Transport coefficients of QCD at NLO

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in collaboration with Guy Moore and Derek Teaney

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Outline

- Transport coefficients: introduction and motivation
- Transport from an effective kinetic theory
- (almost) NLO kinetic theory and first-order coefficients

  (full) NLO kinetic theory for jets: pedagogical review in JG Teaney **QGP5** (2015), gritty details in JG Moore Teaney **JHEP1603** (2015)

  (almost) NLO 1st-order JG Moore Teaney, **JHEP1803** (2018)
- Second-order relaxation: results and bounds

  JG Moore Teaney, **1805.02663**

Overview


PLAN
du Siége de la Ville de
BARCELONE
Avec la Carte de la côte de la Mer depuis le
Cap de Cervere jusqu'aux environs
de Llobregat.
DEDIE AU ROY
Campement des Troupes Françoises
BARCELONE
ARMEE NAVALLE
Mediter

Hydrodynamics

- Field theories admit a long-wavelength hydrodynamical limit. Hydrodynamics: Effective Theory based on a gradient expansion of the flow velocity

- For hydro fluctuations with local flow velocity **v** around an equilibrium state (with temp. T), at **first order** in the gradients and in **v**

$$T^{00} = e, \qquad T^{0i} = (e+p)v^i$$

$$T^{ij} = (p - \zeta \nabla \cdot \mathbf{v})\delta^{ij} - \eta \left(\partial_i v^j + \partial_j v^i - \frac{2}{3}\delta^{ij} \nabla \cdot \mathbf{v} \right)$$

Navier-Stokes hydro, two *transport coefficients*: bulk and shear viscosity

Estimating η
(or why is η/s natural)

$$T^{00} = e, \qquad T^{0i} = (e+p)v^i$$

$$T^{ij} = (p - \zeta \nabla \cdot \mathbf{v})\delta^{ij} - \eta \left(\partial_i v^j + \partial_j v^i - \frac{2}{3}\delta^{ij} \nabla \cdot \mathbf{v} \right)$$

- Rewrite the first-order T as

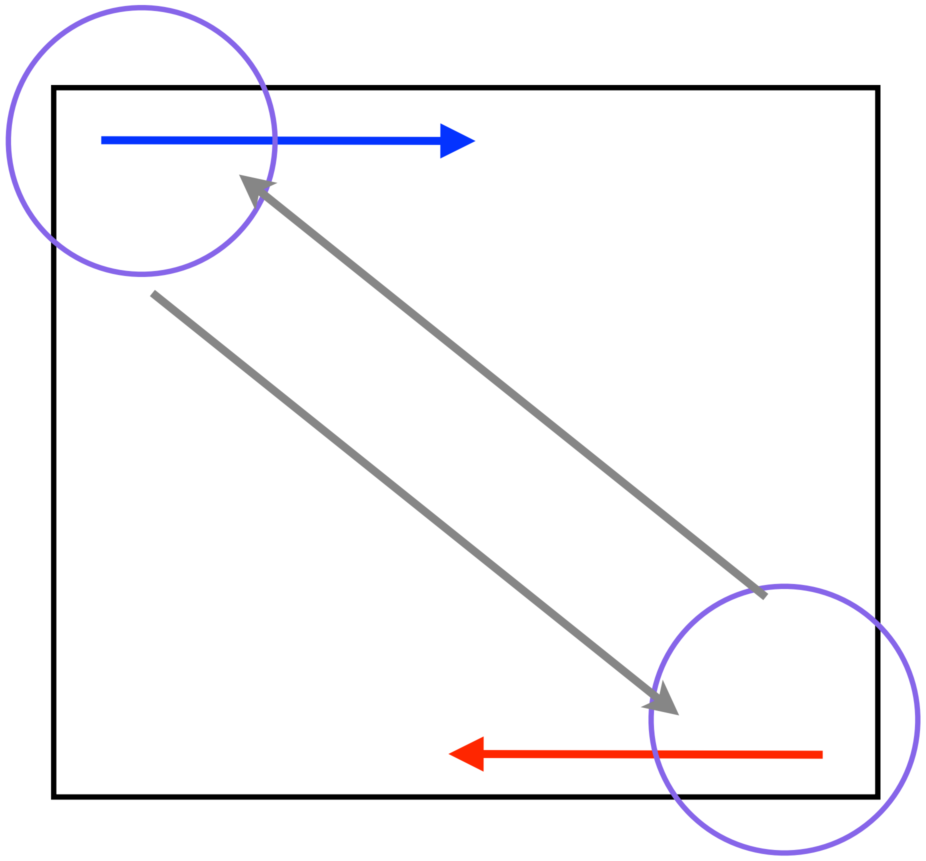
$$T^{xy} = -\frac{\eta}{e+p} \nabla_x T^{0y}$$

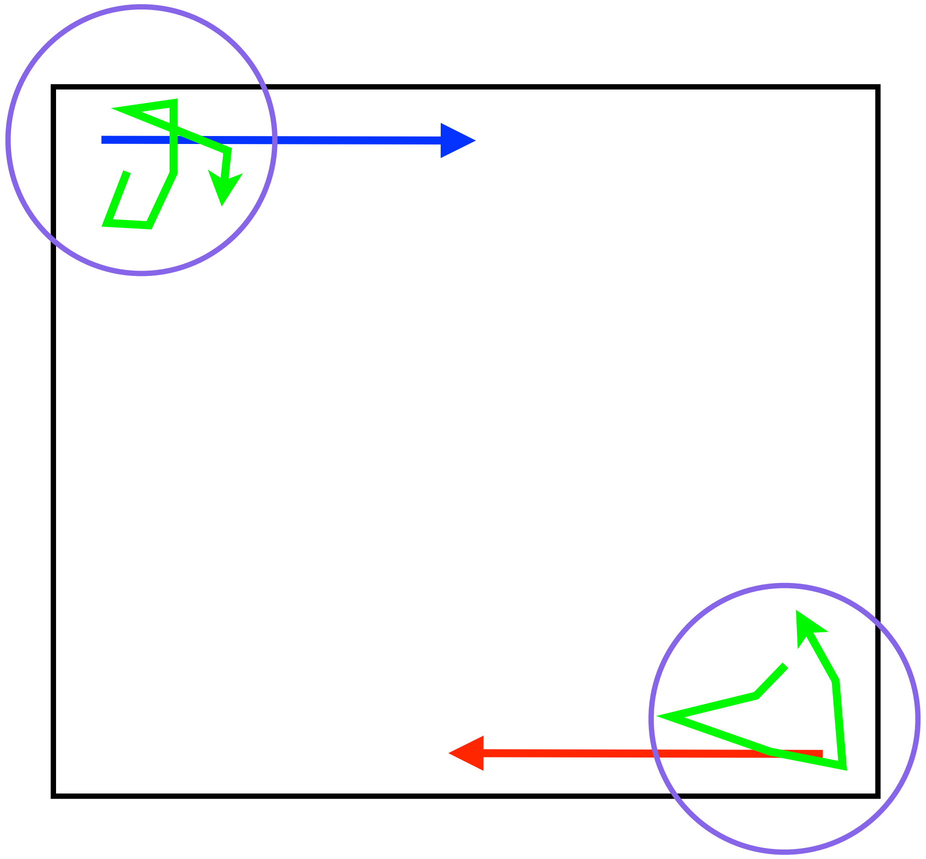
- $\eta/(e+p)$ is a (first-order) relaxation timescale. With $e+p=sT$ one gets to the natural, dimensionless η/s

- In cases with well-defined quasi-particles one has naturally

$$\frac{\eta}{s} \sim T l_{\mathrm{mfp}}$$

Estimating η
(or why is η/s natural)





- Weak coupling: long distances between collisions, easy diffusion. **Large** η/s
- Strong coupling: short distances between collisions, little diffusion. **Small** η/s

Estimating η
(or why is η/s natural)

- (Mean free path)$^{-1}$~ cross section x density

$$\frac{\eta}{s} \sim T l_{\rm mfp} \sim \frac{T}{n\sigma} \sim \frac{1}{T^2\sigma}$$

- Cross section in a perturbative gauge theory (T only scale*)

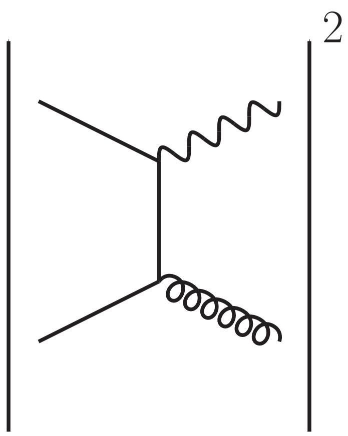
$$\sigma \sim \frac{g^4}{T^2} \qquad \frac{\eta}{s} \sim \frac{1}{g^4}$$

* Coulomb divergences and screening scales (m_D~gT) in gauge theories

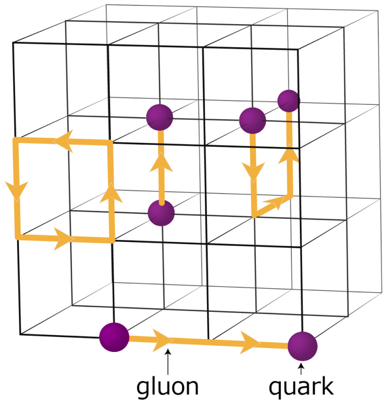
$$\sigma \sim \frac{g^4}{T^2}\ln(1/g) \qquad \frac{\eta}{s} \sim \frac{1}{g^4\ln(1/g)}$$

- From holography one instead has η/s=1/(4π) (for $\mathcal{N}=4$ SYM) and a conjectured lower limit
  Kovtun Son Starinets Policastro **PRL87** (2001) **PLR94** (2004)

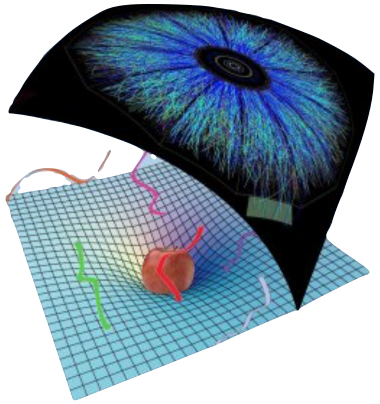
Theory approaches to (QCD) transport coefficients



pQCD: QCD action (and EFTs and kinetic theories thereof). Real world: extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$

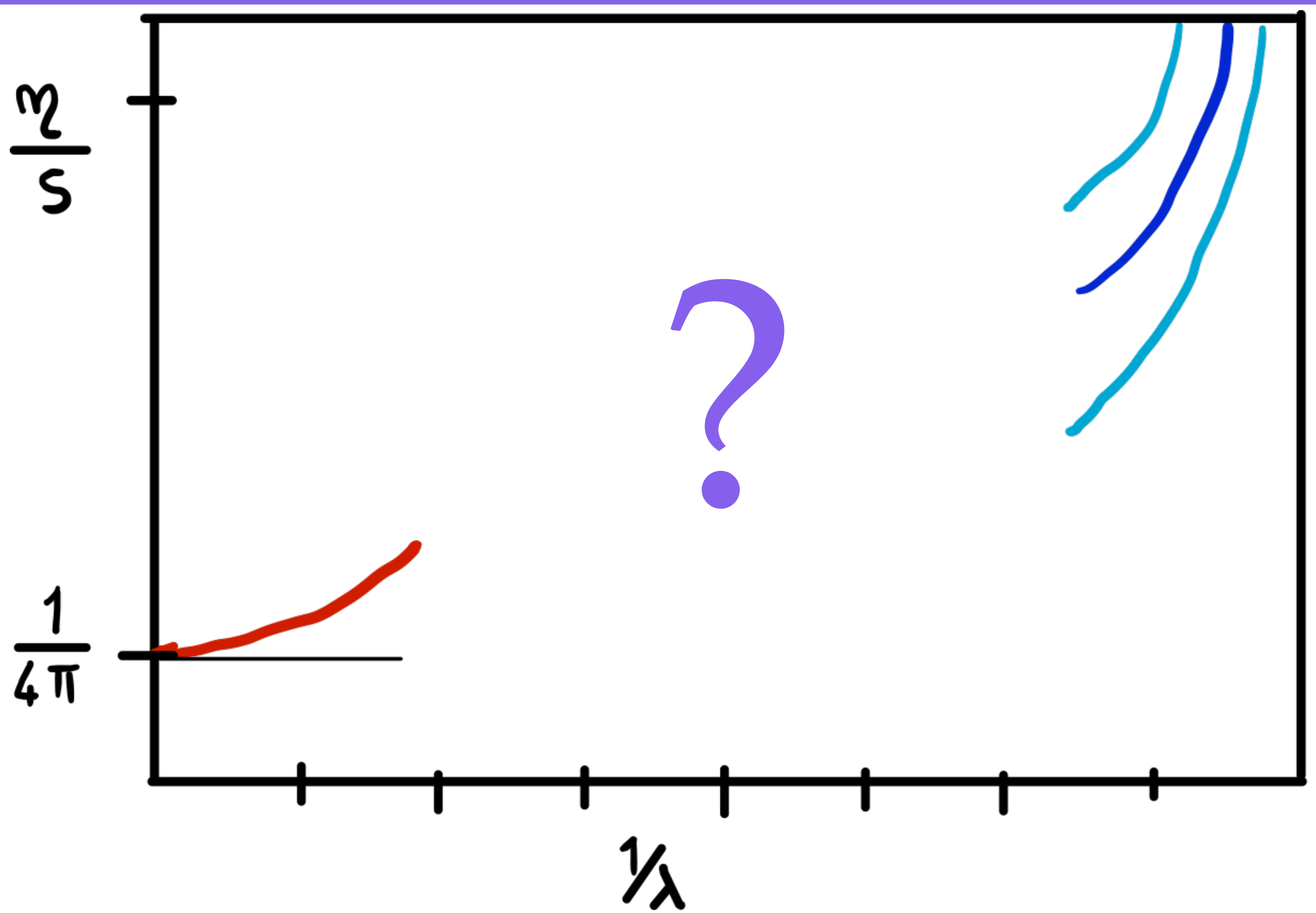


lattice QCD: Euclidean QCD action. Real world: analytically continue to Minkowskian domain (see **H. Meyer's talk** for a related topic)



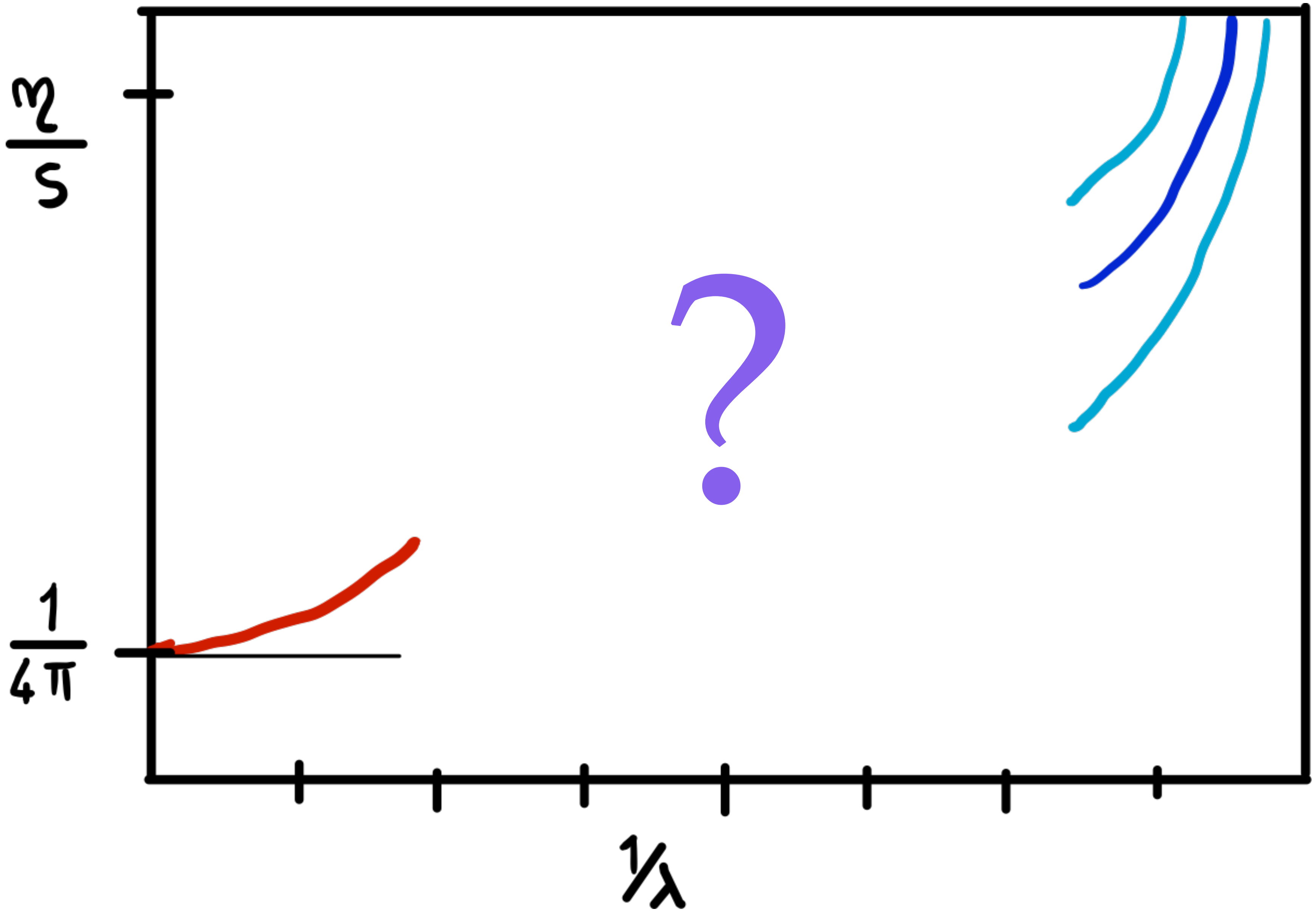
AdS/CFT: $\mathcal{N}=4$ action, weak and strong coupling. Real world: extrapolate to QCD

Motivation



- Holography: Kovtun Son Starinets Policastro **PRL87** (2001) **PLR94** (2004), $\lambda^{-3/2}$ corrections: Buchel et al (2005-2008)
- pQCD: Arnold Moore Yaffe (AMY) (2000-2003)

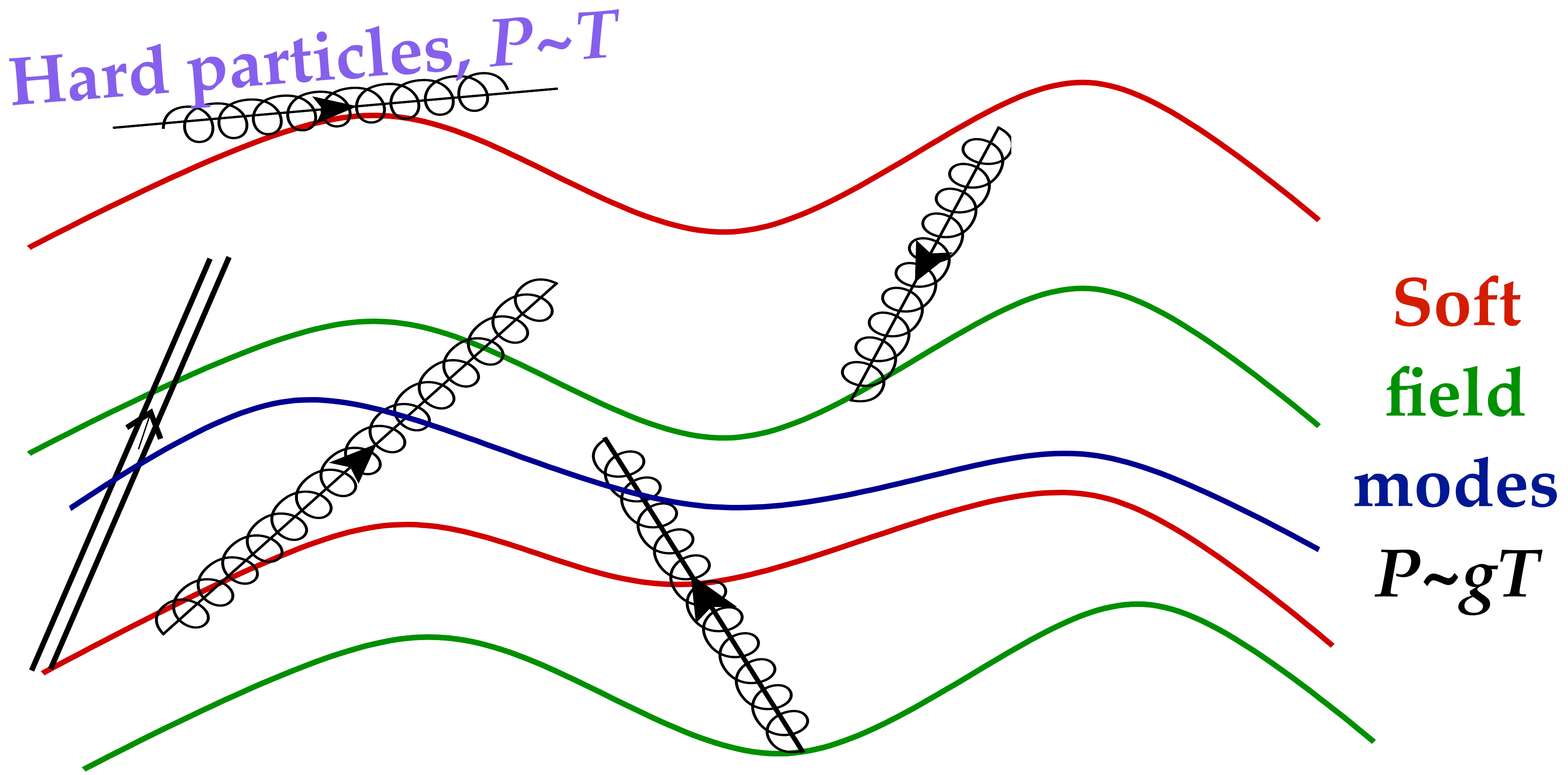
Motivation



- Add NLO to the right, understand kinetic theory beyond LO

The effective kinetic theory

The weak-coupling picture

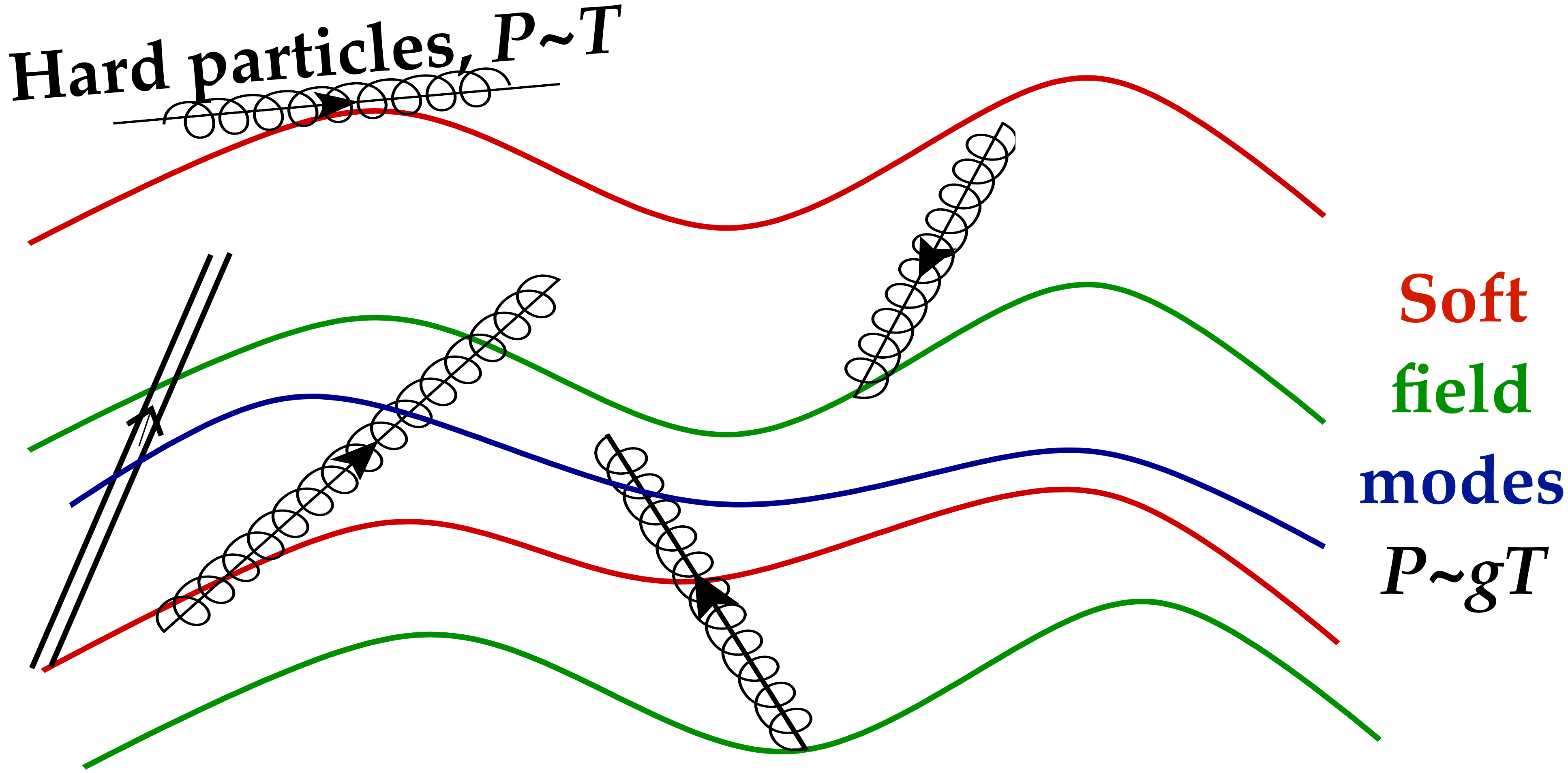


$$\alpha_s = \frac{g^2}{4\pi}$$

Figure by D. Teaney

- Hard (quasi)-particles carry most of the stress-energy tensor. (Parametrically) largest contribution to thermodynamics

The weak-coupling picture



$$\alpha_s = \frac{g^2}{4\pi}$$

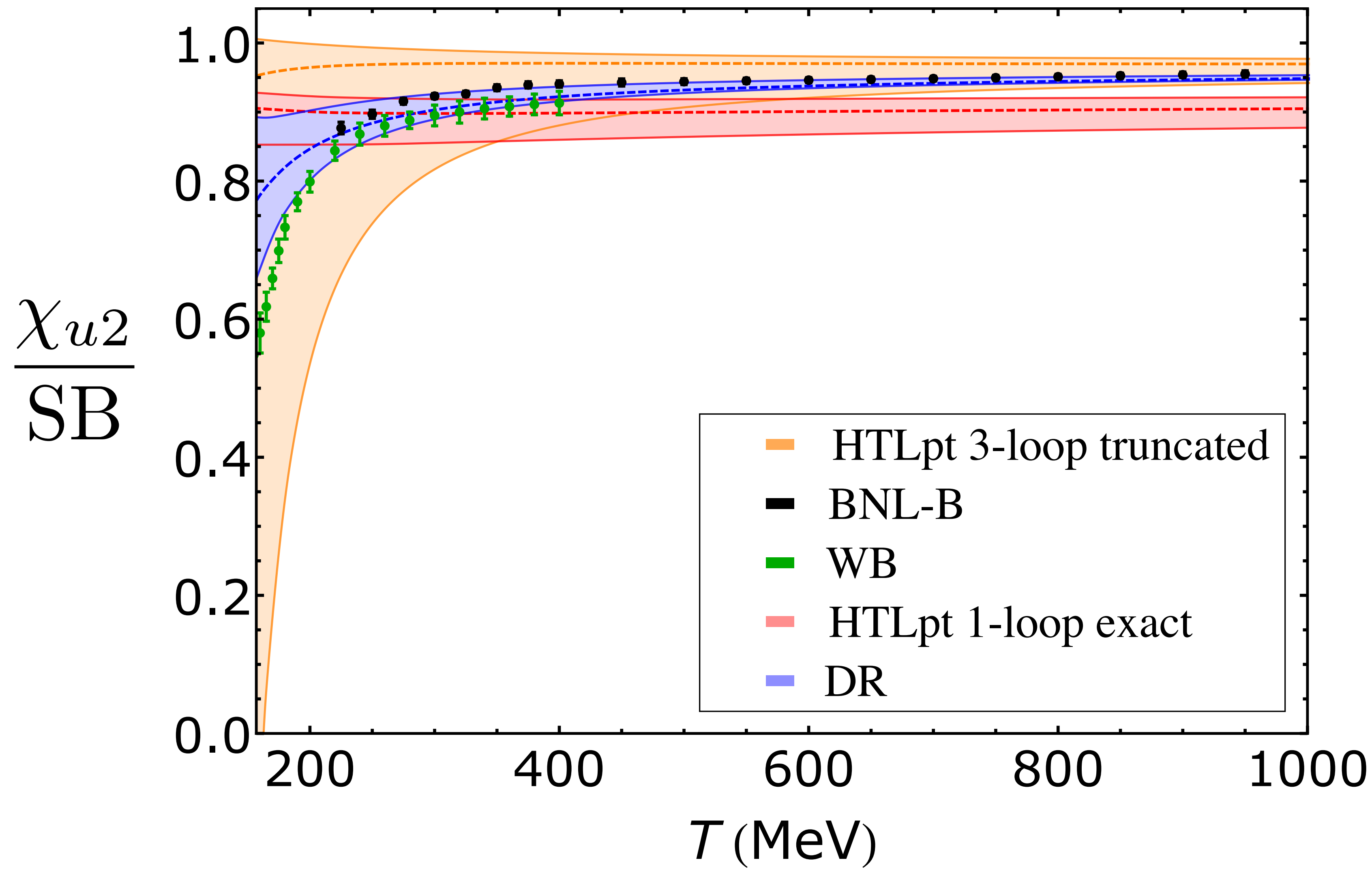
Figure by D. Teaney

- The gluonic soft fields have large occupation numbers ⇒ they can be treated classically

$$n_B(\omega) = \frac{1}{e^{\omega/T} - 1} \overset{\omega \sim gT}{\simeq} \frac{T}{\omega} \sim \frac{1}{g}$$

Weak-coupling thermodynamics

$$\chi_{u2} = \frac{\partial^2 p(T,\mu)}{\partial \mu_u^2}$$

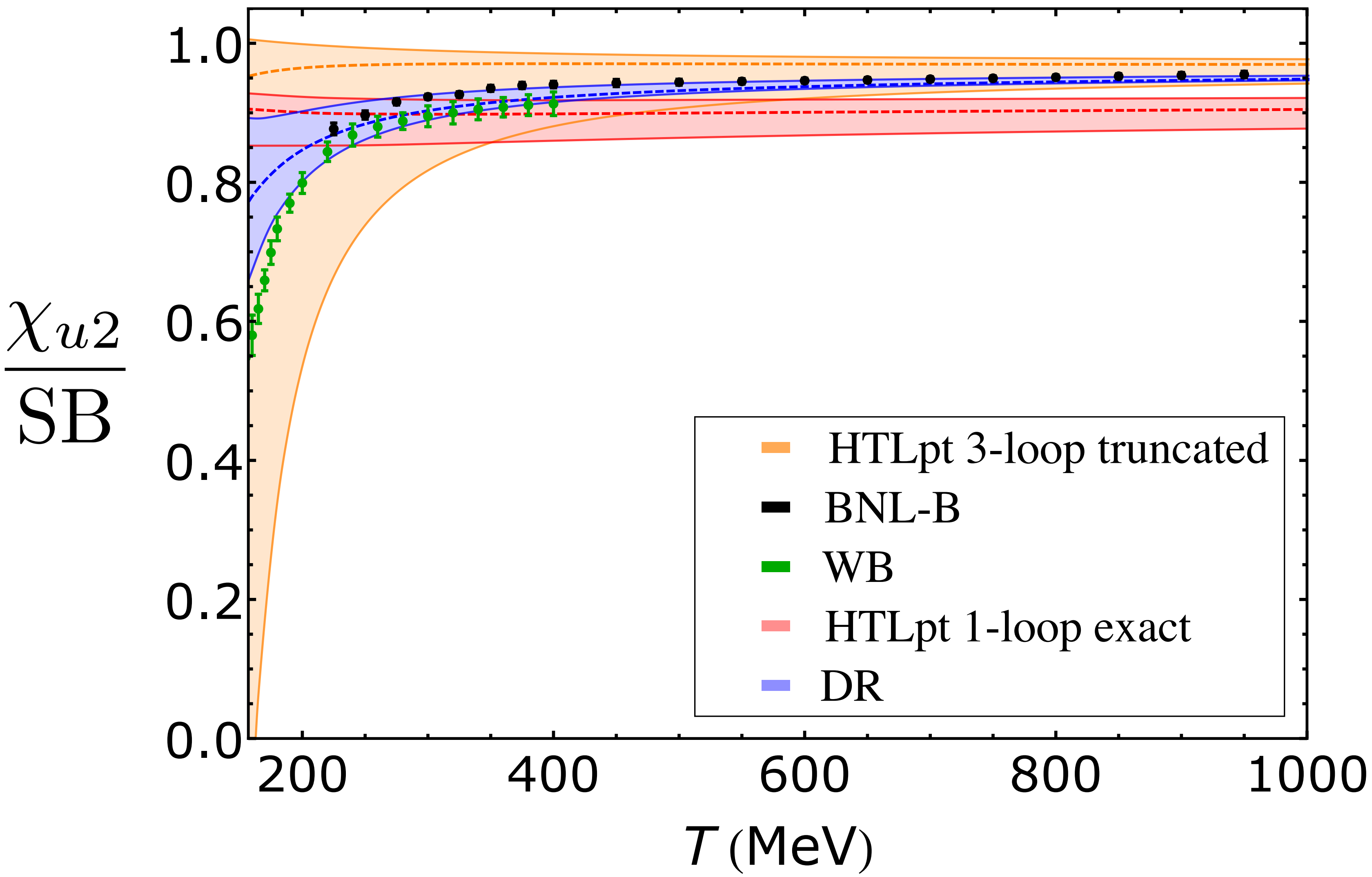


Mogliacci Andersen Strickland Su Vuorinen **JHEP1312** (2013)

- Successful for static (thermodynamical) quantities. Possibility of solving the soft sector non-perturbatively (3D theory on the lattice). See talk by **P. Schicho** tomorrow

Weak-coupling thermodynamics

$$\chi_{u2} = \frac{\partial^2 p(T,\mu)}{\partial \mu_u^2}$$



Mogliacci Andersen Strickland Su Vuorinen **JHEP1312** (2013)

- Extra motivation: understand how to set up a similar resummation program for kinetics

The effective kinetic theory

Baym Braaten Pisarski Arnold Moore Yaffe Baier Dokshitzer Mueller Schiff Son Peigné Wiedemann Gyulassy Wang Aurenche Gelis Zaraket Blaizot Iancu . . .

The effective kinetic theory

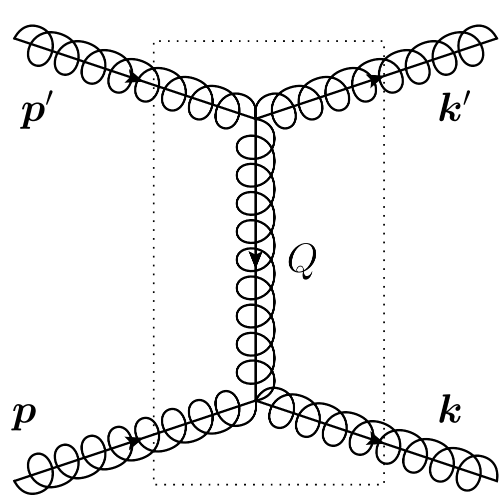
- The effective theory is obtained by integrating out (off-shell) quantum fluctuations
- Boltzmann equation for the single-particle phase space-distribution: its convective derivative equals a collision operator $$(\partial_t + \mathbf{v_p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C[f]$$
- In other words at weak coupling the underlying QFT has well-defined quasi-particles. These are weakly interacting with a mean free time ($1/g^4T$) larger than the actual duration of an individual collision ($1/T$)
- Justified at weak coupling, can be extended to factor in non-perturbative contributions

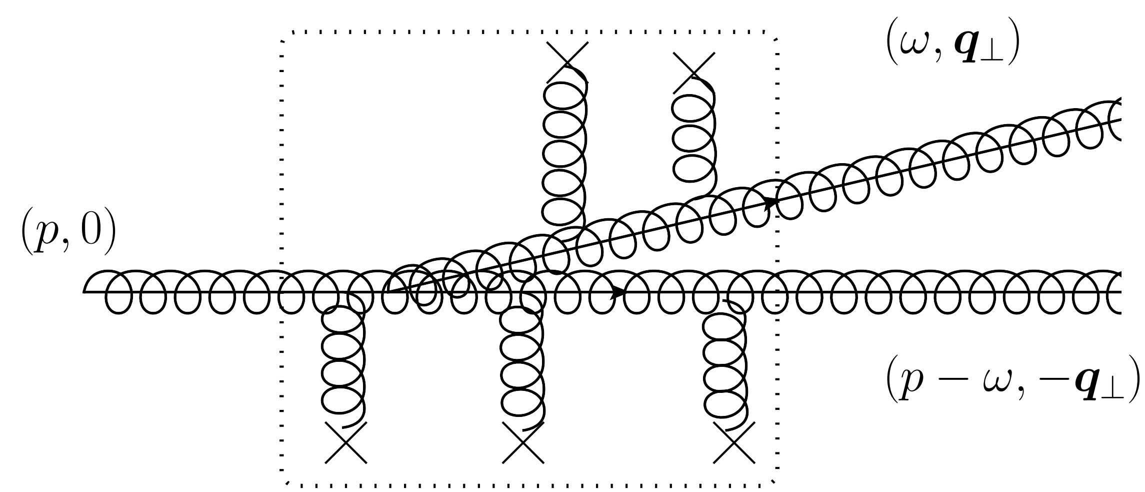
The AMY kinetic theory

- Effective Kinetic Theory (EKT) for the phase space density of quarks and gluons

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

- At **leading order**: elastic, number-preserving 2↔2 processes and collinear, number-changing effective 1↔2 processes AMY (2003)





Landau-Pomeranchuk-Migdal resummation
(Wait for the **next talk**)

Transport coeffs from the EKT

Transport coeffs from the EKT

- To obtain the transport coefficients linearize the theory

$$f(\mathbf{p}) = f_{\mathrm{EQ}}(\mathbf{p}) + \sum_{\ell} \delta f_{\ell}(\mathbf{p})$$

- Source term equates linearized collision operator

$$\mathcal{S}_\ell = \mathcal{C}\delta f_\ell$$

$$\mathcal{S}_\ell \equiv \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \right) f_{\mathrm{EQ}}(\mathbf{p}, u, \beta, \mu)$$

- Since $\langle T^{i \neq j} \rangle \propto \eta$, $\langle \mathbf{J}_q \rangle = -D_q \boldsymbol{\nabla} \langle n_q \rangle$ (light flavor diffusion) η requires ℓ=2, D_q ℓ=1

- Transport coefficients obtained by the kinetic theory definitions of T, J once δf_ℓ has been obtained

Transport coeffs from the EKT

- To obtain the transport coefficients linearize the theory

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- Source term equates linearized collision operator

$$\mathcal{S}_{\ell} = \mathcal{C}\delta f_{\ell}$$

- To solve the linear equation, introduce the inner product

$$(f, g) \equiv \int_{\mathbf{p}} f(\mathbf{p})\, g(\mathbf{p})$$

and minimize

$$(\delta f_{\ell}, \mathcal{S}_{\ell}) - \frac{1}{2}(\delta f_{\ell}, \mathcal{C} f_{\ell})$$

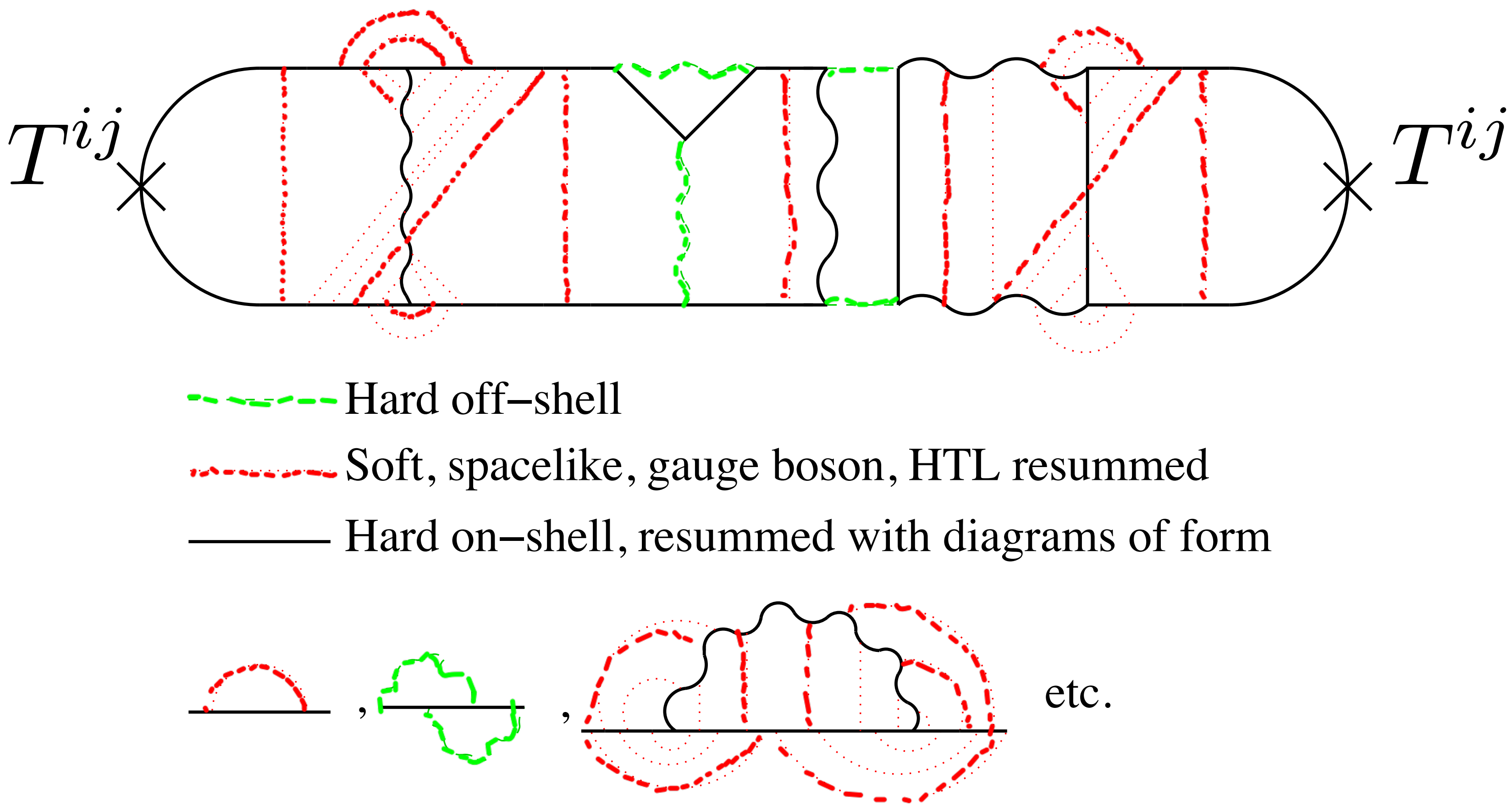
AMY (2000-03)

Transport coeffs from the EKT

- Linearized EKT equivalent to Kubo formula

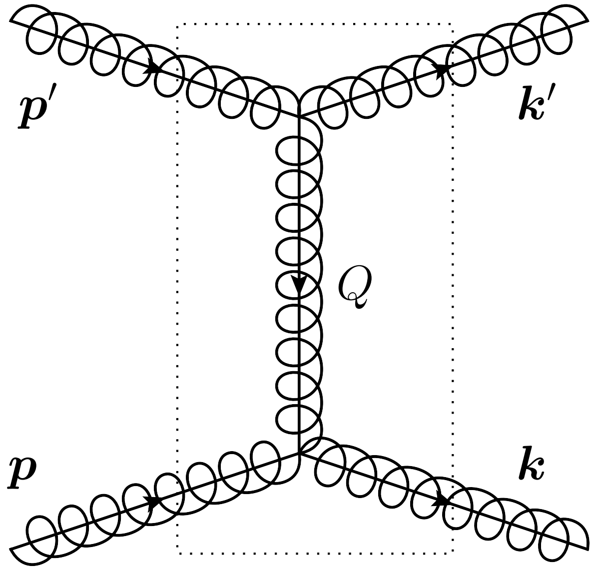
$$\eta = \frac{1}{20} \lim_{\omega \to 0} \frac{1}{\omega} \int d^4x \, e^{i\omega t} \left\langle \left[T^{ij}(t, \mathbf{x}), T^{ij}(0, \mathbf{0}) \right] \right\rangle \theta(t)$$

- Not practical at weak coupling: loop expansion breaks down AMY (2000-2003)



Reorganization

- The NLO corrections come from regions sensitive to soft gluons (no quarks in this illustration)
- Before we get there, let's have a reorganized perspective on these regions at LO
- Look at 2↔2 scattering

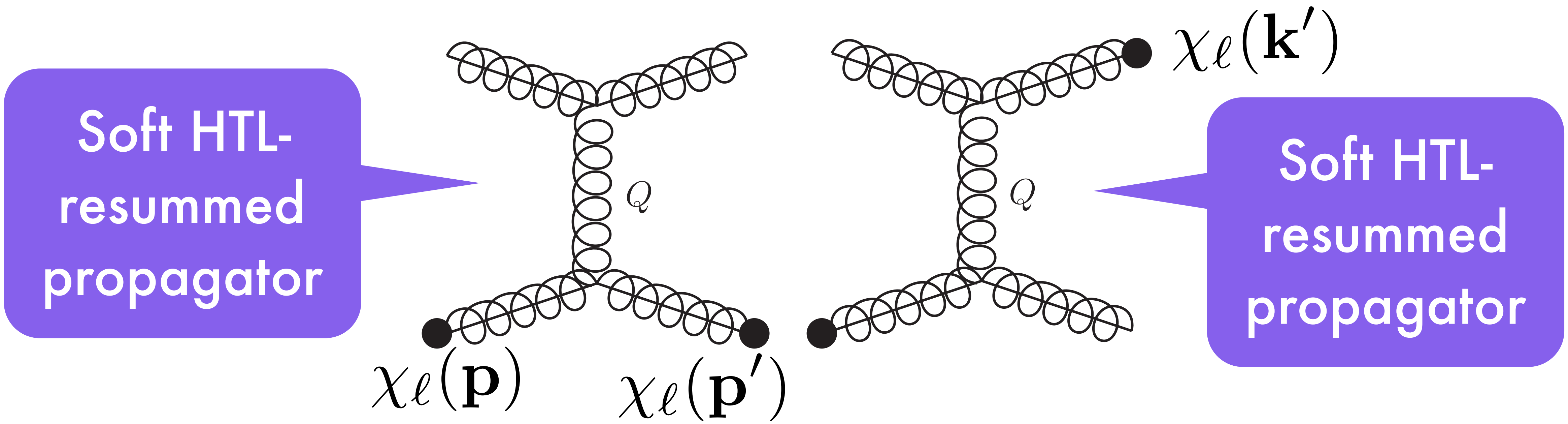


$$\int_{\mathbf{pkp'k'}} \left|\mathcal{M}(\mathbf{p},\mathbf{k};\mathbf{p}',\mathbf{k}')\right|^2 (2\pi)^4\,\delta^{(4)}(P+K-P'-K')$$
$$\times\, f_{\rm EQ}(p)\, f_{\rm EQ}(k)\,[1+f_{\rm EQ}(p')]\,[1+f_{\rm EQ}(k')]$$
$$\times \Big[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\Big]^2$$

$$\delta f_\ell(\mathbf{p}) \equiv f_{\rm EQ}(\mathbf{p})(1+f_{\rm EQ}(\mathbf{p}))\chi_\ell(\mathbf{p})$$

LO soft gluon scattering

- When Q=P'-P becomes soft there are two possibilities for
$$\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2 \qquad (\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$$



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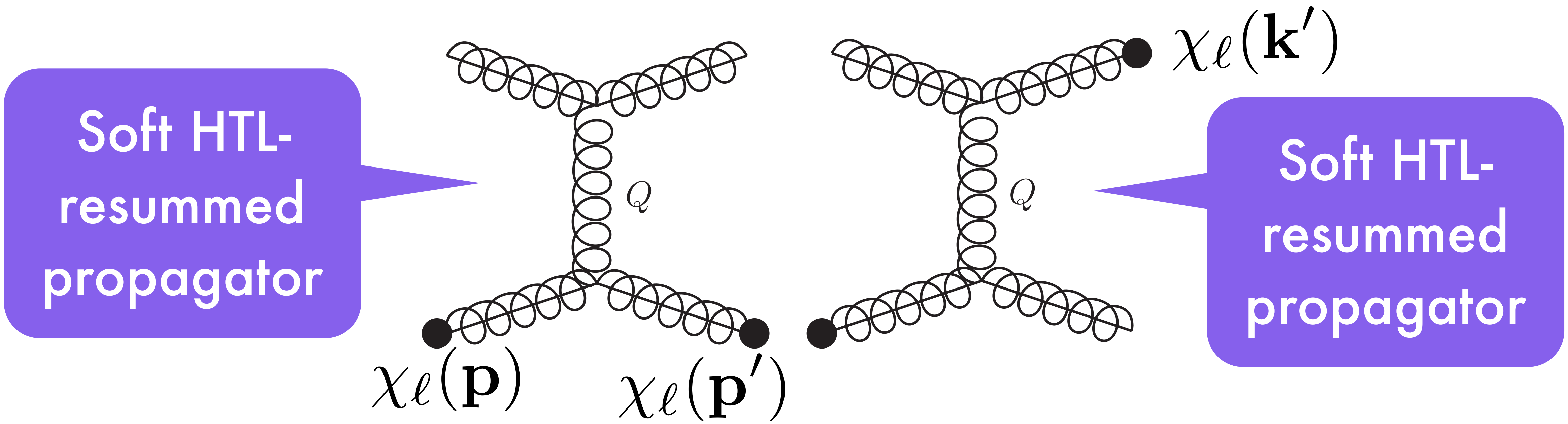


- Left: diffusion terms, **p** and **p'** strongly correlated

$$\left(\chi_\ell(\mathbf{p}) - \chi_\ell(\mathbf{p}')\right)^2 = (\hat{\mathbf{p}}\cdot\mathbf{q})^2[\chi'(p)]^2 + \frac{\ell(\ell+1)}{2}\frac{q^2-(\hat{\mathbf{p}}\cdot\mathbf{q})^2}{p^2}[\chi(p)]^2$$

identify a longitudinal and a transverse momentum broadening contribution, $\hat{q}_L$ and $\hat{q}$

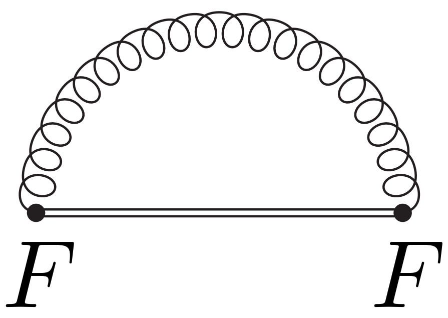
Light-cone techniques

- Key advancement over the past decade: analytical properties of soft thermal amplitudes at light-like separations
  Caron-Huot **PRD82** (2008) *Review*: JG Teaney **QGP5** (2015)

- In a nutshell, retarded functions are analytical in the upper half plane in **any** time-like variable. In the soft sector to NLO also for **light-like variables** ($q^+ = (q^0 + q^z)/2$).

$$\hat{q}(\mu_\perp) = g^2 C_A \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}_{\perp} \rangle_{q^-=0}$$

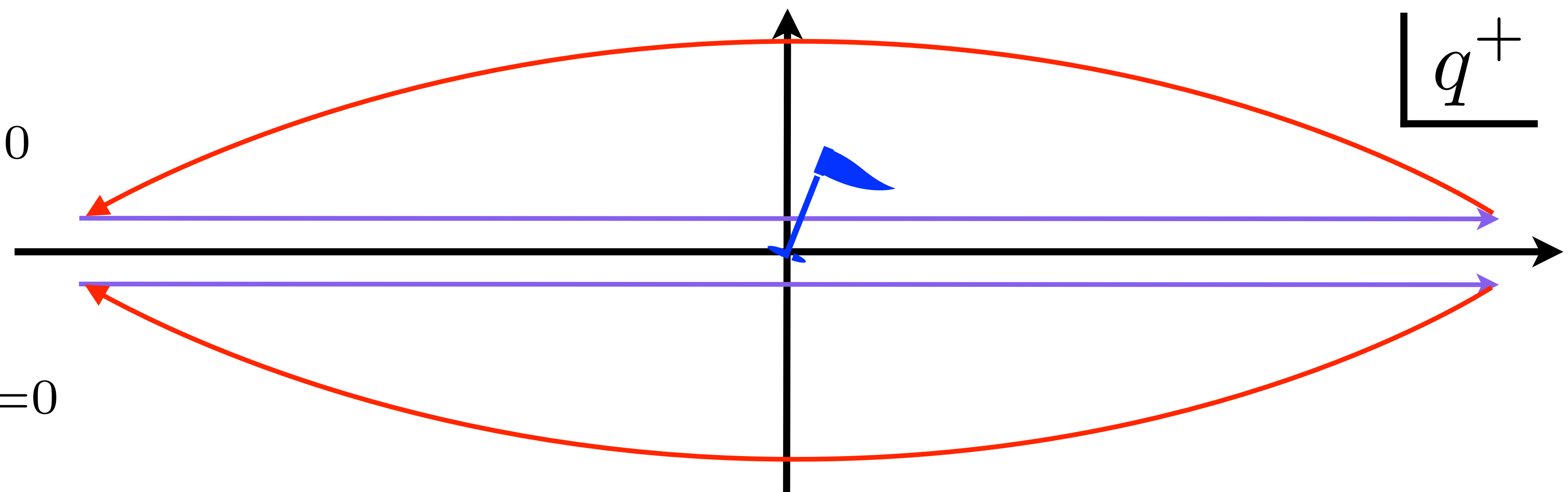
$$\hat{q}_L(\mu_\perp) = g^2 C_A \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-+}(Q) F^{-+} \rangle_{q^-=0}$$



Light-cone techniques

$$\int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}_{\perp} \rangle_{q^-=0}$$

$$\int \frac{dq^+}{2\pi} \langle F^{-+}(Q) F^{-+} \rangle_{q^-=0}$$



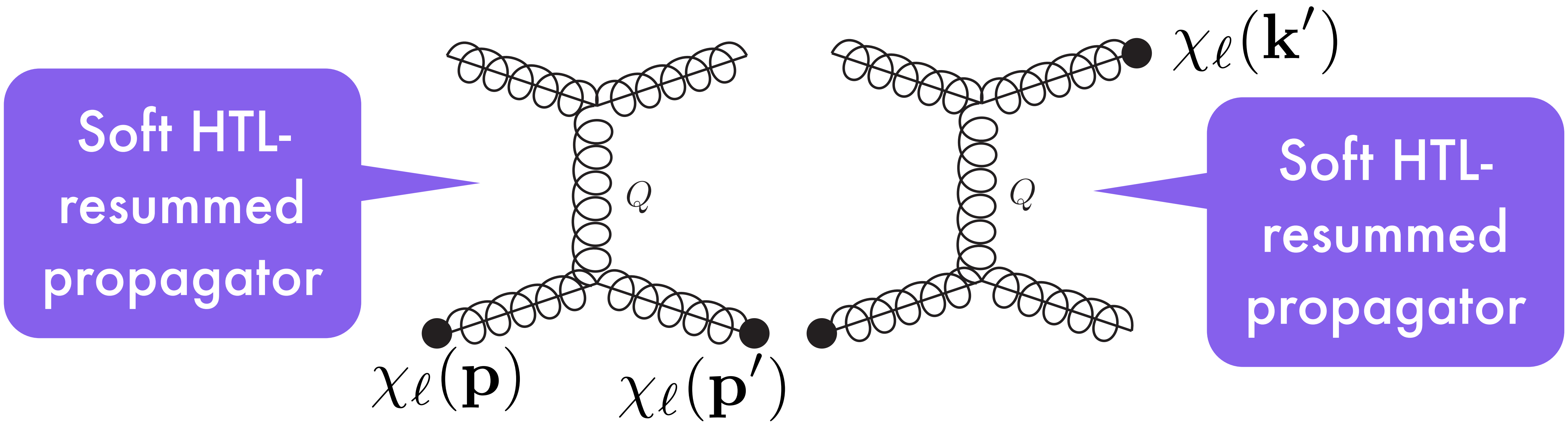
$$\langle F^{-\mu}(Q) F^{-\nu} \rangle_{q^-=0} \propto \frac{T}{q^+} \left(G_R^{-\mu;-\nu}(Q) - G_A^{-\mu;-\nu}(Q) \right)$$

- Only non-analytic feature in each half-plane: **Matsubara zero mode** (q^+=0, all other frequencies far away)
  ⇒ Transverse function (⊥⊥) becomes **Euclidean (EQCD)**

- The longitudinal function (++) does not feel the pole, but it does not vanish fast enough on the **arcs at large** $|q^+|$
  ⇒ Only sensitive to the **light-cone dispersion relation**

LO soft gluon scattering

- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p}) + \chi_\ell(\mathbf{k}) - \chi_\ell(\mathbf{p}') - \chi_\ell(\mathbf{k}')\right]^2$ $\quad (\chi_\ell(\mathbf{p}) = f_\ell(\hat{\mathbf{p}})\chi(p))$



- **Diffusion terms:** summing up, light-cone techniques

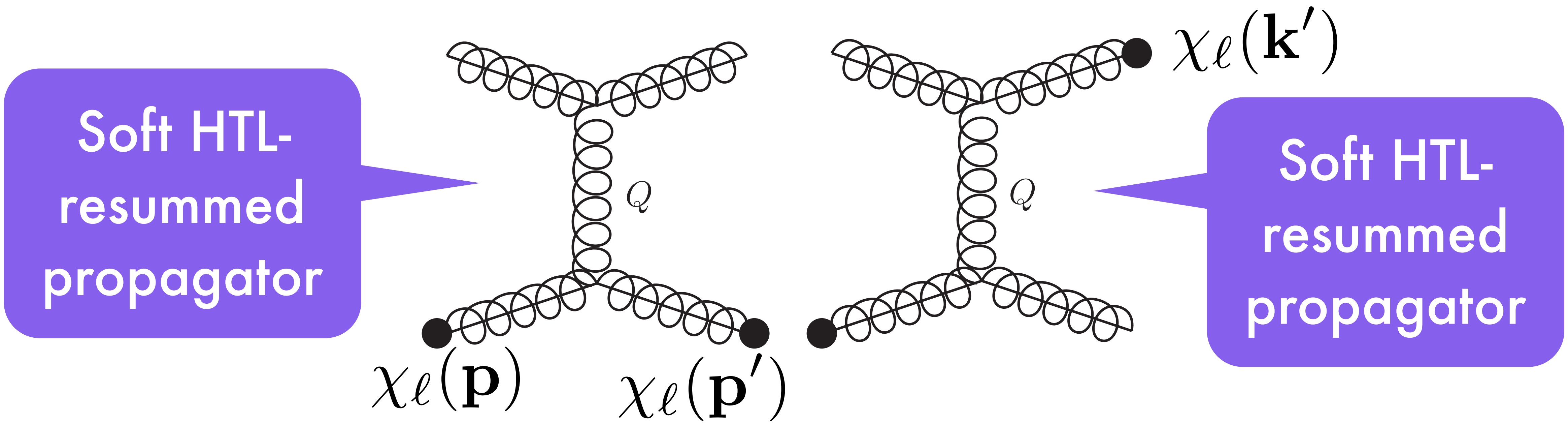
$$\hat{q}_L^a\Big|_{\mathrm{soft}} = \frac{g^2 C_{R_a} T m_D^2}{4\pi} \ln \frac{\sqrt{2}\mu_\perp}{m_D} \qquad \hat{q}^a\Big|_{\mathrm{soft}} = \frac{g^2 C_{R_a} T m_D^2}{2\pi} \ln \frac{\mu_\perp}{m_D}$$

give rise to the leading log contribution

Caron-Huot **PRD82** (2008) JG Moore Teaney **JHEP1603** (2015)

LO soft gluon scattering

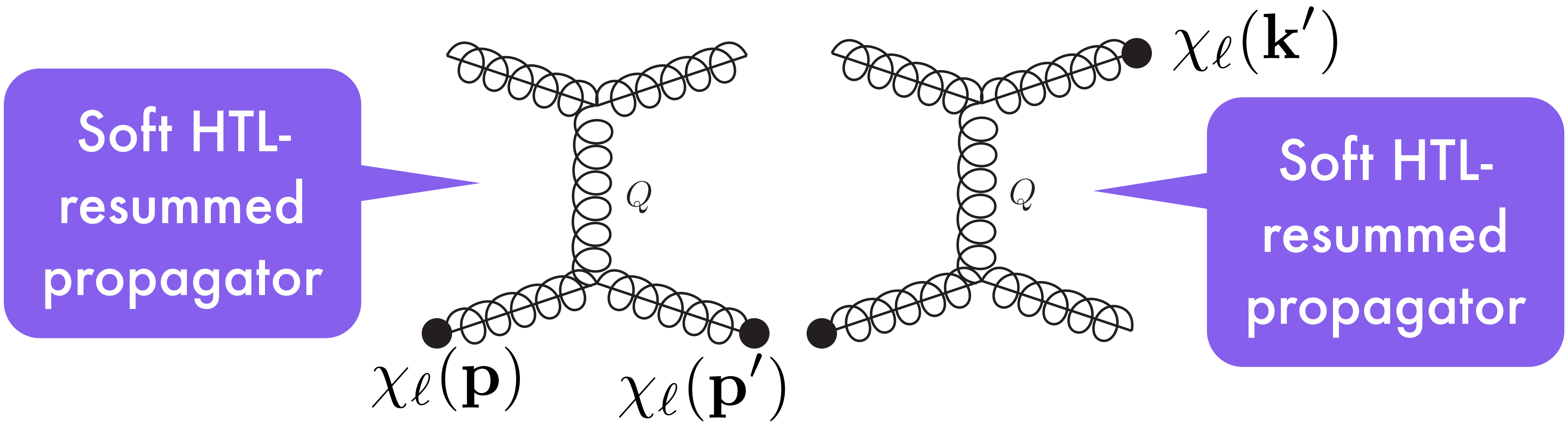
- When Q=P'-P becomes soft there are two possibilities for $\Big[\chi_\ell(\mathbf{p}) + \chi_\ell(\mathbf{k}) - \chi_\ell(\mathbf{p}') - \chi_\ell(\mathbf{k}')\Big]^2$ $\quad (\chi_\ell(\mathbf{p}) = f_\ell(\hat{\mathbf{p}})\chi(p))$



- Right: cross terms, **p**,**p′** and **k**,**k′** not correlated. Two-point function of two uncorrelated deviations from equilibrium
  (diffusion was the response of an off-eq leg to the equilibrium bath)

LO soft gluon scattering

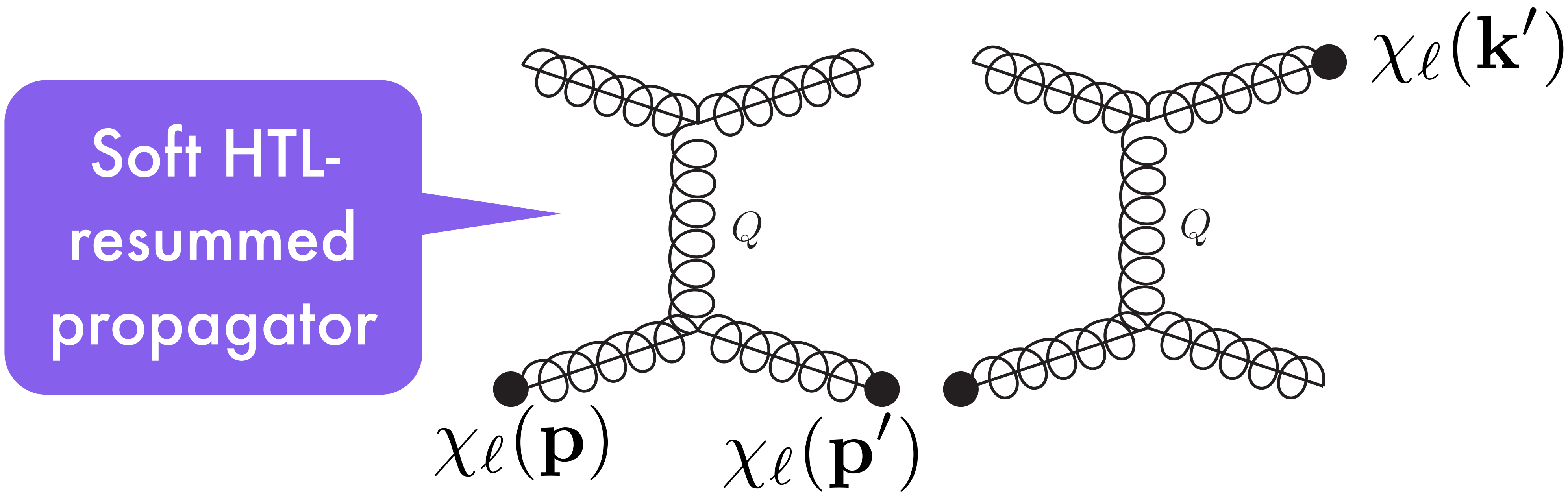
- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p}) + \chi_\ell(\mathbf{k}) - \chi_\ell(\mathbf{p}') - \chi_\ell(\mathbf{k}')\right]^2$ $\quad (\chi_\ell(\mathbf{p}) = f_\ell(\hat{\mathbf{p}})\chi(p))$



- Right: cross terms, **p**,**p'** and **k**,**k'** not correlated. Light-cone techniques not applicable, have to use numerical integration. Easy at LO, where they are finite (no leading log contribution)

LO soft gluon scattering

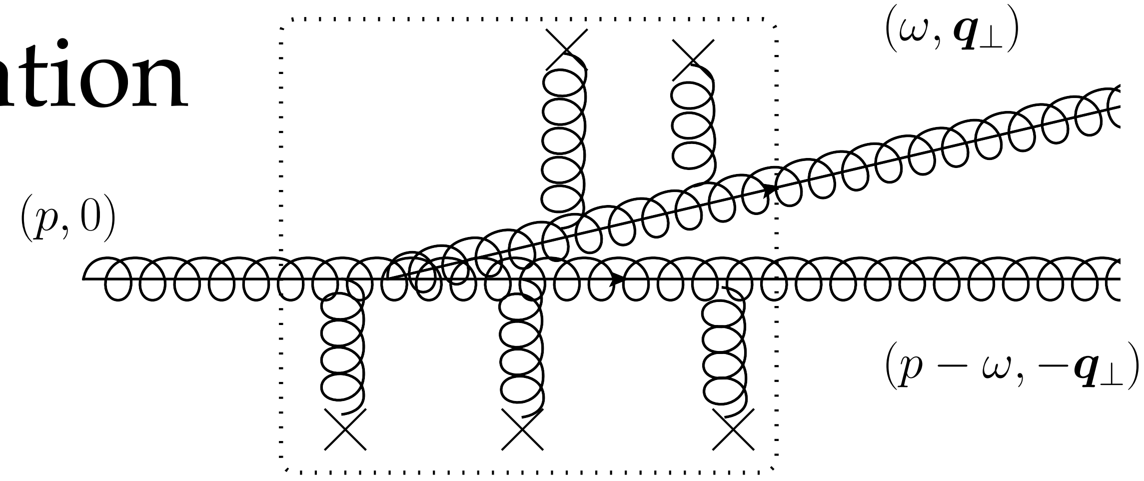
- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p}) + \chi_\ell(\mathbf{k}) - \chi_\ell(\mathbf{p}') - \chi_\ell(\mathbf{k}')\right]^2$ $\quad (\chi_\ell(\mathbf{p}) = f_\ell(\hat{\mathbf{p}})\chi(p))$



- Right: cross terms, **p**,**p′** and **k**,**k′** not correlated. The original Boltzmann equation becomes a Fokker-Planck equation for soft scattering. The diffusion part is there a loss term and these terms are gain terms

Reorganization

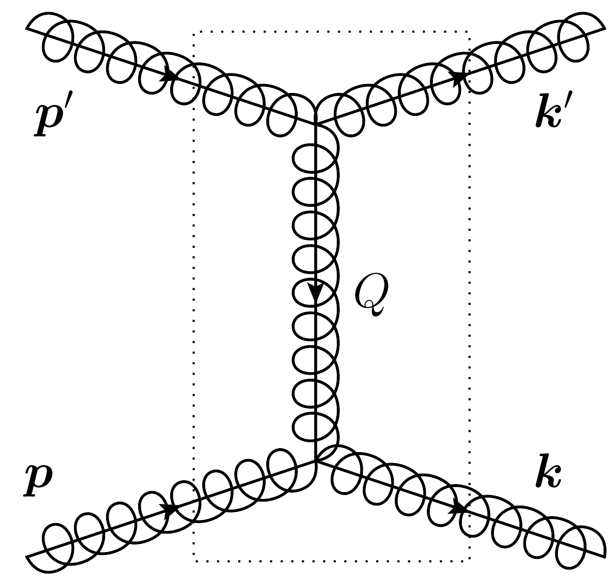
- 1↔2 processes: strictly collinear kinematics, ~unaffected by reorganization



- Reorganization of the LO collision operator

$$\int_{\mathbf{p}} \delta f_\ell(\mathbf{p}) \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \right) f_{\mathrm{EQ}}(\mathbf{p}, u, \beta, \mu) = \int_{\mathbf{p}} \delta f_\ell(\mathbf{p}) \left[C^{\mathrm{large}}[\mu_\perp] + C^{\mathrm{diff}}[\mu_\perp] + C^{\mathrm{cross}} + C^{\mathrm{coll}} \right]$$

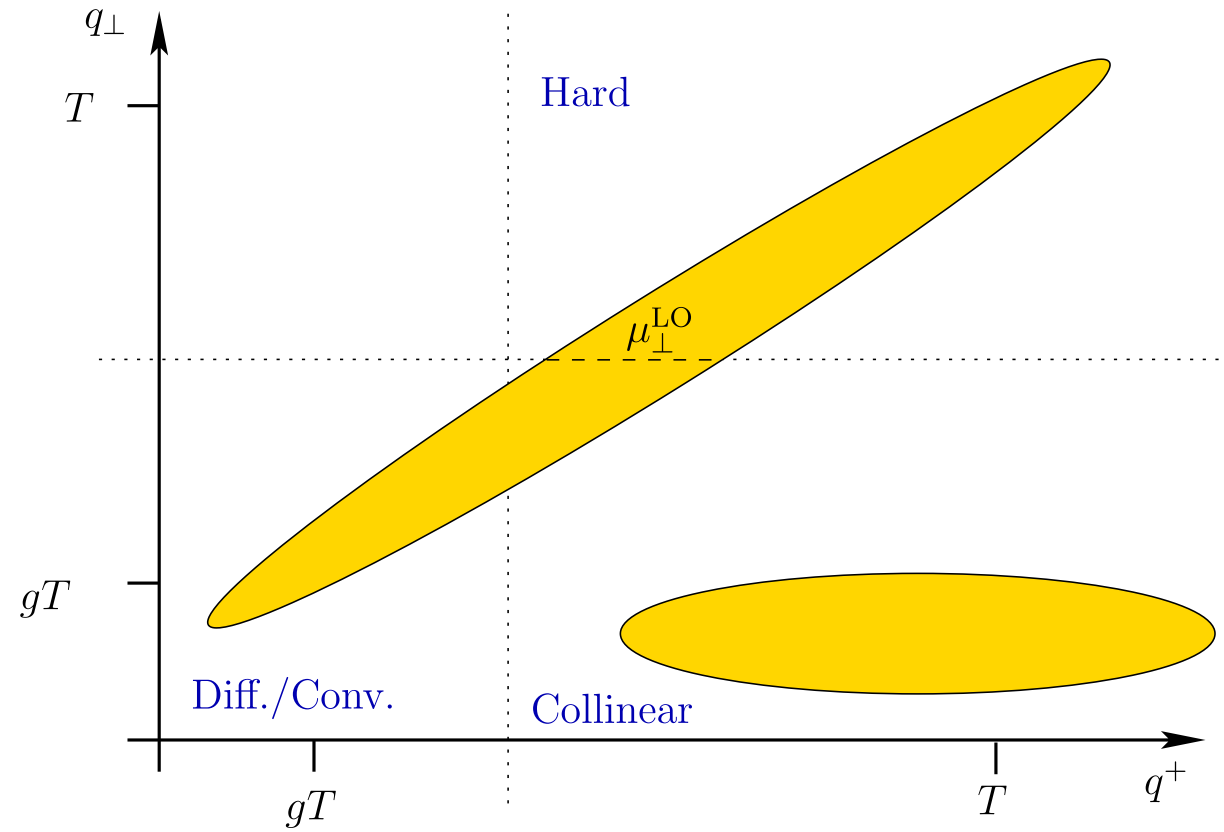
- Final ingredient: 2↔2 large angle scatterings, IR-regulated to avoid the soft region

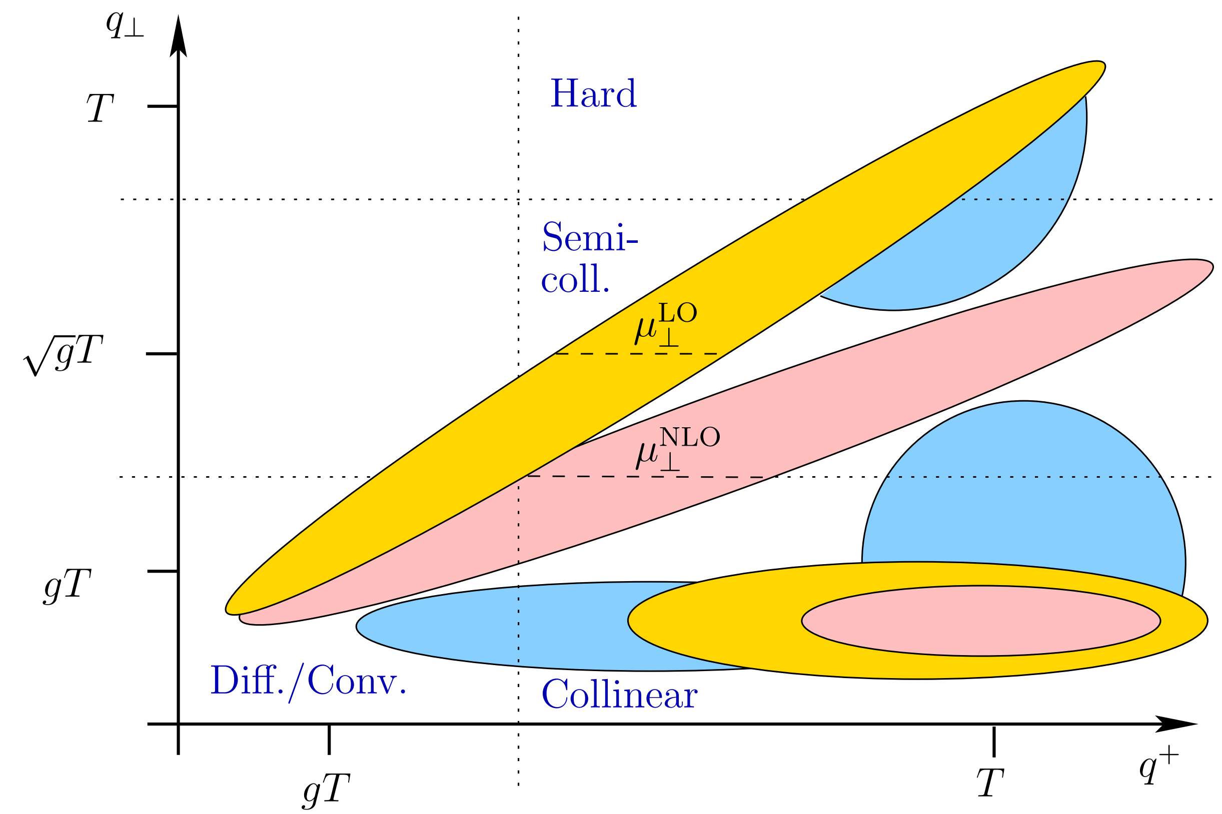


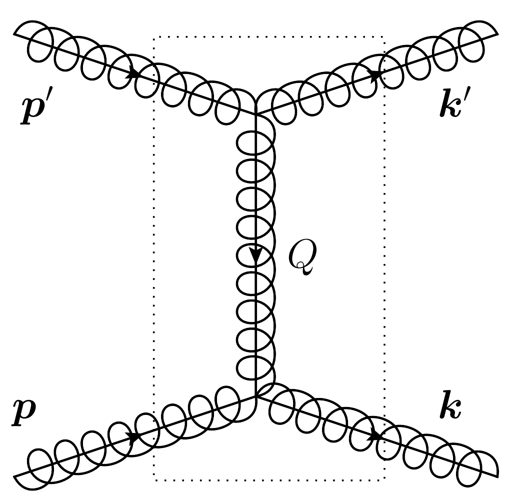
Going to NLO

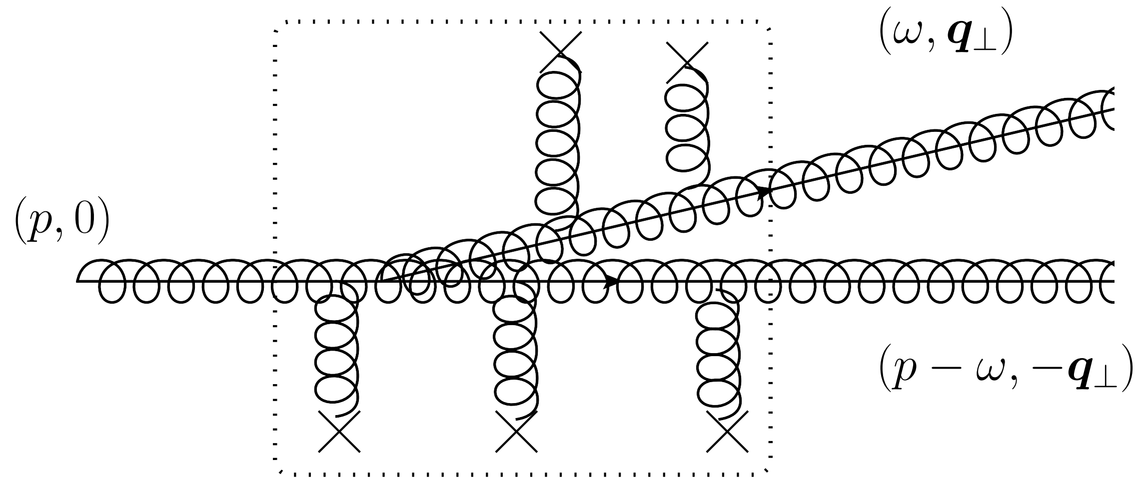
- The diffusion, cross and collinear terms receive $O(g)$ corrections
- There is a new semi-collinear region

Going to NLO


$q_\perp$
T
Hard
$\mu_\perp^{\rm LO}$
gT
Diff./Conv.
Collinear
gT
T
q^+


$q_\perp$
T
Hard
Semi-
coll.
$\mu_\perp^{\rm LO}$
$\sqrt{g}T$
$\mu_\perp^{\rm NLO}$
gT
Diff./Conv.
Collinear
gT
T
q^+

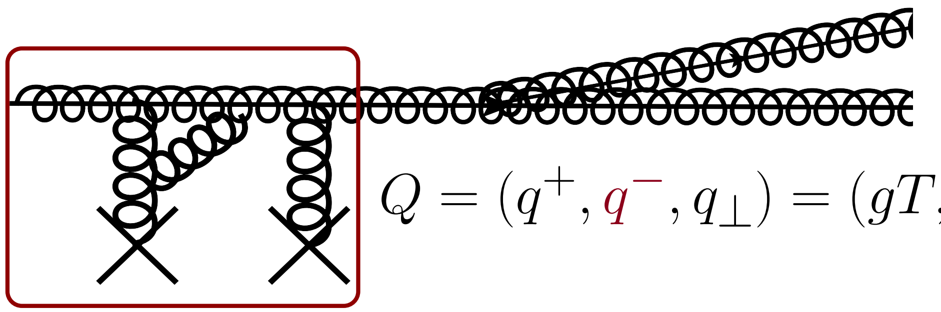

$\boldsymbol{p}'$
$\boldsymbol{k}'$
Q
$\boldsymbol{p}$
$\boldsymbol{k}$


$(\omega, \boldsymbol{q}_\perp)$
$(p, 0)$
$(p-\omega, -\boldsymbol{q}_\perp)$

Collinear corrections

- The differential eq. for LPM resummation gets correction from NLO *C($q_\perp$)* and from the thermal asymptotic mass at NLO (Caron-Huot 2009)

$$\frac{d\Gamma}{d^2q_\perp} = \mathcal{C}_{\rm LO}(q_\perp) = \frac{g^2 C_A T m_D^2}{q_\perp^2(q_\perp^2+m_D^2)}$$

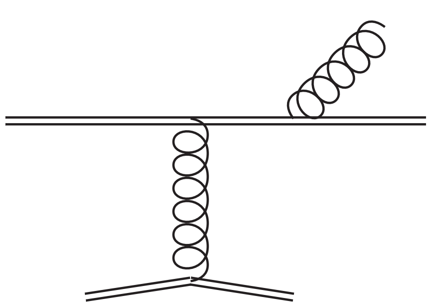


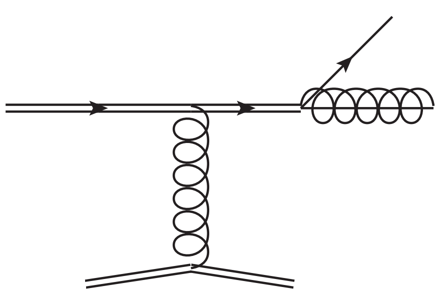
$$Q = (q^+, q^-, q_\perp) = (gT,\, g^2T,\, gT)$$

$\mathcal{C}_{\rm NLO}(q_\perp)$ complicated but analytical (Euclidean tech)

Caron-Huot **PRD79** (2009), Lattice: Panero *et al.* (2013)

- Regions of overlap with the diffusion and semi-collinear regions need to be subtracted



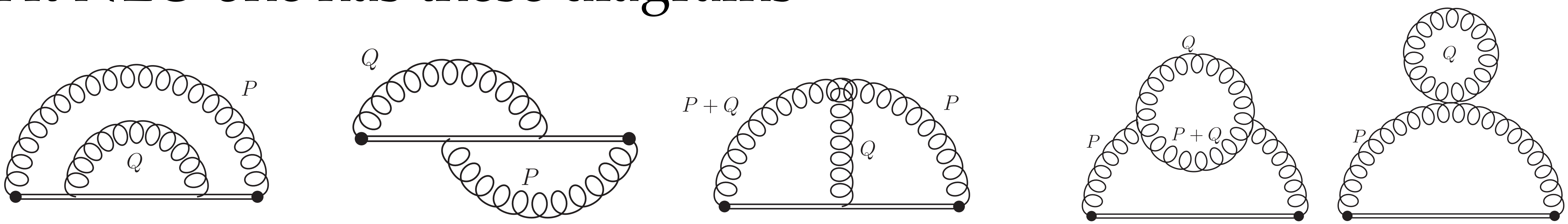


NLO diffusion and cross

- At NLO one needs to understand diffusion and cross/gain with either extra soft gluons in intermediate legs or with external soft scatterers
- For diffusion: application of light-cone techniques still possible, huge simplification and closed-form results

Diffusion corrections

- At NLO one has these diagrams



- For transverse: Euclidean calculation Caron-Huot **PRD79** (2009)

$$\hat{q}_{\mathrm{NLO}} = \hat{q}_{\mathrm{LO}} + \frac{g^4 C_A^2 T^3}{32\pi^2} \frac{m_D}{T} \left(3\pi^2 + 10 - 4\ln 2\right)$$

- For longitudinal:

$$\hat{q}_L(\mu_\perp)_{\mathrm{LO}} = g^2 C_A T \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_\infty^2}{q_\perp^2 + m_\infty^2}$$

$$\hat{q}_L(\mu_\perp)_{\mathrm{NLO}} = g^2 C_A T \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_\infty^2 + \delta m_\infty^2}{q_\perp^2 + m_\infty^2 + \delta m_\infty^2} \approx g^2 C_A T \int \frac{d^2 q_\perp}{(2\pi)^2} \left[\frac{m_\infty^2}{q_\perp^2 + m_\infty^2} + \frac{q_\perp^2 \, \delta m_\infty^2}{(q_\perp^2 + m_\infty^2)^2} \right]$$

after collinear subtraction light-cone sum rule still sees only dispersion relation ($O(g)$ correction). NLO still UV-log sensitive

NLO diffusion and cross

- For cross/gain: no diffusion picture = no "easy" light-cone sum rules, only way would be bruteforce HTL (and understanding how to deal with soft legs in kinetic theory).

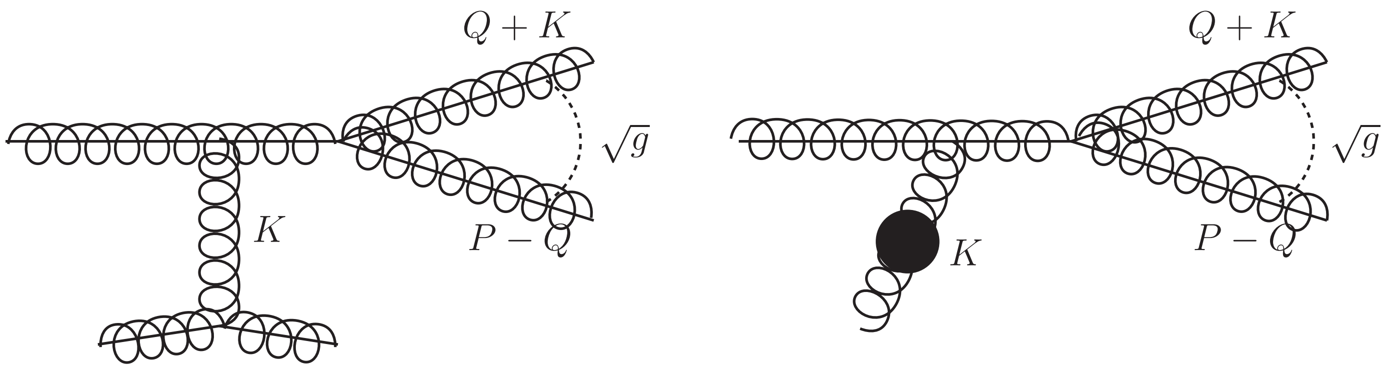
- **Missing**, but silver lining: they're finite, so just estimate the number and vary it

- NLO test ansatz: LO cross x m_D/T(~g) x arbitrary constant that we vary

$$C_{\rm NLO}^{\rm cross} = C_{\rm LO}^{\rm cross} \times \frac{m_D}{T} \times c_{\rm cross}$$

Semi-collinear processes

- Seemingly different processes boiling down to wider-angle radiation



K soft cut, spacelike

K soft plasmon, timelike

- Evaluation: introduce "*modified* $\hat{q}$" tracking the changes in the small light-cone component p^- of the gluons. Can be evaluated in EQCD

"standard"
$$\hat{q} = g^2 C_A \int \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}{}_{\perp} \rangle_{q^-=0}$$

"modified"
$$\hat{q}(\delta E) = g^2 C_A \int \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}{}_{\perp} \rangle_{q^-=\delta E}$$

- Rate ∝ "*modified* $\hat{q}$" x DGLAP splitting. IR log divergence makes collision operator finite at NLO

Results

JG Ghiglieri Moore **JHEP1803** (2018)

Results (and their fine print)

- Inversion of the collision operator using variational Ansatz
- At NLO just add $O(g)$ corrections to the LO collision operator, do not treat them as perturbations in the inversion

$$\mathcal{S}_\ell = [\mathcal{C} + \delta\mathcal{C}]\delta f_\ell \implies \delta f_\ell = \frac{1}{\mathcal{C} + \delta\mathcal{C}}\mathcal{S}_\ell$$

- First perturbative corrections to s are of order g^2. Including them would be inconsistent with the treatment of the collision operator
- New implementation for semi-collinear processes. Better behaved as g grows due to resummations of N^nLO terms ($n \geq 2$)

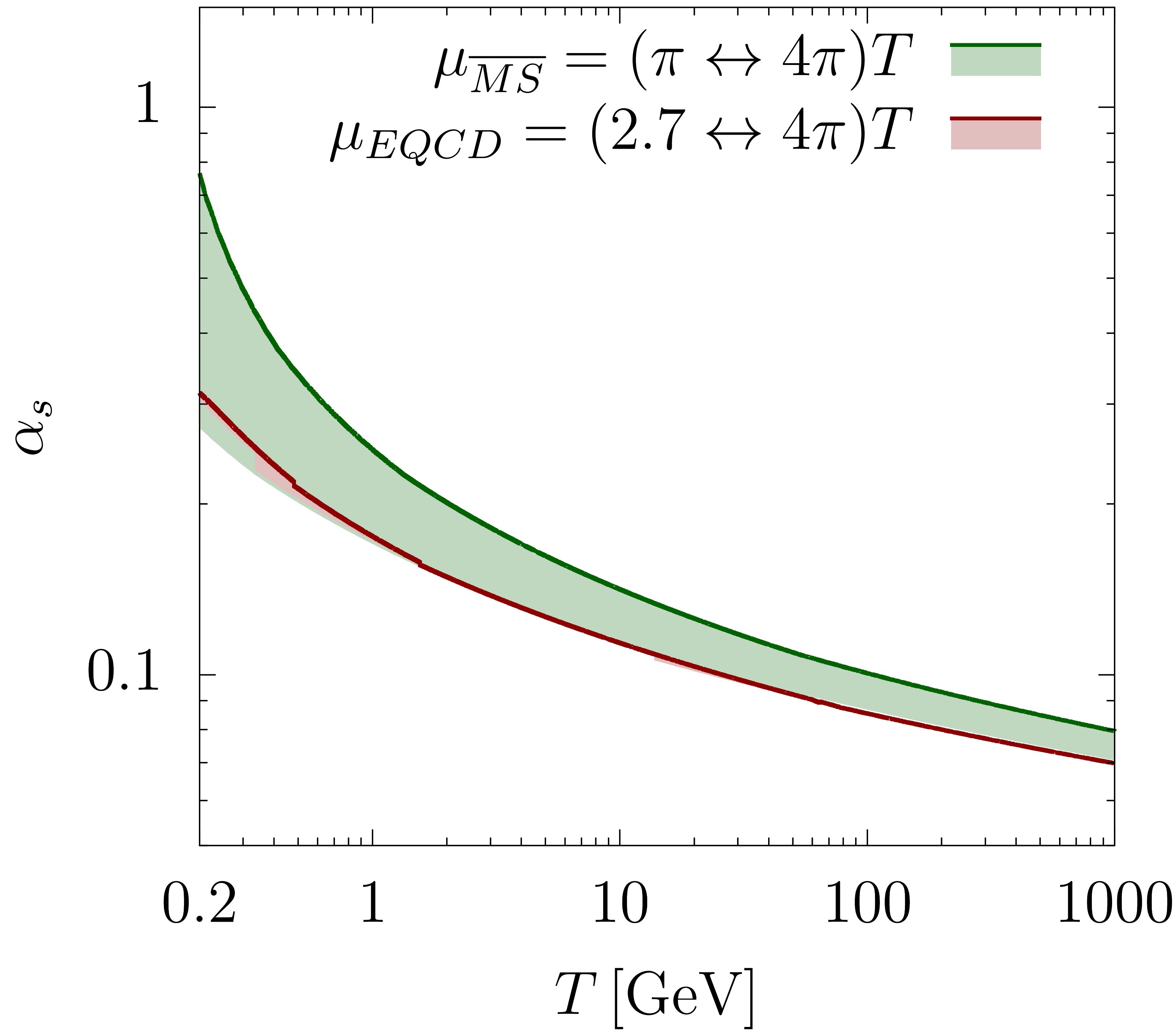
JG Ghiglieri Moore **JHEP1803** (2018)

Results (and their fine print) (and their fine print)

- We want to plot $\eta(T)$ and need $g(T)$. But kinetic theory with massless quarks still conformal to NLO: no vacuum UV divergences and no guidance from the calculation on how to set the scale
- Relate parameter $m_D/T \sim g$ to temperature through
  - Two-loop EQCD $g(T)$ as in Laine Schröder **JHEP0503** (2005)
  - Simple two-loop MSbar with various μ/T
- Extra degree of arbitrariness in the relation of quark mass thresholds with T

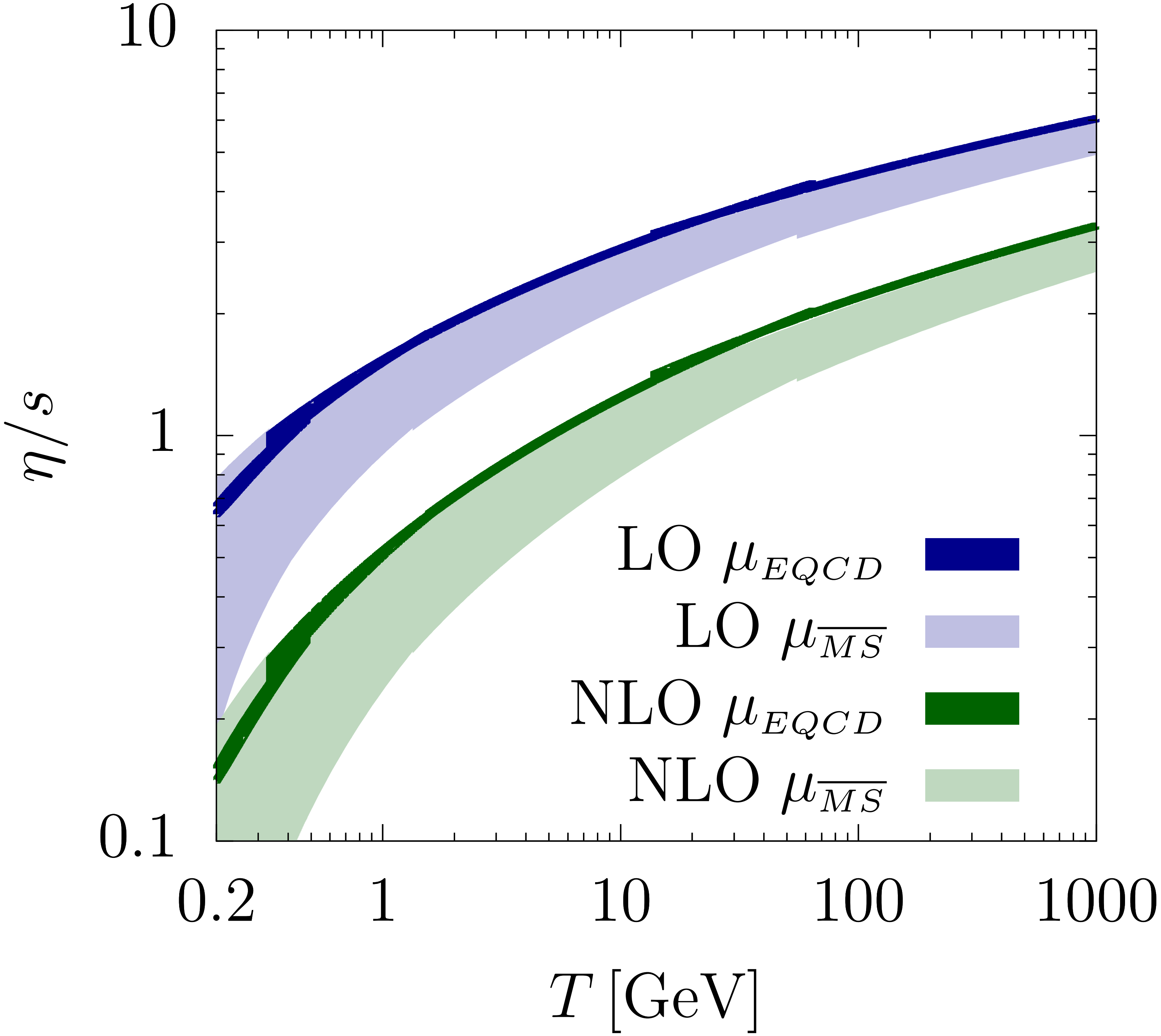
JG Ghiglieri Moore **JHEP1803** (2018)

Results



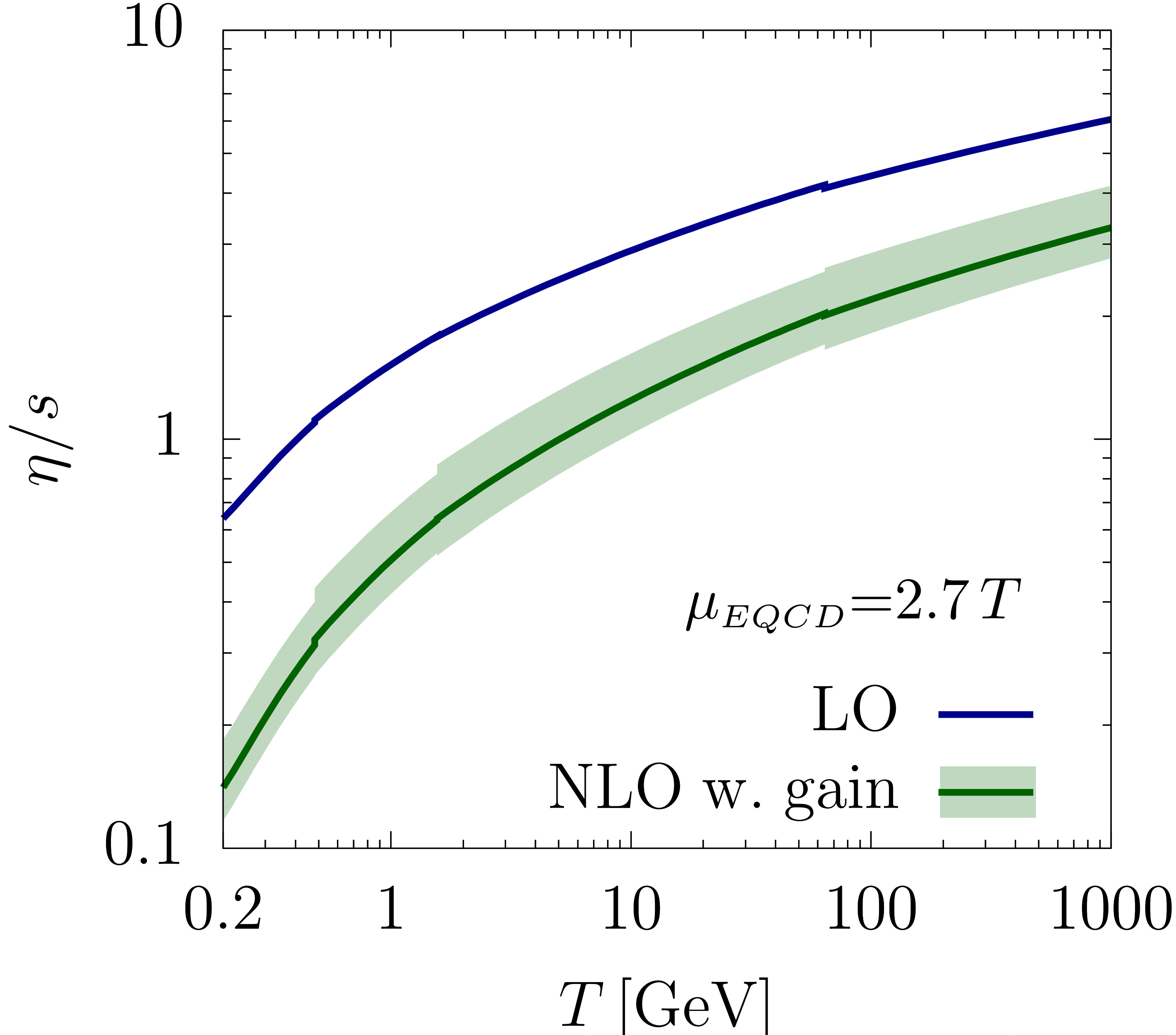
$\mu_{\overline{MS}} = (\pi \leftrightarrow 4\pi)T$
$\mu_{EQCD} = (2.7 \leftrightarrow 4\pi)T$
1
0.1
α_s
0.2
1
10
100
1000
$T\,[\mathrm{GeV}]$

$\eta/s(T)$ of QCD



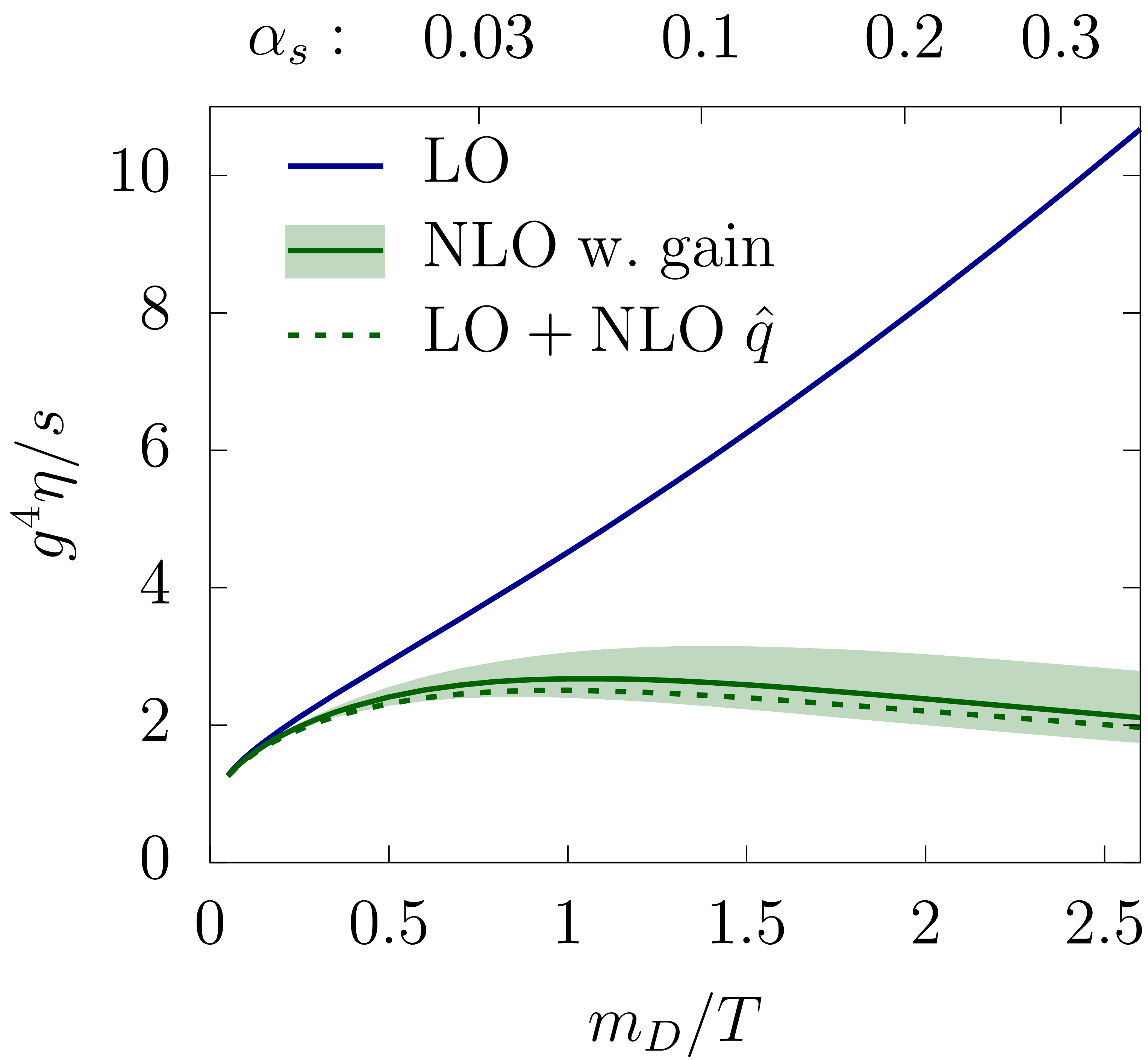
- (almost) NLO has larger effect than running (NNLO)

η/s(T) of QCD



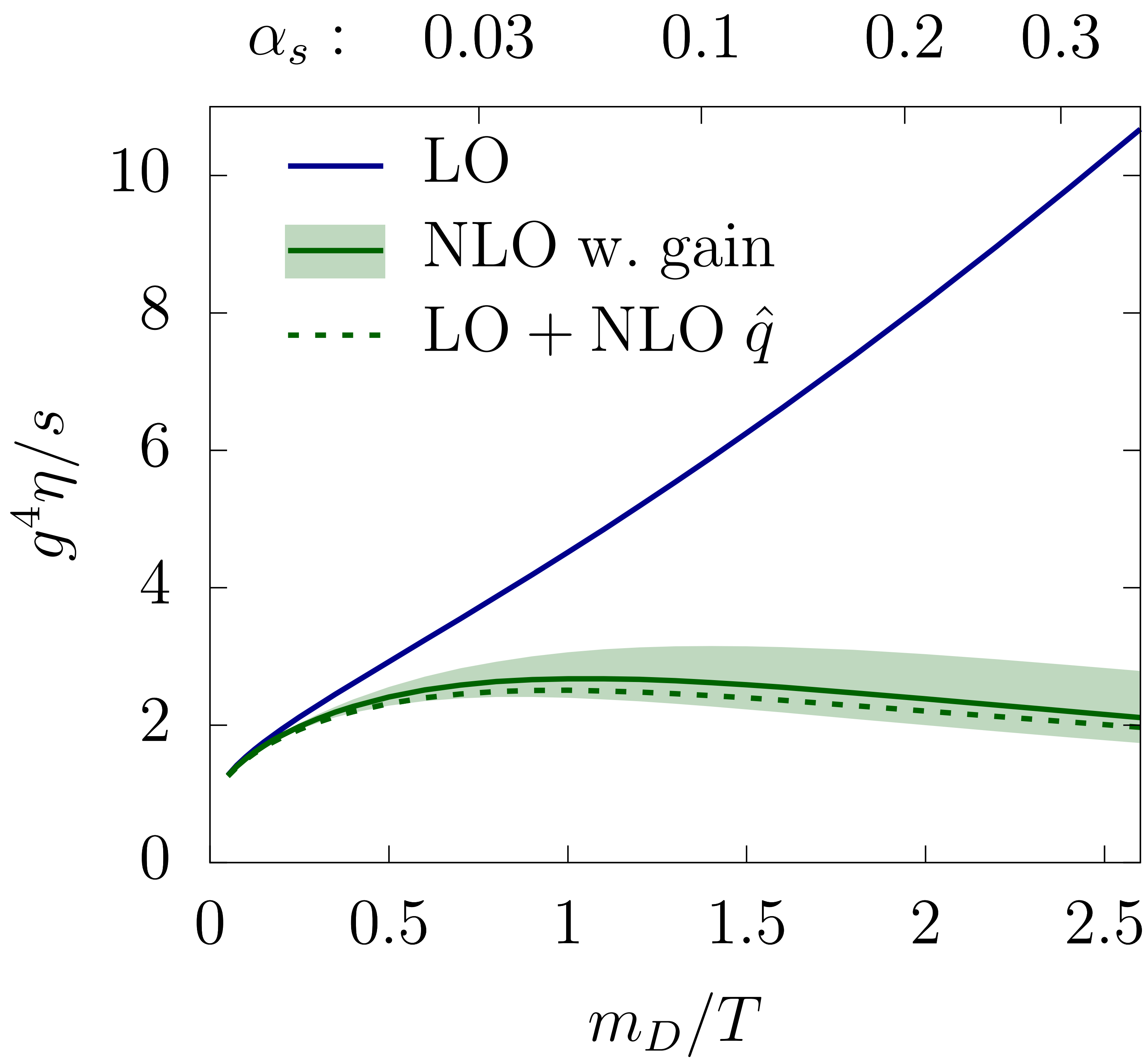
- Cross / gain ansatz (-2<C_{cross}<2) introduces $O(\pm 30\%)$ uncertainty

η/s convergence



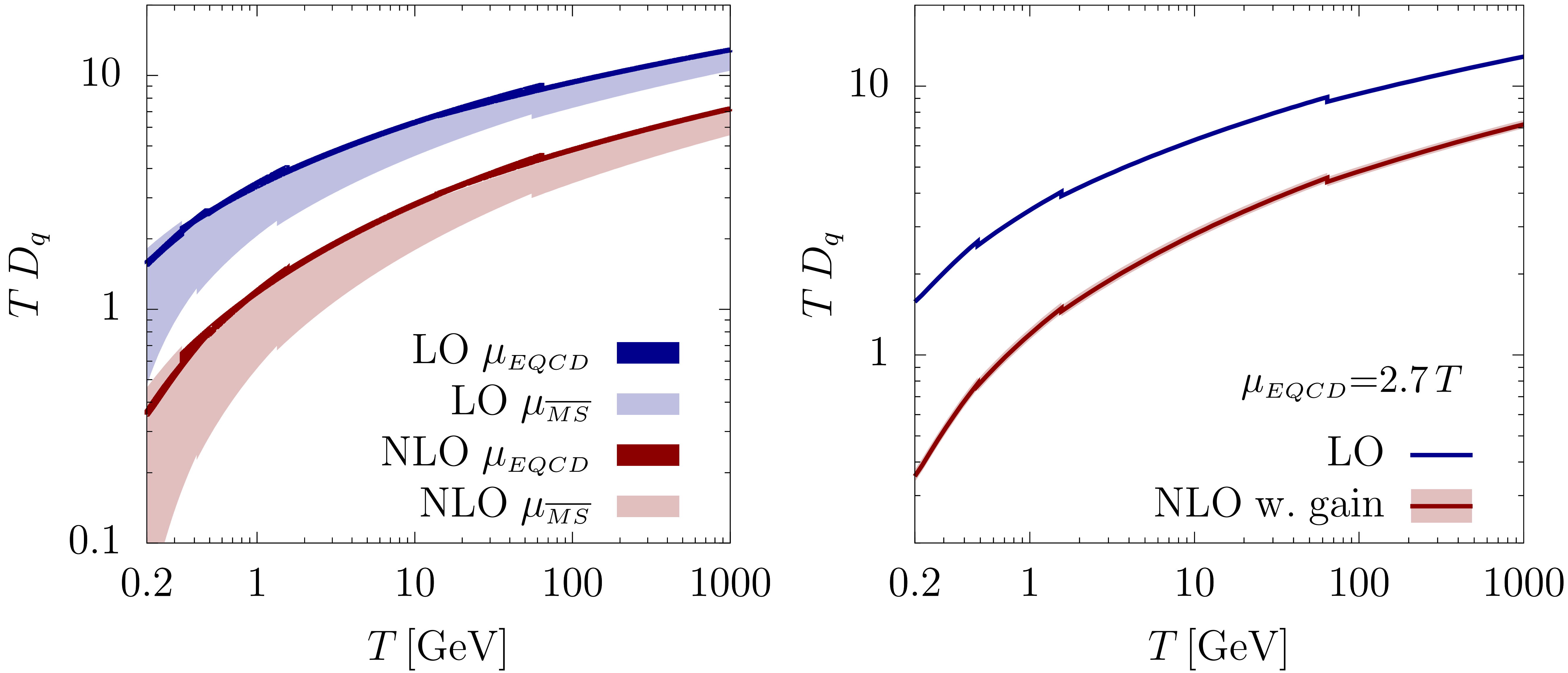
- Convergence realized at m_D~$0.5T$

η/s convergence



- The ~entirety of the downward shift comes from NLO $\hat{q}$

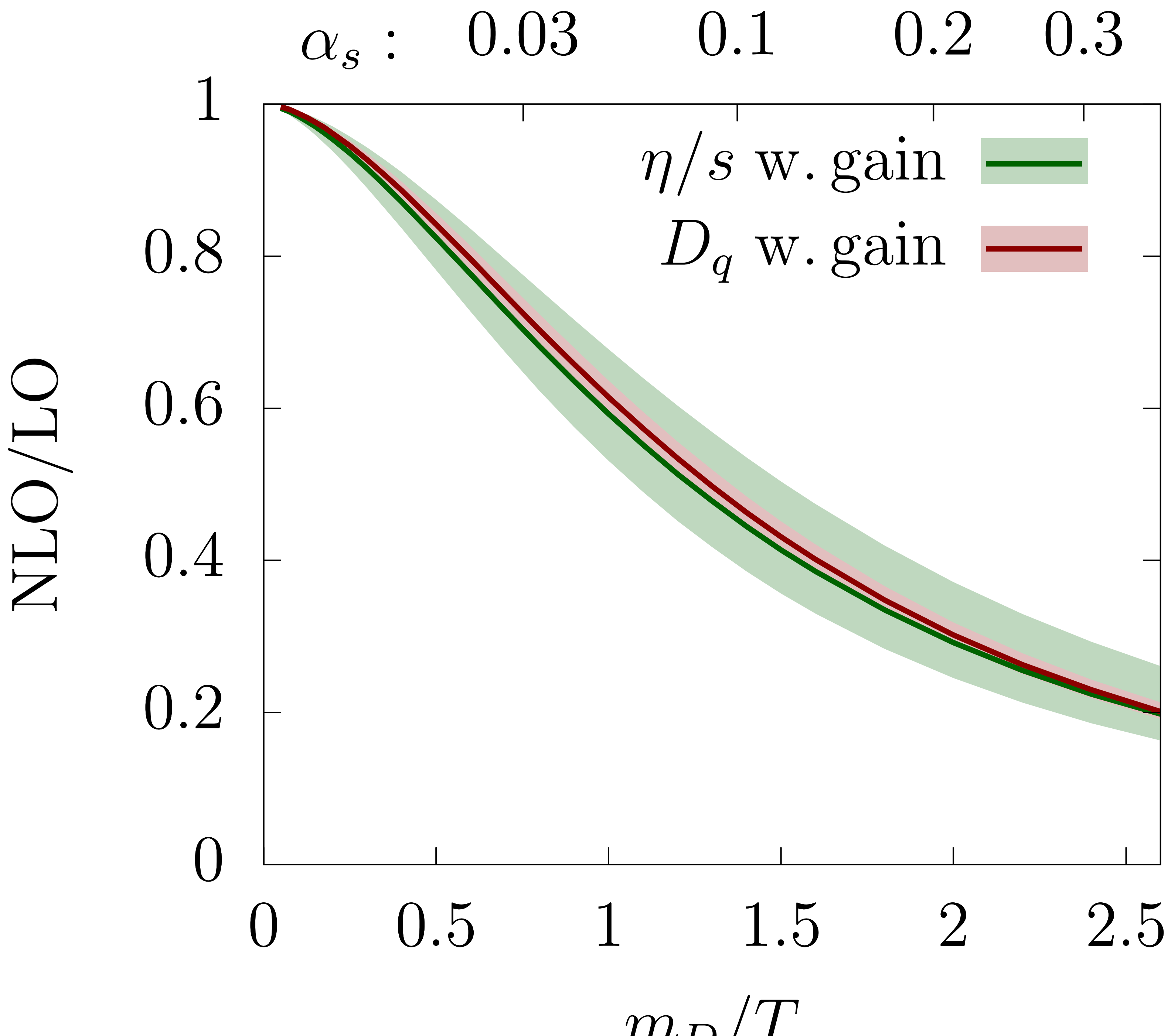
$D_q T(T)$ of QCD



$$\langle \mathbf{J}_q \rangle = -D_q \boldsymbol{\nabla} \langle n_q \rangle$$

- Cross ansatz uncertainty much smaller (soft quarks here)

Ratios



- NLO $\hat{q}$ domination makes ratios similar

Second-order relaxation

- To be causal and stable, second order in the gradients is required. There, lots of work on writing down/computing all the new transport coefficients that pop up
  Baier *et al.* **JHEP0804** (2007), Denicol *et al.* **PRD85** (2012), Battacharyya *et al.* **JHEP0802** (2007)

- We look at the second-order relaxation τ_π of the shear stress tensor to its Navier-Stokes form

$$\tau_\pi \partial_t \pi^{ij} = \pi_1^{ij} - \pi^{ij}$$

- Similarly for a flavor current $\tau_j \partial_t \boldsymbol{j} = \boldsymbol{j}_1 - \boldsymbol{j}$
  recalling that $\langle \mathbf{J}_q \rangle = -D_q \boldsymbol{\nabla} \langle n_q \rangle$

- Hydro practitioners usually fix $2^{\rm nd}$-order coeffs through the ratio to the $1^{\rm st}$-order ones. What can we say about that?

Second-order relaxation

- In the kinetic theory, second-order coefficients require second-order expansion of *f*. τ_π obtained from first-order δf acting as source term

- From the properties of the collision operator and inner product τ_π is given by the square of the first-order departure

$$\eta\tau_\pi = \frac{1}{15T}(\chi,\chi)$$

Moore York **PRD79** (2009) (I have slightly redefined the inner product)

- Two consequences

Can obtain (a)NLO results from the same setup

Can think for one more second about this inner product

Second-order relaxation

- The shear viscosity and enthalpy can be written as
$$\eta\tau_\pi = \frac{1}{15T}(\chi,\chi) \qquad \eta = \frac{1}{15}(\chi,1) \qquad e+p = \frac{T}{3}(1,1)$$
- Recall that $\mathcal{S}_\ell = \mathcal{C}\delta f_\ell \qquad \delta f_\ell(\mathbf{p}) \equiv f_{\rm EQ}(\mathbf{p})(1+f_{\rm EQ}(\mathbf{p}))\chi_\ell(\mathbf{p})$
- The linearized collision operator is symmetric wrt the inner product and is positive-definite (in the ℓ=1,2 channels)
- We have all the spectral ingredients for a triangular inequality
$$\frac{\tau_\pi}{\eta/(e+p)} = 5\frac{(\chi,\chi)\,(1,1)}{(\chi,1)^2} \geq 5$$
- Generic bound in **any kinetic theory**, as long as enthalpy / charge susceptibility determined consistently within it

Second-order relaxation

- We have all the spectral ingredients for a triangular inequality

$$\frac{\tau_\pi}{\eta/(e+p)} = 5\frac{(\chi,\chi)\,(1,1)}{(\chi,1)^2} \geq 5 \qquad \frac{\tau_j}{D_q} \geq 3$$

- Generic bound in **any kinetic theory**, as long as enthalpy/charge susceptibility determined consistently within it

- At infinite coupling in $\mathcal{N}=4$ (finite coupling correction also known for τ_π)

$$\frac{\tau_\pi}{\eta/(e+p)} = 4 - 2\ln(2) \approx 2.6 \qquad \frac{\tau_j}{D_{U(1)}} = \frac{\pi}{2}$$

Baier *et al* **JHEP0804** (2007), Bu Lublinsky Sharon **JHEP1604** (2016)

- Strongly-coupled holographic theories are **very far** from having a kinetic quasiparticle description

Second-order relaxation

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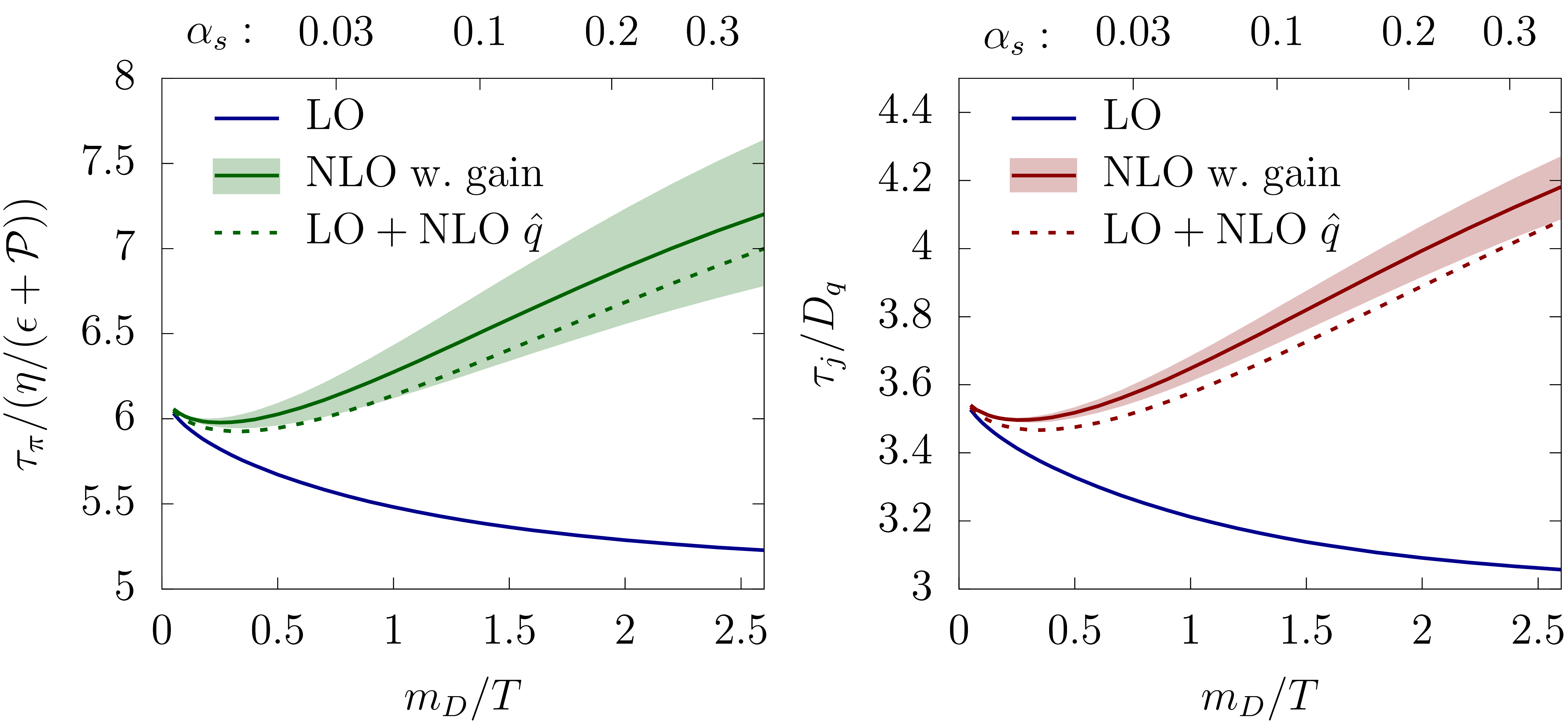
Baier *et al* **JHEP0804** (2007), Bu Lublinsky Sharon **JHEP1604** (2016)

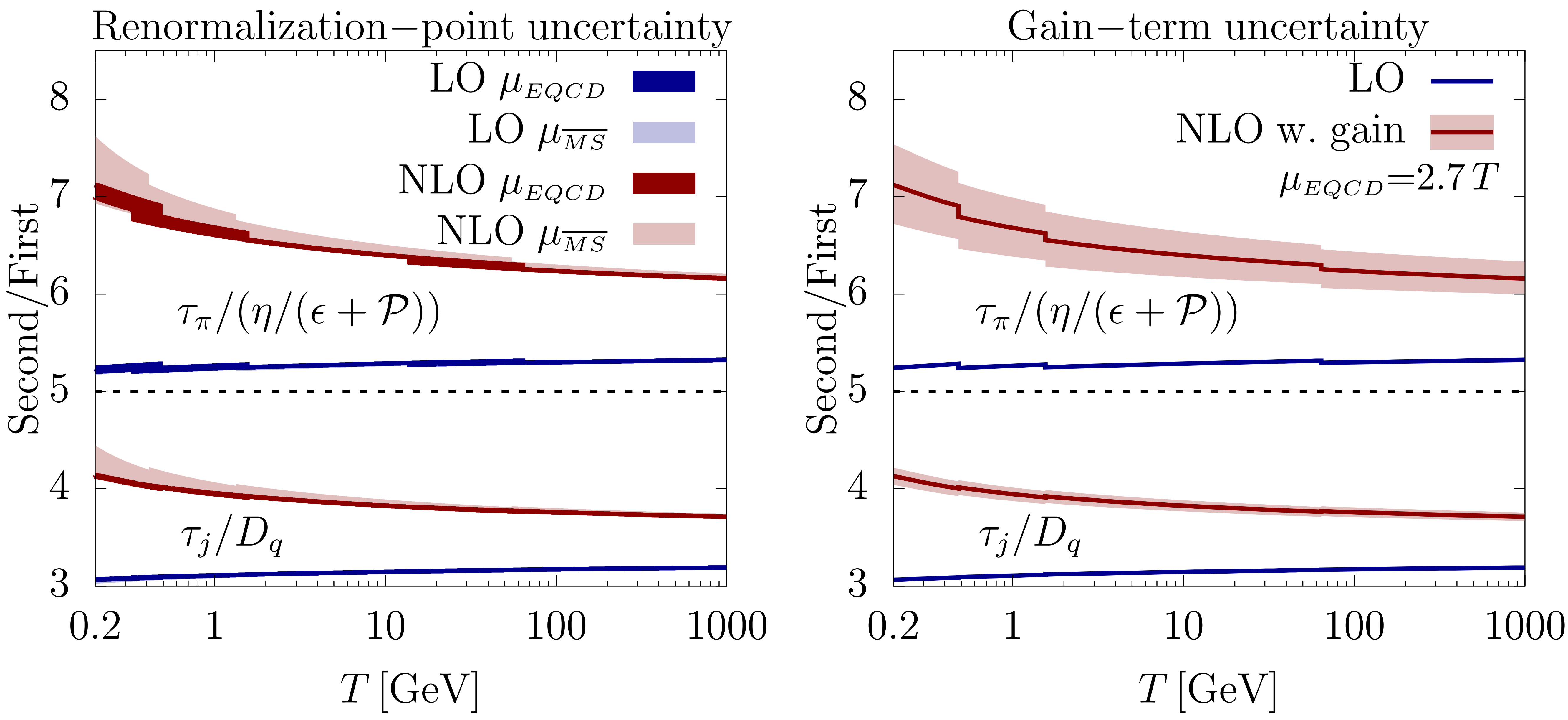
- NLO kinetic theory and holography can give comparable 1st order coefficients, but they differ fundamentally at 2nd order

2nd-order relaxation: (a)NLO results



- ~40% increase from LO and robustness of these second/first-order coeffs. ratios

2nd-order relaxation: (a)NLO results



- Temperature dependence much milder than for the first- or second-order coefficients alone

Summary

- The effective kinetic theory of QCD has been extended to (almost) NLO
- NLO corrections are large, η and D_q down by a factor of ~4-5 in the phenomenological region
- Convergence below m_D~0.5T (or at T well above the TeV scale)
- Second-order τ_π is bounded from below in kinetic theory. Strong-coupling holography violates the bounds.
- Milder increase at (a)NLO for the second-order coefficients, weak temperature dependence for the second/first order ratios, good news for hydro simulations.

Conclusions and outlook

- The techniques used/introduced here can be applied for the transport properties of all sorts of SEWM with ultrarelativistic quasi-particle d.o.f.s

- Light-cone techniques are an essential tool. Now being applied to Early Universe physics as well
  Anisimov Besak Bödeker (2011-12) JG Laine (2015-18)

- Corrections dominated by NLO $\hat{q}$. Can we identify the physics responsible for this and reorganize the perturbative expansion?

- NLO kinetic theory and holography can give comparable 1st order coefficients, but they differ fundamentally at 2nd order: strongly-coupled thys very far from having quasi-particles

Backup

Euclideanization of light-cone soft physics

- For $t/x_z = 0$: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n, p) e^{i\mathbf{p}\cdot\mathbf{x}}$$

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

- For $t/x_z = 0$: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \SumInt_p G_E(\omega_n, p) e^{i\mathbf{p}\cdot\mathbf{x}}$$

- Consider the more general case $|t/x^z| < 1$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0 x^0)} \left(\frac{1}{2} + n_{\mathrm{B}}(p^0)\right)(G_R(P) - G_A(P))$$

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- Change variables to $\tilde{p}^z = p^z - p^0(t/x^z)$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 d\tilde{p}^z d^2 p_\perp e^{i(\tilde{p}^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)} \left(\frac{1}{2} + n_{\mathrm{B}}(p^0)\right) (G_R(p^0, \mathbf{p}_\perp, \tilde{p}^z + (t/x^z)p^0) - G_A)$$

Caron-Huot **PRD79** (2009)

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- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

Caron-Huot **PRD79** (2009)

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- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

$$G_{rr}(t,\mathbf{x}) = T\sum_n\int dp^z d^2p_\perp e^{i(p^z x^z+\mathbf{p}_\perp\cdot\mathbf{x}_\perp)}G_E(\omega_n,p_\perp,p^z+i\omega_n t/x^z)$$

- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

- For t/x_z =0: equal time Euclidean correlators.

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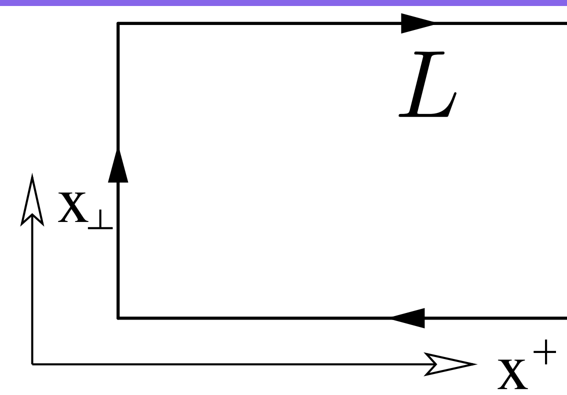
- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

$$G_{rr}(t,\mathbf{x})_{\mathrm{soft}} = T\int d^3p\, e^{i\mathbf{p}\cdot\mathbf{x}}\, G_E(\omega_n=0,\mathbf{p})$$

- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

LPM resummation



$\propto e^{\mathcal{C}(x_\perp)L}$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- All points at spacelike or lightlike separation, only preexisting correlations
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008)
  - Can be "easily" computed in perturbation theory
  - Possible lattice measurements Laine **EPJC72** (2012) Laine Rothkopf **JHEP1307** (2013) Panero Rummukainen Schäfer **1307.5850**

Semi-collinear processes

- Important technical detail: strict *O(g)* vs resummed N^{n}LO
- Strict *O(g)* semicollinear rate actually involves subtraction of collinear and hard limits ,i.e. $\hat{q}(\delta E) - \hat{q}(0) - \hat{q}(\delta E, m_D \to 0)$
- This makes it mostly negative: when extrapolating to larger *g* we risk a negative collision operator
- We devised a new implementation that, while equivalent at *O(g)*, is better behaved when extrapolating due to resummations of N^{n}LO terms ($n \geq 2$)
- In a nutshell, make $C(q_\perp)$ δE-dependent in the first-order of the LPM ladder resummation. Importance for jet physics?