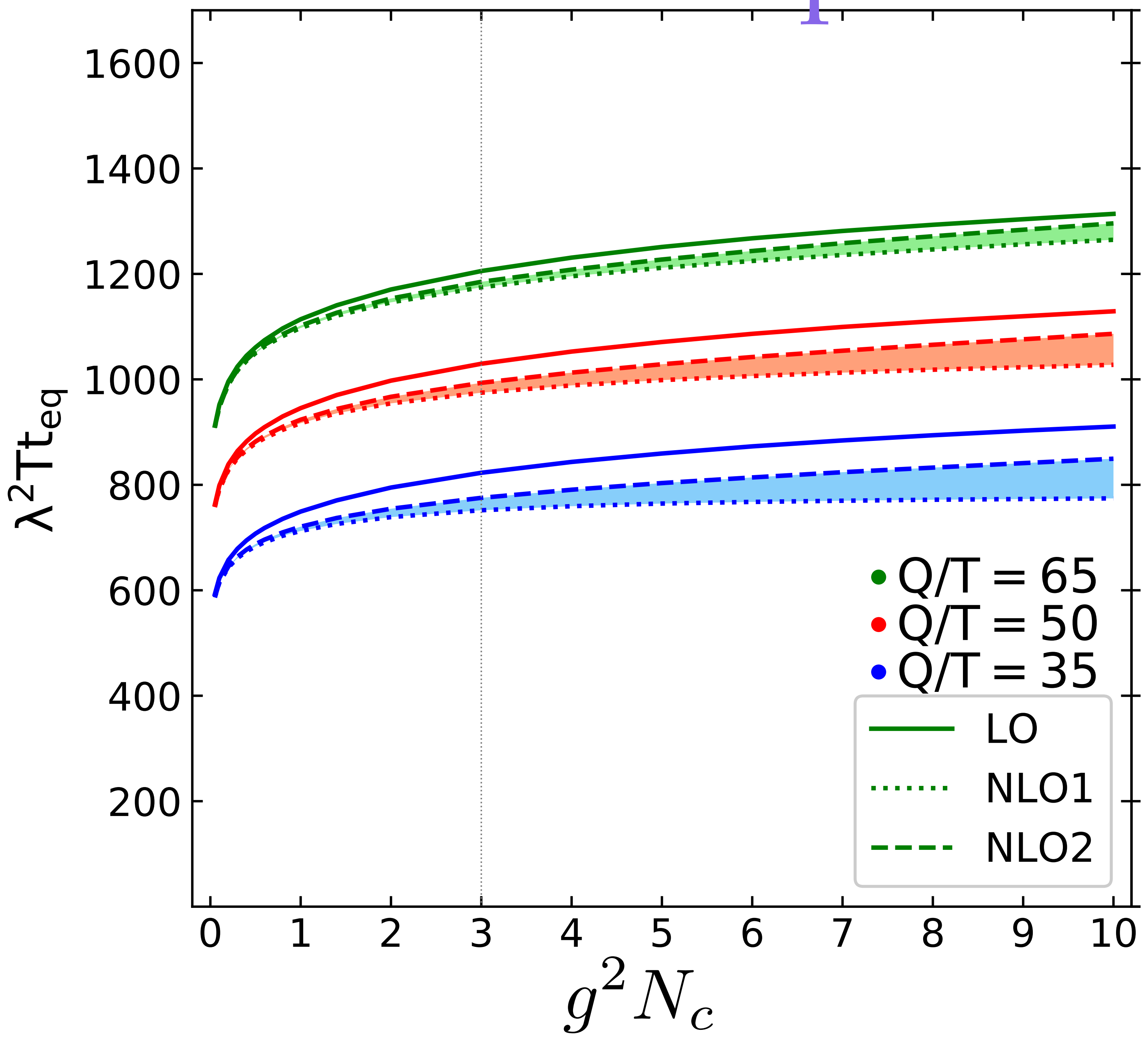
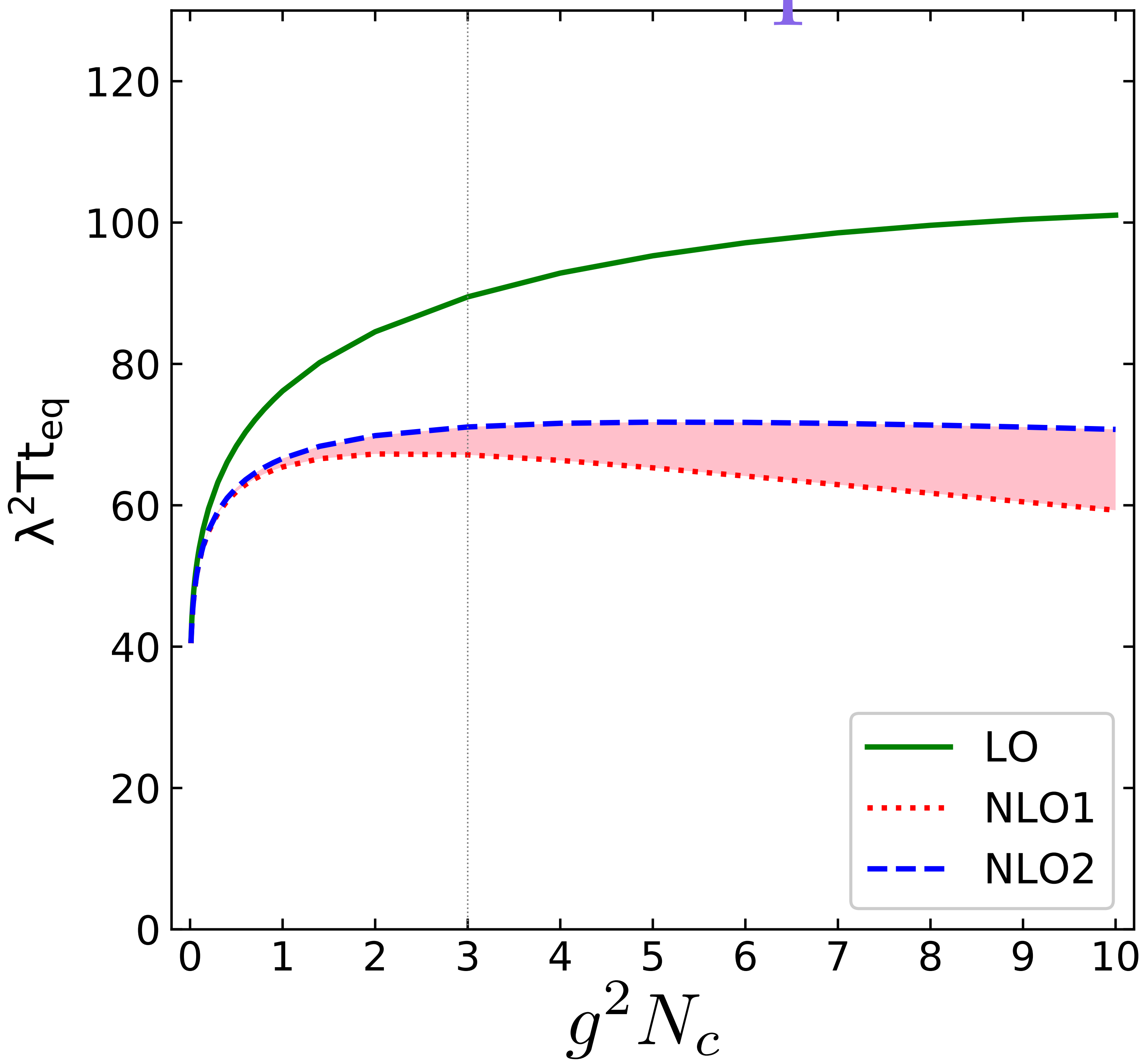


Overoccupied



- Robust behaviour but in this case no isotropizing effect of transverse momentum broadening

The kinetic theory approach: thermalisation

- Yet another (classical) complication arises in the IR in the case of anisotropies: **plasma instabilities** Mrowczynski, Romatschke Strickland, Arnold Lenaghan Moore
- Recently, instability subtracted momentum broadening kernel, together with a recipe for dealing with the instabilities, was provided in Hauksson Jeon Gale **PRC105** (2022). **Poster** by **Hauksson** Wednesday also discusses anisotropy effects on jet
- Anisotropy found to *reduce* the scattering kernel in the QGP phase. An important step towards a comprehensive kinetic treatment of anisotropic plasmas, work in progress
- Large&anisotropic transverse momentum broadening in the glasma, **posters** by **Czajka** and **Schuh** Wednesday, **talk** by **Carrington** Thursday 11:30

Conclusions

- Many recent improvements in the determination of transverse momentum broadening and medium-induced radiation in the QCD plasma are instrumental in better quantifying theory uncertainties and narrowing the gap between Lagrangians and phenomenology
    - Improved approximations and numerical solutions for the radiation rates
    - Non-perturbative determination of the broadening kernel
    - Quantum corrections: anomalous diffusion and double splitting
    - Improved understanding of the systematics of extrapolations to intermediate couplings for transport&thermalisation
- LOts and (N)LOts of progress, (N)LOts and (NN?)LOts and lat(tices) still to do

Extra slides

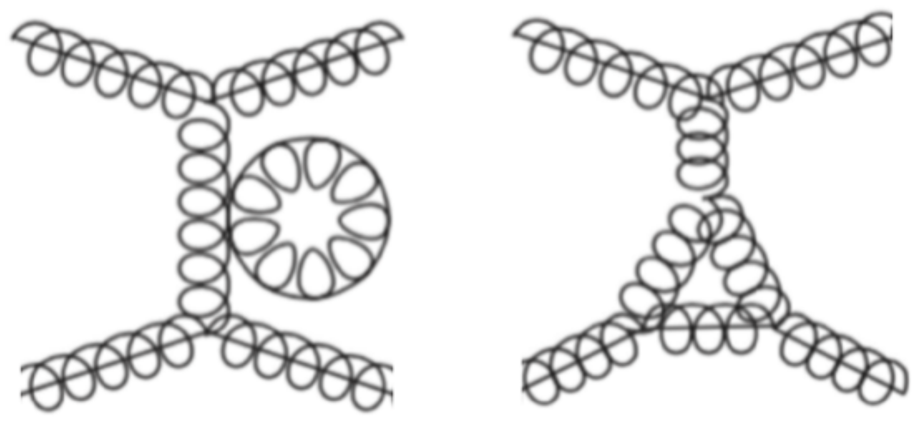
Istropic thermalisation at NLO

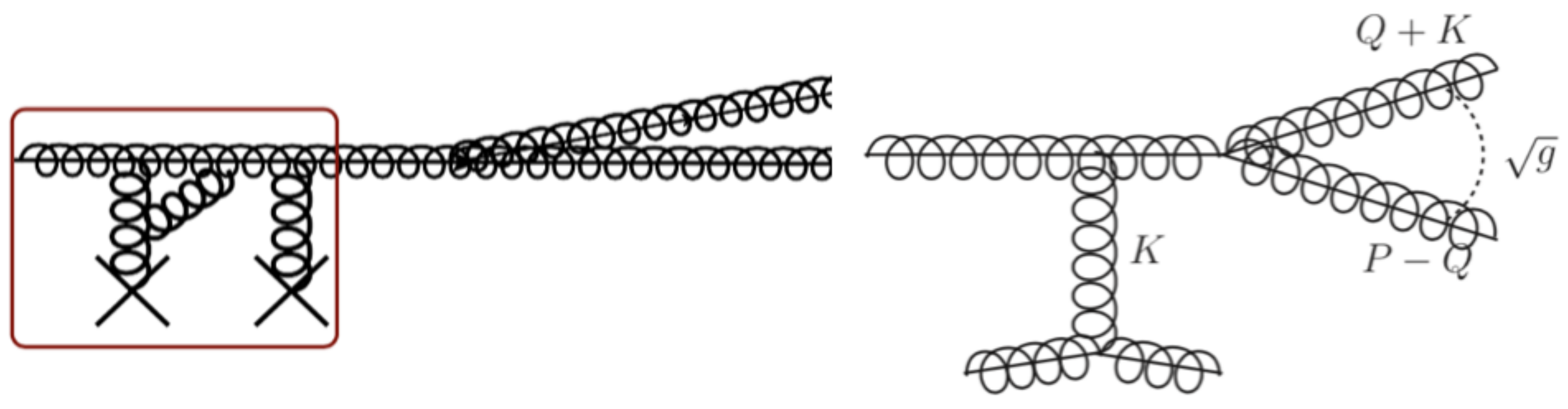
Slide from
Y. Fu's talk
Wed 9:20

Ghiglieri,Moore,Teaney 1509.07773

$O(g)$ corrections come from soft gluon:

- For $2 \to 2$: soft gluon legs or soft gluon loops.





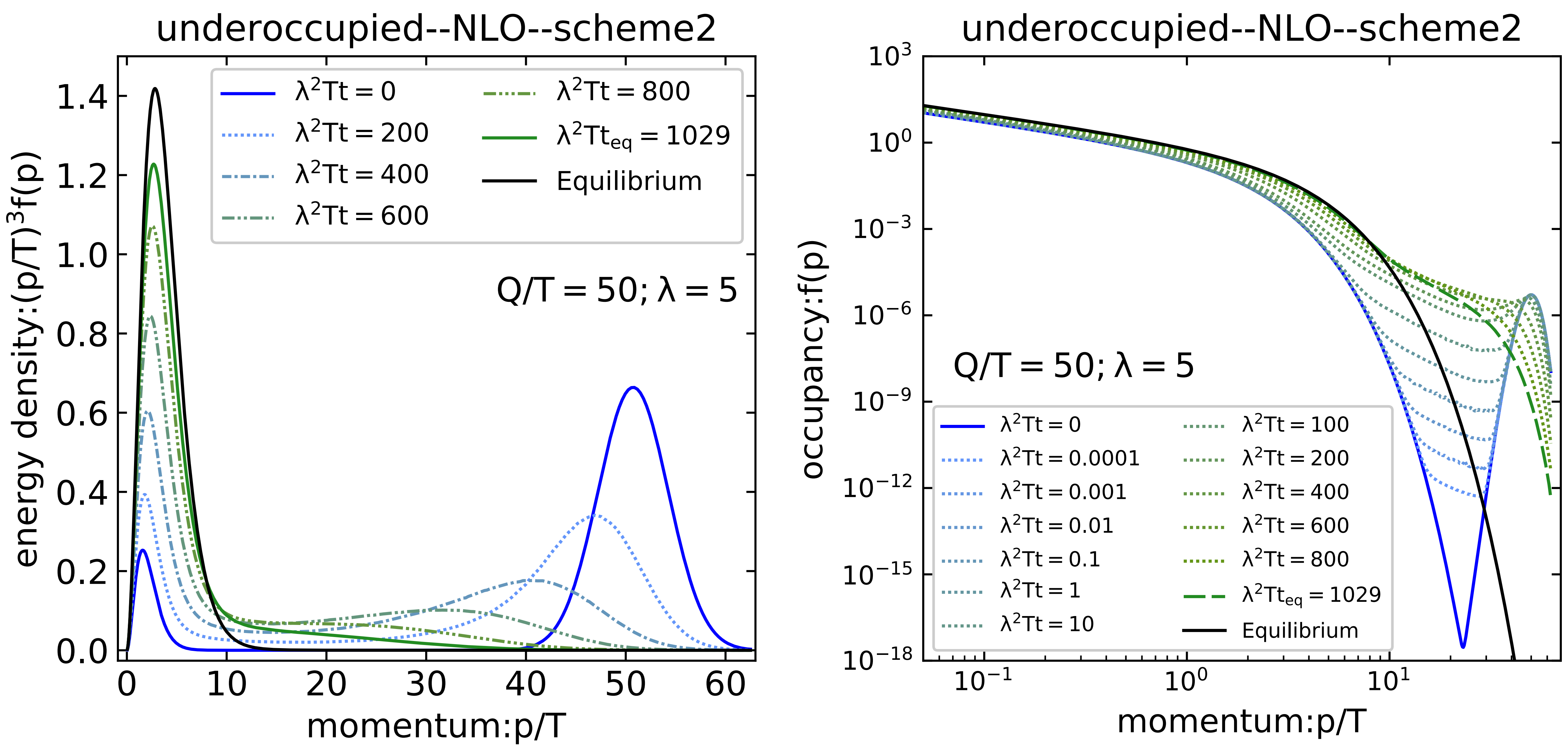
- For $1 \leftrightarrow 2$:
  - one-loop soft scatterings from the medium;
  - wider-angle semi-collinear radiation.

We can construct collision operators that are equivalent up to NLO, with ambiguities at NNLO. (A general property of kinetic theory resummations.)

- use different results arising from these collision operators and their spread from LO to estimate of the uncertainty of NLO corrections.

Istropic thermalisation at NLO

Slide from
Y. Fu's talk
Wed 9:20



NLO qualitatively similar to LO:(LO: Kurkela,Lu 1405.6318)

- the hard particles lose energy through the radiational cascade heating the soft thermal bath;
- the system thermalizes as the hard particles are quenched in the thermal bath.

Istropic thermalisation at NLO

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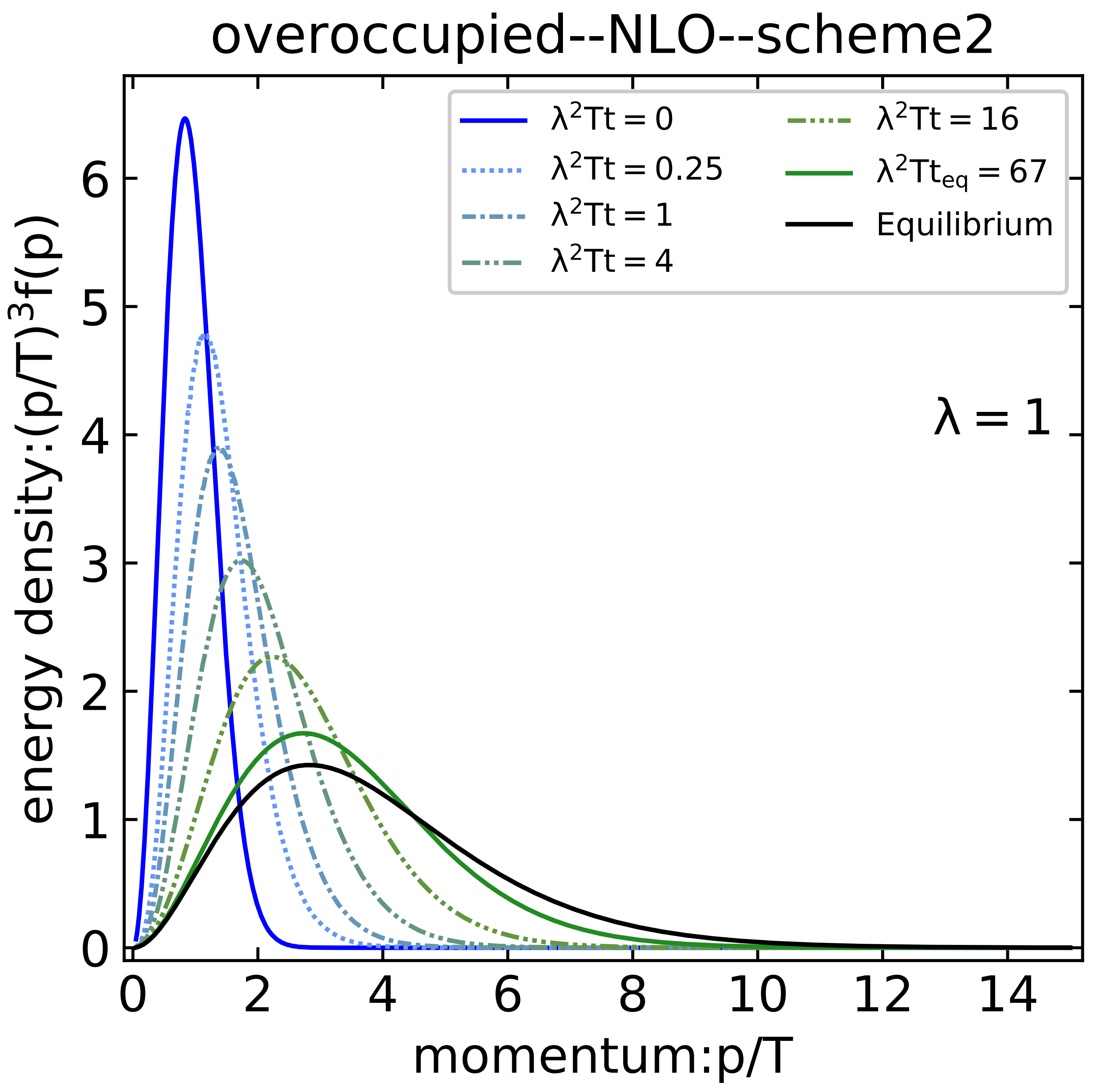
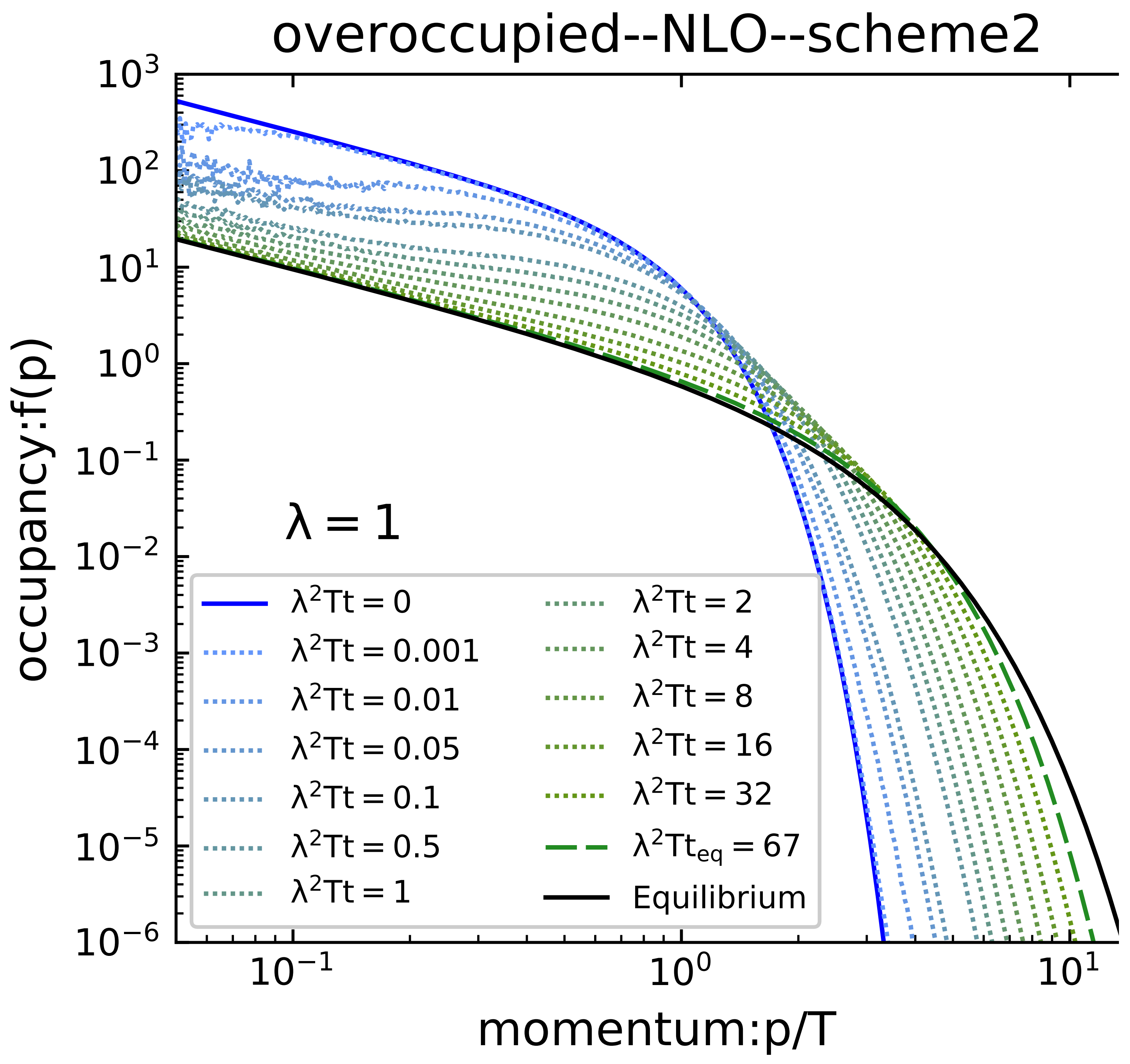




NLO qualitatively similar to LO:(LO: Kurkela,Lu 1405.6318)

- self-similar direct energy cascade to UV.

The photon rate and the 4D lattice

Polynomial ansatz of the spectral function $\rho_H(\omega, \vec{k}) = 2(\rho_T(\omega, \vec{k}) - \rho_L(\omega, \vec{k}))$,

J. Ghiglieri, O. Kaczmarek, M. Laine, and F. Meyer, Phys. Rev. D 94, 016005.

For $\omega < \omega_0$

$$\rho^H_{fit} = \frac{\beta(\omega_0)\omega^3}{2\omega_0^3}\left(5 - 3\frac{\omega^2}{\omega_0^2}\right) - \frac{\gamma(\omega_0)\omega^3}{2\omega_0^2}\left(1 - \frac{\omega^2}{\omega_0^2}\right) + \left(\delta_0\left(\frac{\omega}{\omega_0}\right) + \delta_1\left(\frac{\omega}{\omega_0}\right)^3\right)\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2$$

Slide courtesy of D. Balа

See also Cè Harris Meyer Steinberg Toniato **PRD102** (2020)

- For $\omega > \omega_0$, NLO-LPM re-summed spectral function has been used.

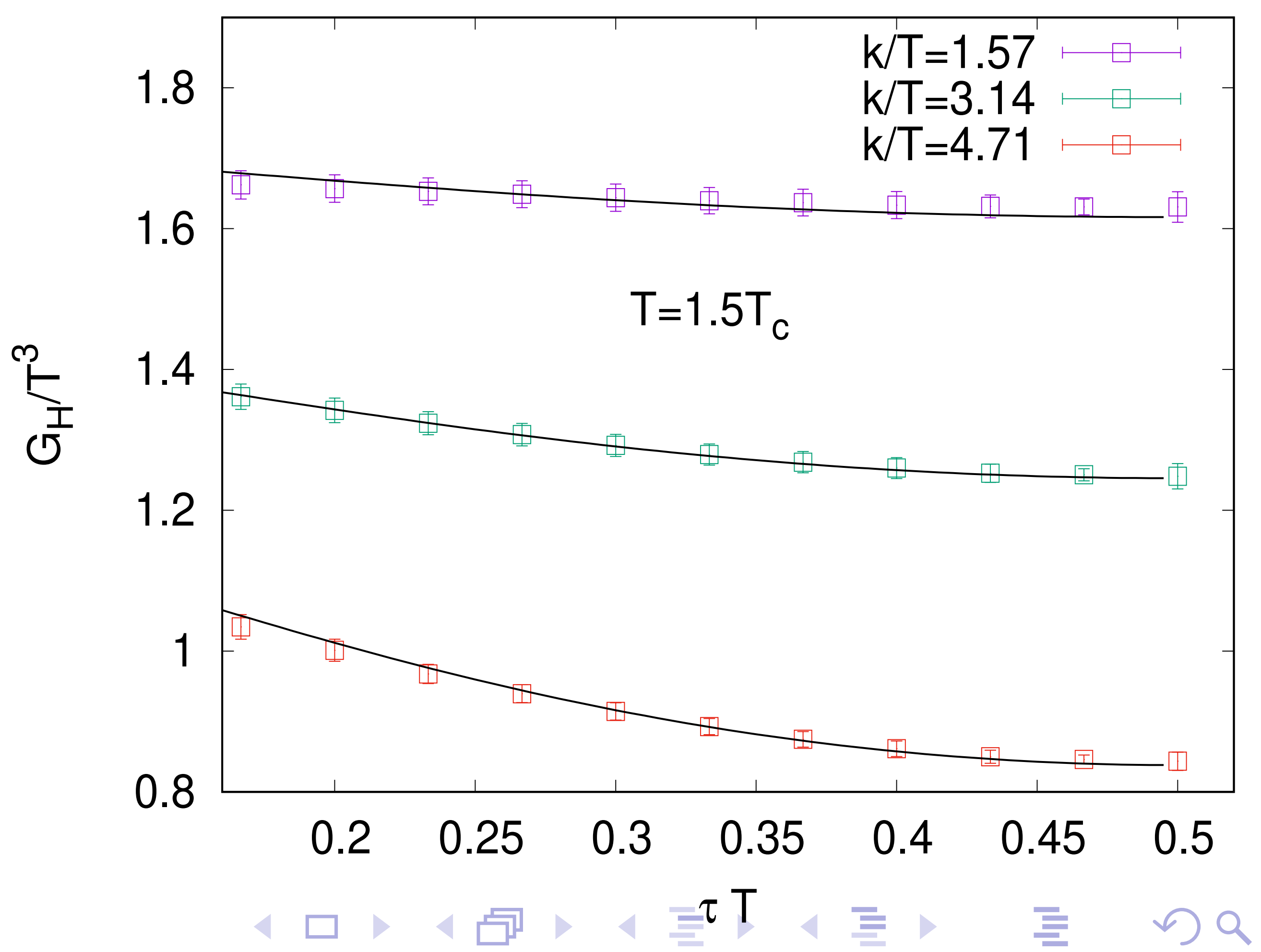
  G. Jackson & M. Laine, J. High Energy. Phys. 2019, 144
- β and γ is determined from the perturbative spectral function at ω_0.
- The parameter δ_0 is determined in terms of δ_1 to satisfy,

$$\int_0^\infty d\omega\, \omega\, \rho_H(\omega, \vec{k}) = 0$$

  M. Ce et al. Phys. Rev. D 102, 091501(R)
- The ω_0 should be chosen deep into the time-like region $\omega_0 = \sqrt{k^2 + (\pi T)^2}$.

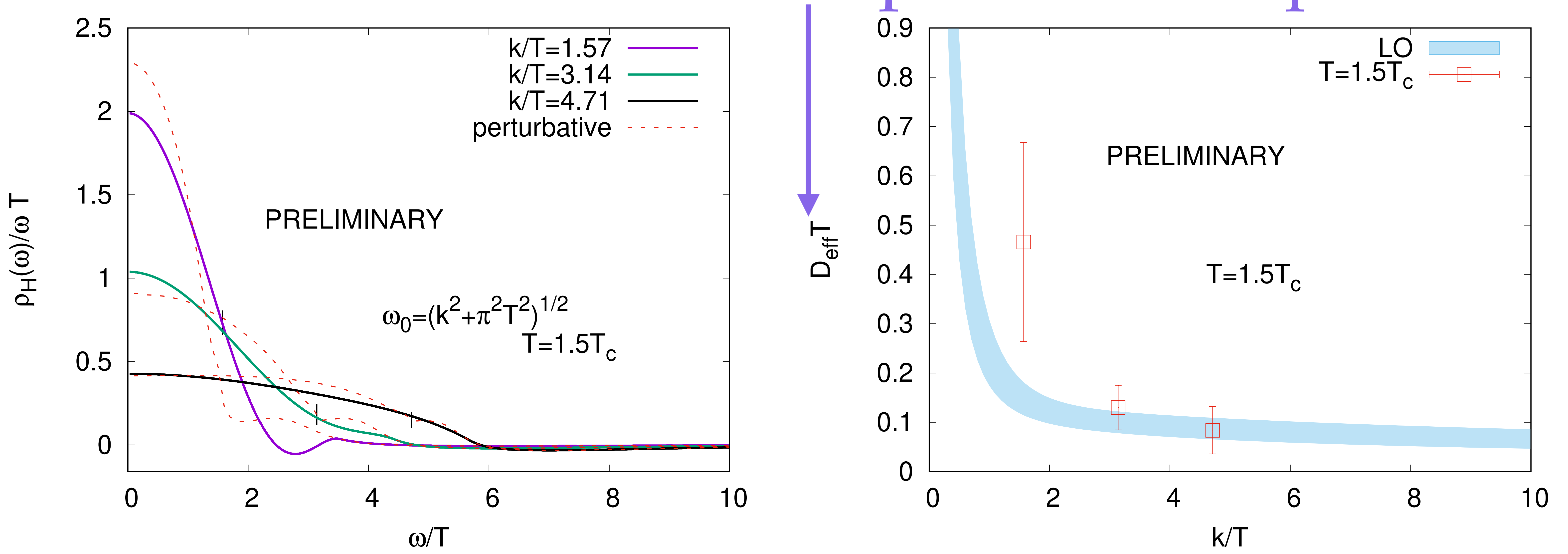
- The lattice $T - L$ correlator is fitted with respect to δ_1 for light quark at $1.5T_c$ in SU(3) plasma.

  Bala, Jackson, Kaczmarek (Poster).



The photon rate and the 4D lattice

Proportional to photon rate



Bala, Jackson, Kaczmarek (Poster).

Slide courtesy of D. Bala

- Low-ω part of the spectral function is estimated from lattice data.
- Effective diffusion coefficient $D_{eff}(k) = \frac{\rho_H(|\vec{k}|,\vec{k})}{2\chi_q|\vec{k}|}$ calculated from these spectral functions.
- The statistical error on D_{eff} is much smaller than the systematic uncertainty which has been obtained by varying ω_0 between $\sqrt{k^2+(\pi T)^2}$ and $\sqrt{k^2+(2\pi T)^2}$.
- At smaller momentum the D_{eff} differs from the perturbative estimate.

See also Cè Harris Meyer Steinberg Toniato **PRD102** (2020)