Theoretical aspects of photon production in high energy nuclear collisions

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INSTITUTE OF PARTICLE PHYSICS

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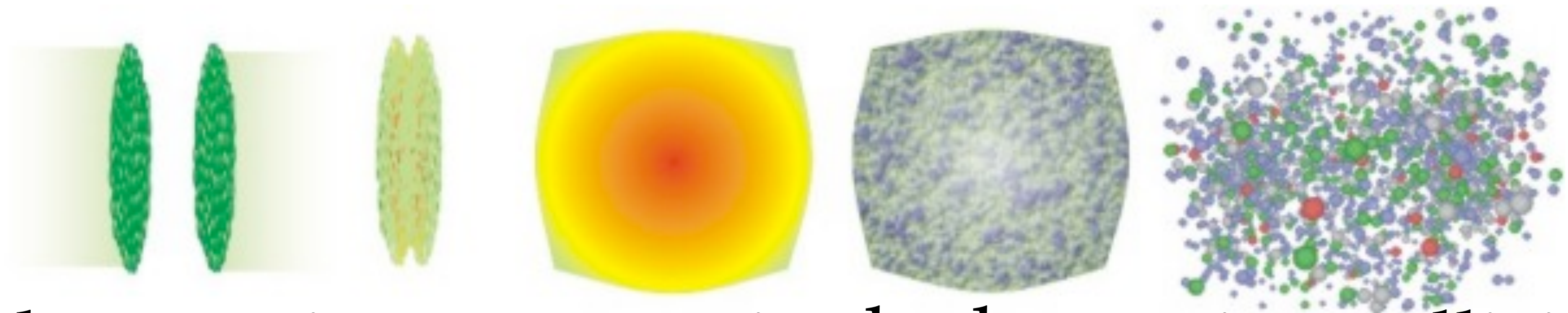
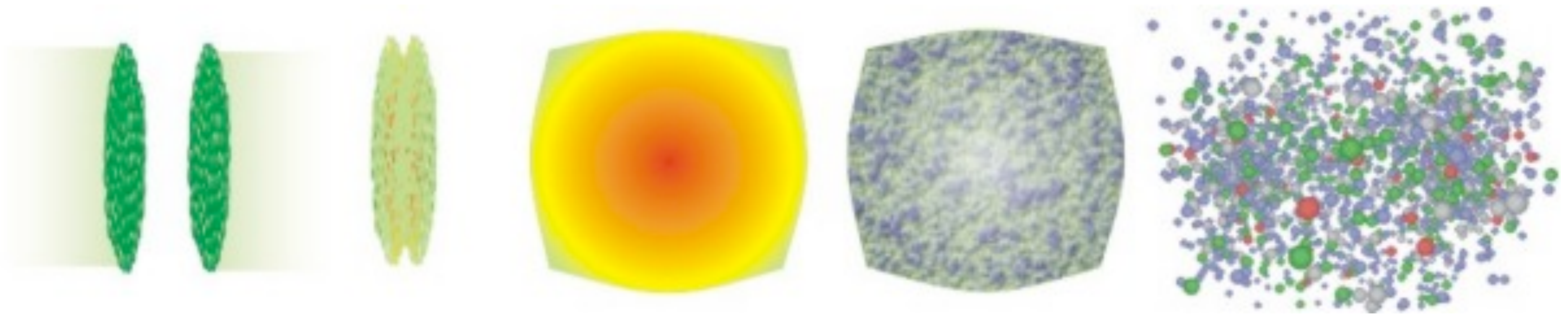
How photons are made



- The hard partonic processes in the heavy ion collision produce quarks, gluons and *prompt/primary photons*
- At a later stage, quarks and gluons form a plasma.
  - Scatterings of thermal partons produce *partonic thermal photons*
  - A jet traveling can radiate *jet-thermal photons*
- Later on, hadronization. *hadron gas photons*
- (Some) hadrons decay into *decay photons*

How photons are made



- Theoretical description: convolution of microscopic rates over the macroscopic evolution of the medium
- In this talk
  - overview and recent results on the microscopic rates, mostly for the *thermal phase*
  - *real photons* and *virtual photons* (dileptons)

How to compute rates

- $\alpha \ll 1$ implies that photon production is a rare event and that rescatterings and back-reactions are negligible: medium is transparent to/not cooled by photons

- At leading order in QED and to all orders in QCD the photon and dilepton rates are given by

$$\frac{d\Gamma_{\gamma}(k)}{d^3k} = -\frac{\alpha}{4\pi^2 k}\int d^4X e^{iK\cdot X}\mathrm{Tr}\rho J^{\mu}(0)\, J_{\nu}(X)$$

$$\frac{d\Gamma_{l+l-}(k)}{dk^0 d^3k} = -\frac{\alpha^2}{6\pi^3 K^2}\int d^4X e^{iK\cdot X}\mathrm{Tr}\rho J^{\mu}(0)\, J_{\nu}(X)$$

The ingredients

$$W^{<}(K) \equiv \int d^4X e^{iK\cdot X} \mathrm{Tr} \rho J^{\mu}(0)\, J_{\nu}(X)$$

- electromagnetic current *J*: how the d.o.f.s couple to photons
- density operator ϱ. In the equilibrium (possibly just local) approximation it becomes the thermal density $\rho \propto e^{-\beta H}$ and the whole thing a thermal average
- The action *S*: how the d.o.f.s propagate and interact

Shopping lists

- pQCD: QCD action (and EFTs thereof), thermal average can be generalized to non-equilibrium. Real world: extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$

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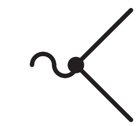
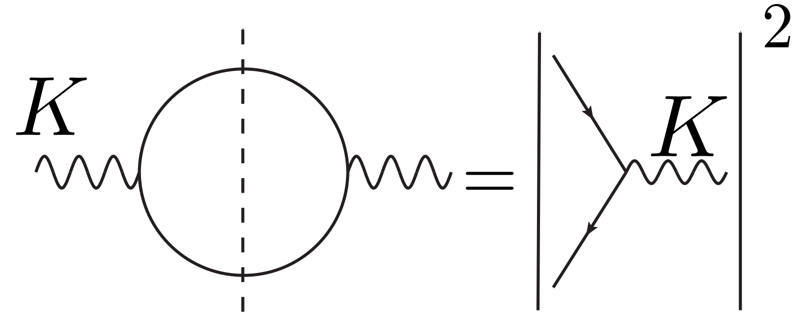
- pQCD: QCD action (and EFTs thereof), thermal average can be generalized to non-equilibrium. Real world: extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$
- lattice QCD: Euclidean QCD action, pure thermal average. Real world: analytically continue to Minkowskian domain
- AdS/CFT: $\mathcal{N}=4$ action, in and out of equilibrium, weak and strong coupling. Real world: extrapolate to QCD

The basics of pQCD photons

$$\frac{d\Gamma_\gamma(k)}{d^3k} = -\frac{\alpha}{4\pi^2 k}\int d^4X e^{iK\cdot X}\,\mathrm{Tr}\rho J^\mu(0)\, J_\nu(X) \qquad J^\mu = \sum_{q=uds} e_q \bar{q}\gamma^\mu q :$$

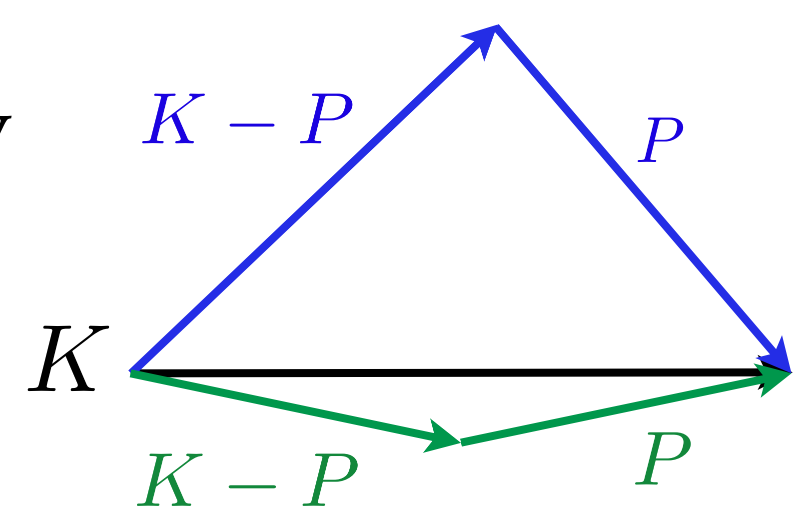


- Real, hard photon: $k^0 = k \gtrsim T$
- At one loop ($\alpha_{\mathrm{EM}}\, g^0$):



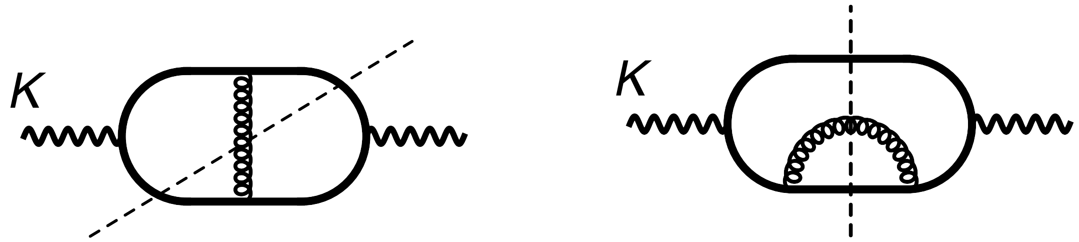
Kinematically forbidden. Need to kick one of the quarks off-shell. Works for dileptons (thermal Drell-Yan)

- Leading order photon is $\alpha_{\mathrm{EM}}\, g^2$
- Strength of the kick (virtuality) naturally divides the calculation in the distinct 2↔2 processes and collinear processes

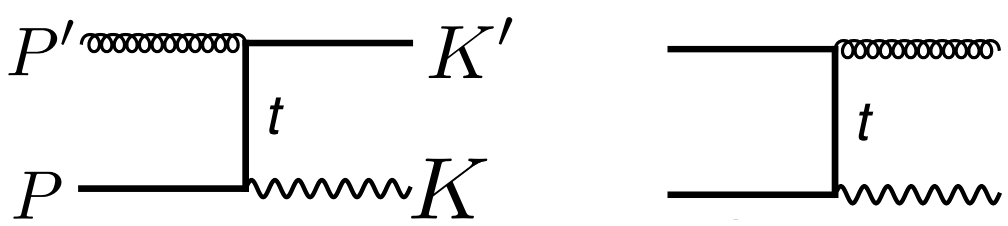


2↔2 processes

- Cut two-loop diagrams ($\alpha_{\mathrm{EM}}\, g^2$)



2↔2 processes (with crossings and interferences):



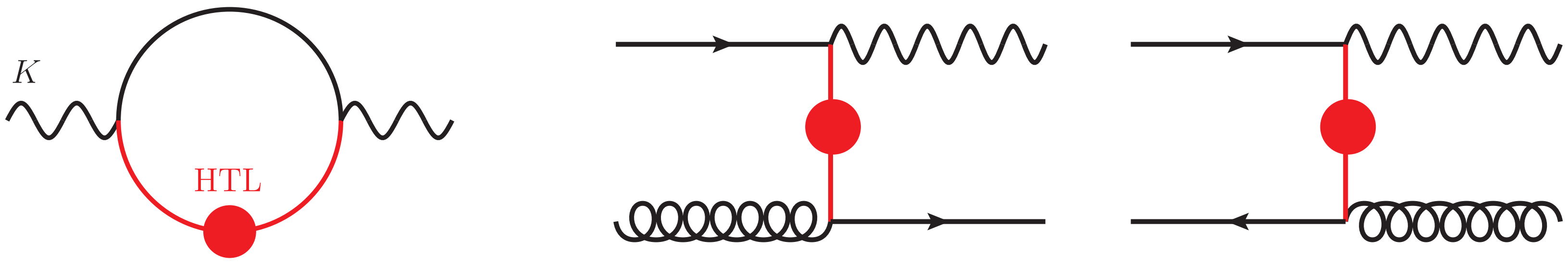
$$\int_{\mathrm{ph.\,space}} f(p) f(p')(1 \pm f(k'))|\mathcal{M}|^2 \delta^4(P + P' - K - K')$$

- Equivalence with kinetic theory: distributions x matrix elements

- IR divergence (Compton) when t goes to zero

$2\leftrightarrow 2$ processes

- The IR divergence disappears when **Hard Thermal Loop** resummation is performed Braaten Pisarski **NPB337** (1990)

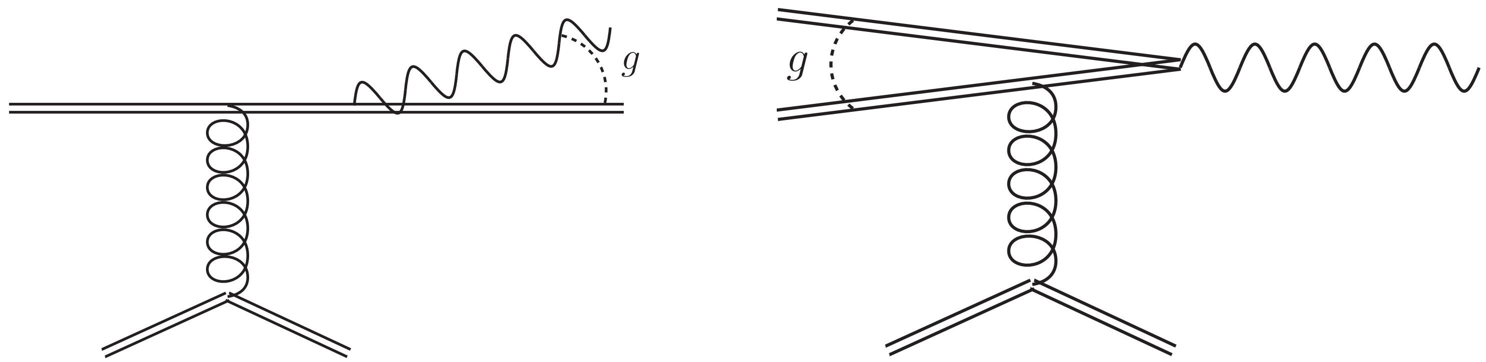


- In the end one obtains the result

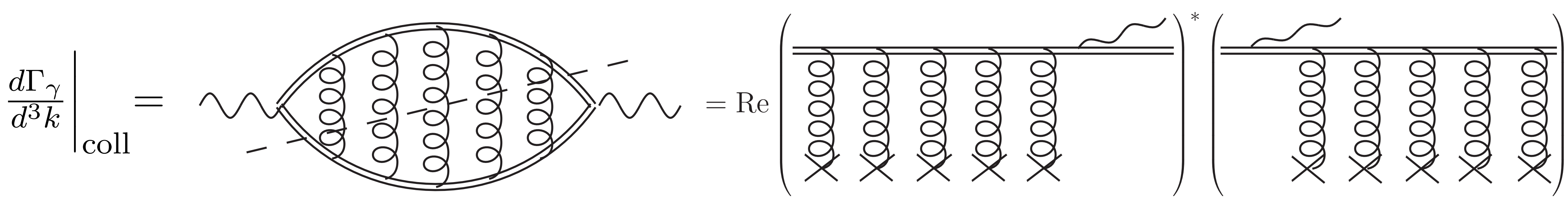
$$\left.\frac{d\Gamma_\gamma}{d^3k}\right|_{2\leftrightarrow 2} \propto e^2 g^2 \left[\log\frac{T}{m_\infty} + C_{2\leftrightarrow 2}\left(\frac{k}{T}\right)\right]$$

Kapusta Lichard Siebert **PRD44** (1991) Baier Nakkagawa Niegawa Redlich **ZPC53** (1992)

Collinear processes

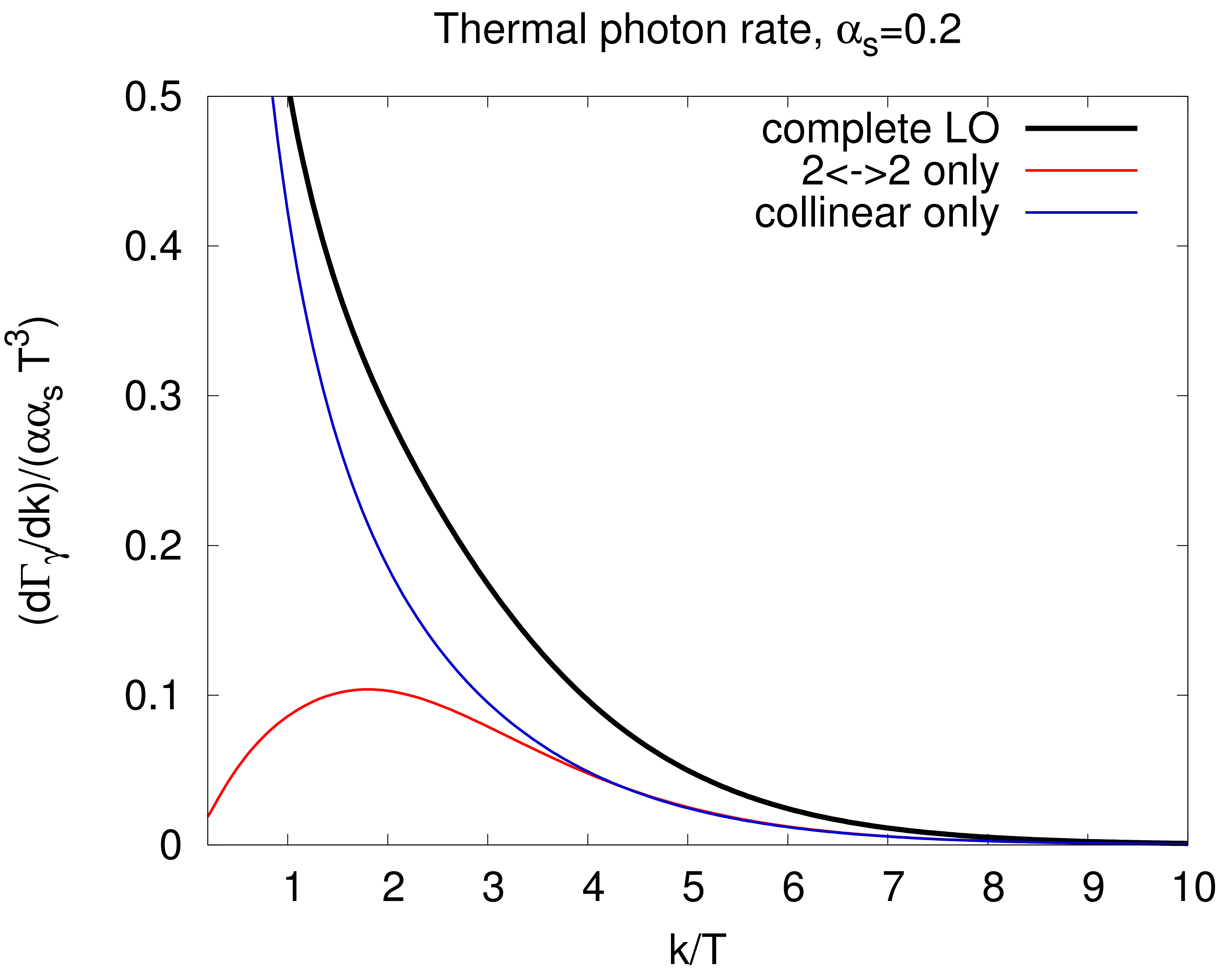


- These diagrams contribute to LO if small (g) angle radiation/annihilation Aurenche Gelis Kobes Petitgirard Zaraket 1998-2000
- Photon formation times is then of the same order of the soft scattering rate $\Rightarrow$ interference: *LPM effect*
- Requires resummation of infinite number of ladder diagrams



AMY (Arnold Moore Yaffe) **JHEP** 0111, 0112, 0226 (2001-02)

LO pQCD photons



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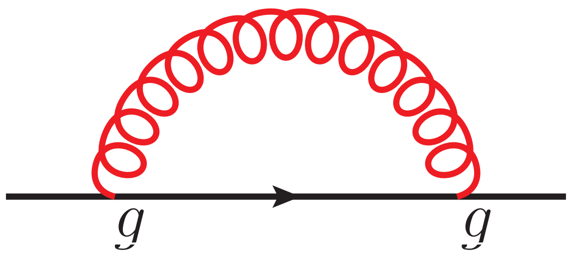
Beyond LO pQCD

- *Beyond leading order*: test the reliability of perturbation theory
- *Beyond thermal equilibrium*: incorporate viscous corrections
- *Beyond* $K^2=0$: dileptons
- *Towards* T_c: talk by Shu Lin

Beyond leading order

- The soft scale gT introduces $O(g)$ corrections

$n_B(p) \sim T/p \sim 1/g$



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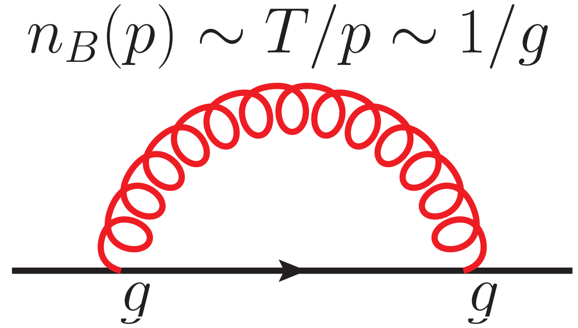
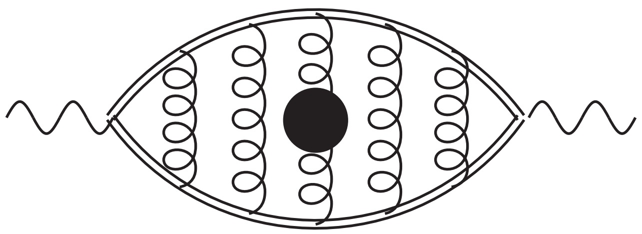


- In the collinear sector: account for 1-loop rungs (related to NLO qhat). Euclidean (EQCD) evaluation
  Caron-Huot **PRD79**, talks by Panero, Meyer



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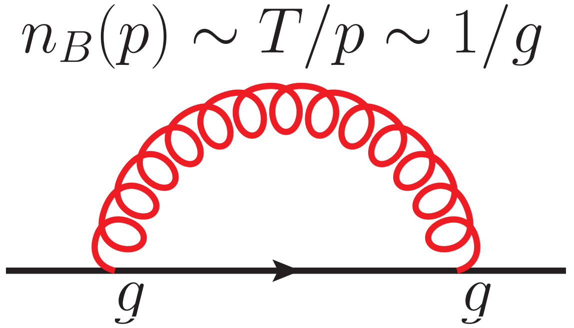

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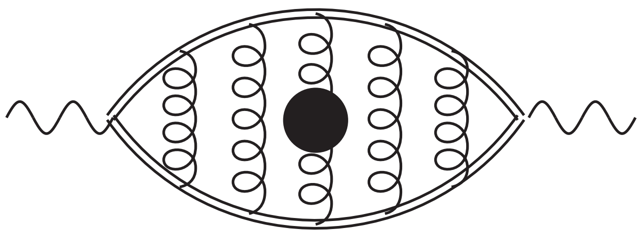
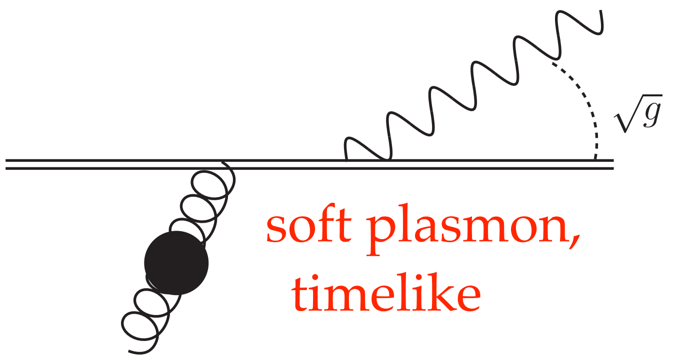
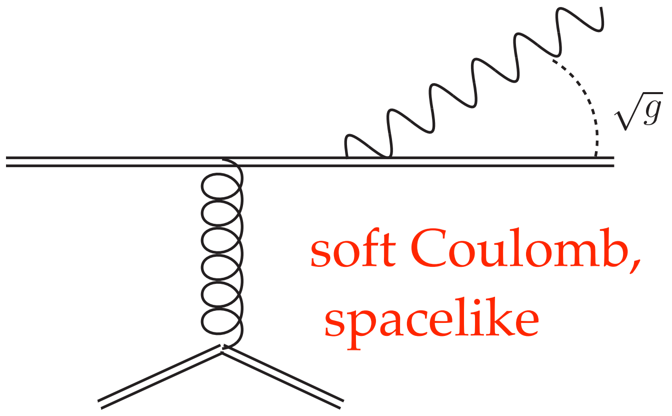
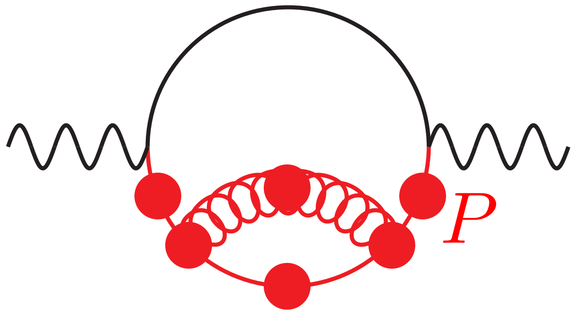


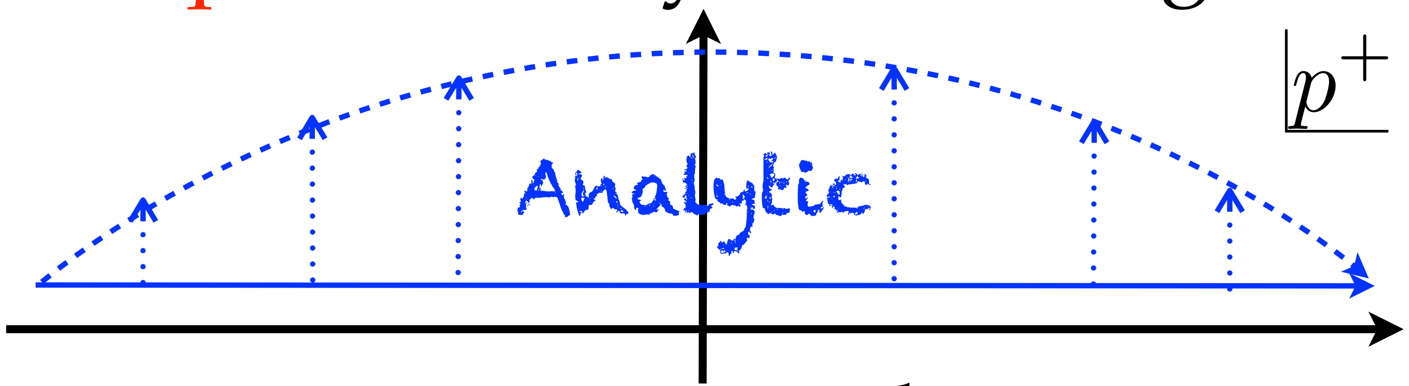
- New semi-collinear processes: larger angle radiation, NLO in collinear radiation approx. Requires a "*modified qhat*", relevance for jets too





- Add soft gluons to soft quarks: nasty all-HTL region





- Analyticity allows us to take a detour in the complex plane away from the nasty region $\Rightarrow$ compact expression

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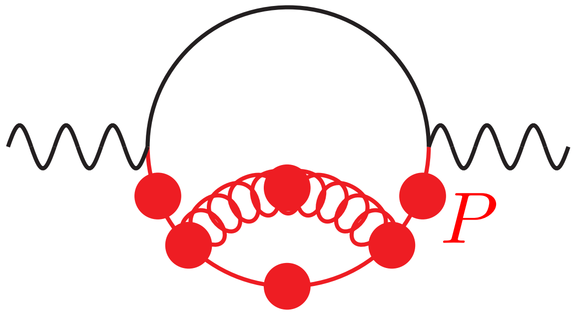
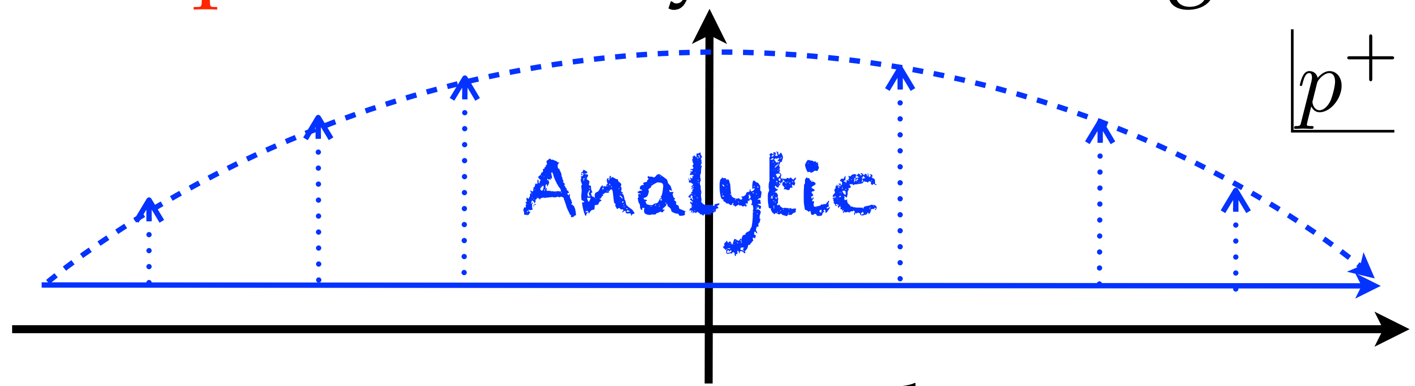
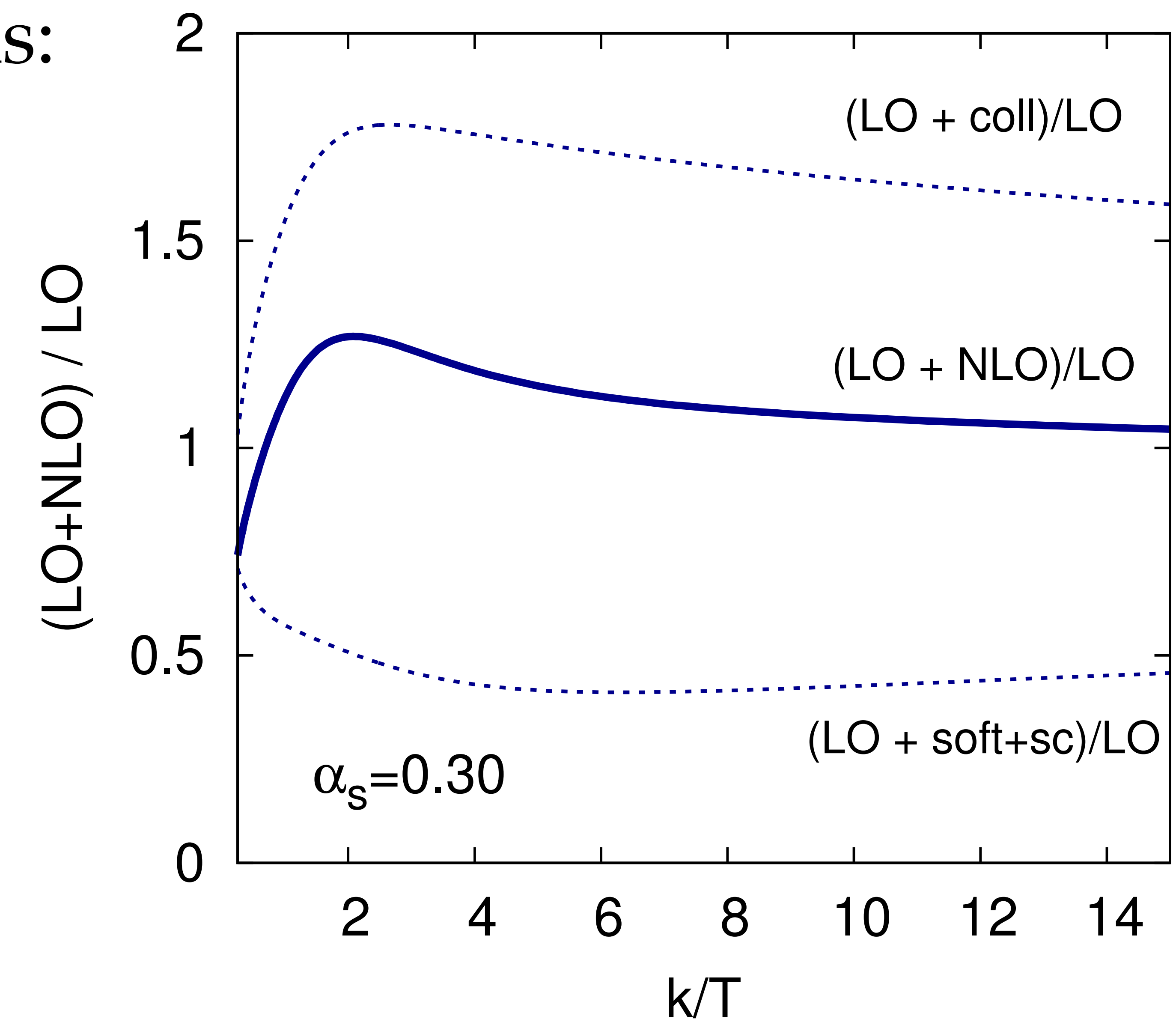


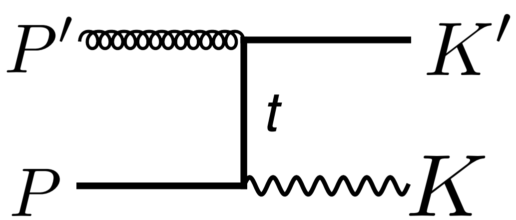


- Analyticity allows us to take a detour in the complex plane away from the nasty region $\Rightarrow$ compact expression

- Summing all contributions: good convergence, but with large cancellations between contributions: error estimate of LO
  JG Hong Kurkela Lu Moore Teaney **JHEP0503**



Beyond thermal equilibrium

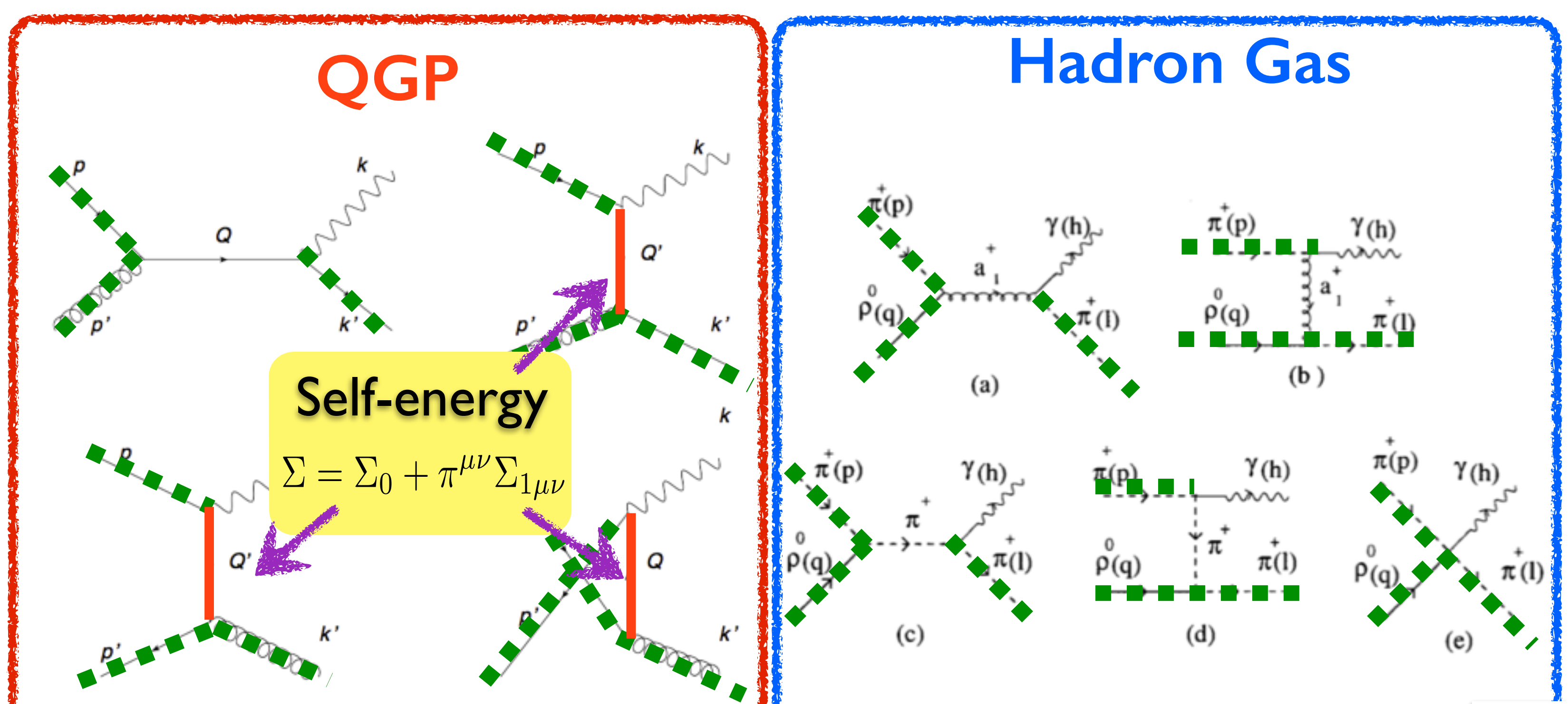


$$\int_{\text{ph. space}} f(p)f(p')(1\pm f(k'))|\mathcal{M}|^2\delta^4(P+P'-K-K')$$

- 2↔2 processes (partonic and hadronic) are easily generalized by introducing viscous distributions

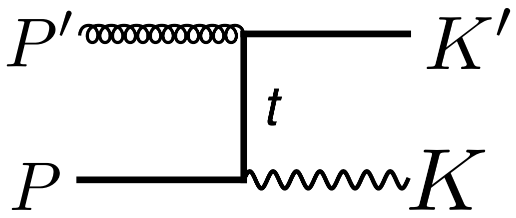
$$f(p^\mu) = f_0(E) + f_0(E)(1\pm f_0(E))\frac{\pi^{\mu\nu}\hat{p}_\mu\hat{p}_\nu}{2(e+p)}\chi\left(\frac{p}{T}\right)$$

Talk by C. Shen,Monday



Shen, Paquet et al. (2014)

Beyond thermal equilibrium



$$\int_{\rm ph.\,space} f(p)f(p')(1\pm f(k'))|\mathcal{M}|^2\delta^4(P+P'-K-K')$$

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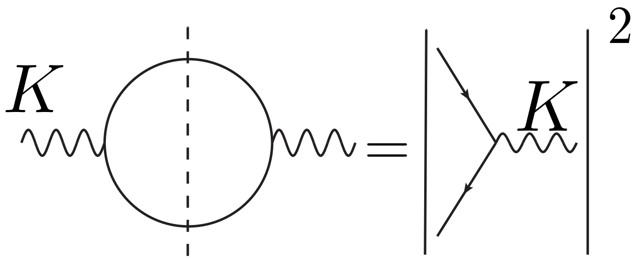
- Small t region: Hard Loop resummation
- Modification of collinear processes will be more complicated, also because of anisotropic gluon Hard Loops

Shen Heinz Paquet Kozlov Gale (2013)

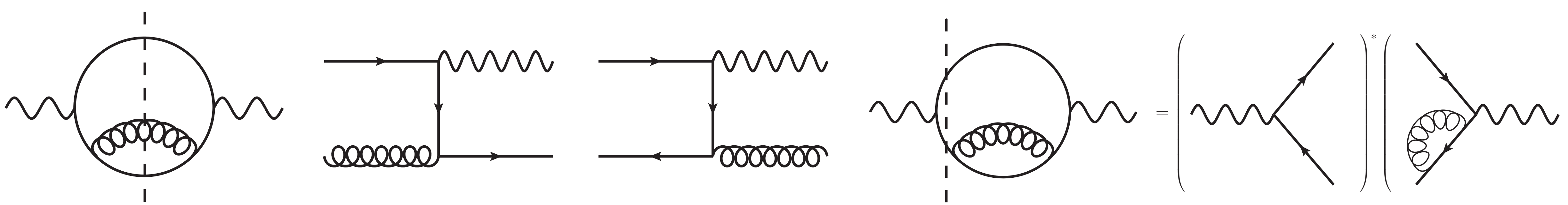
pQCD dileptons

$$\frac{d\Gamma_{l^+l^-}(k)}{dk^0 d^3k} = -\frac{\alpha^2}{6\pi^3 K^2}\int d^4X e^{iK\cdot X}\,\mathrm{Tr}\rho J^\mu(0)\, J_\nu(X)$$

- Consider non-zero virtuality $k^0 > k \geq 0$.
  - Drell-Yan contribution present



  - loop corrections: real and virtual (with IR cancellations)

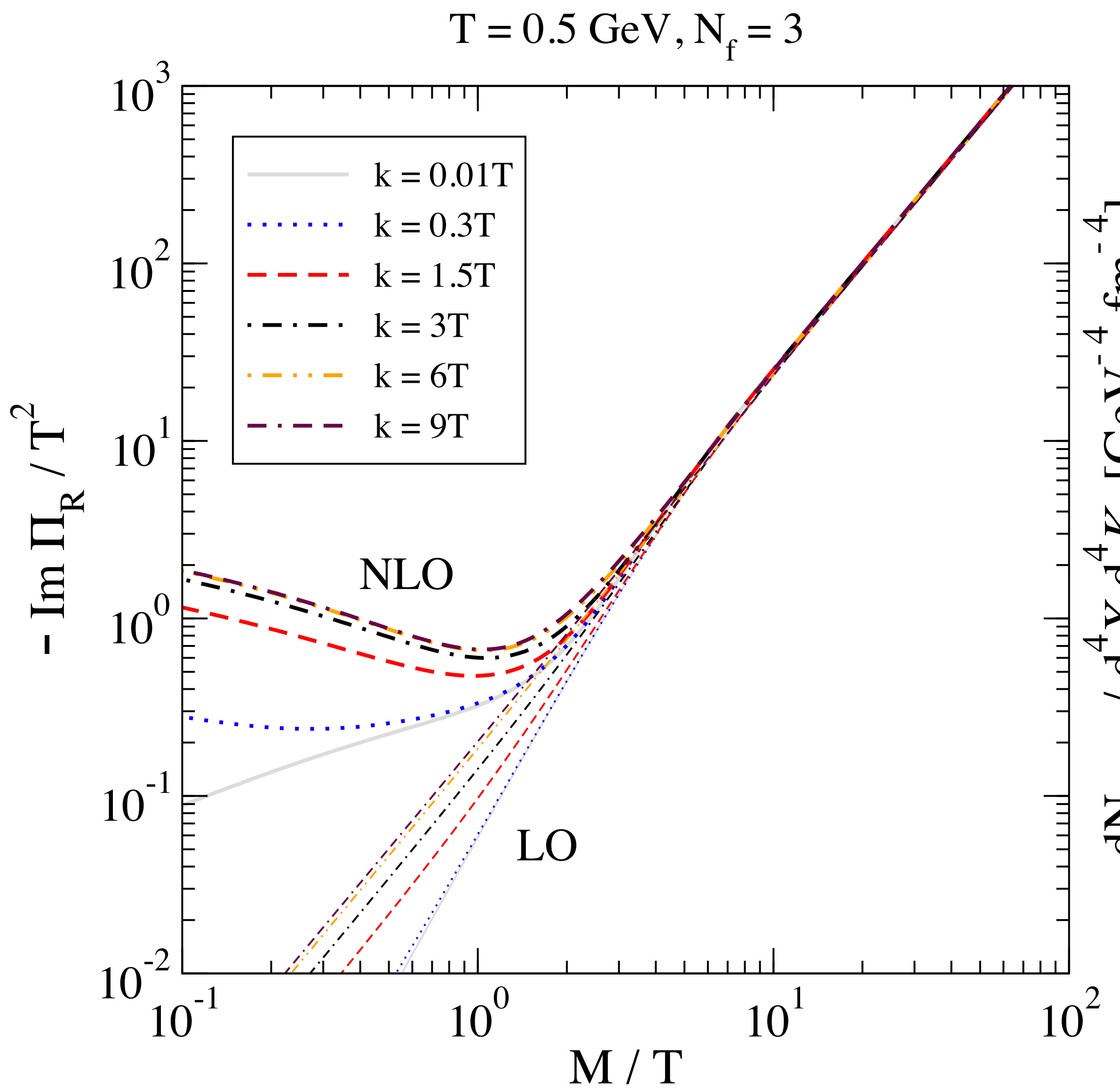


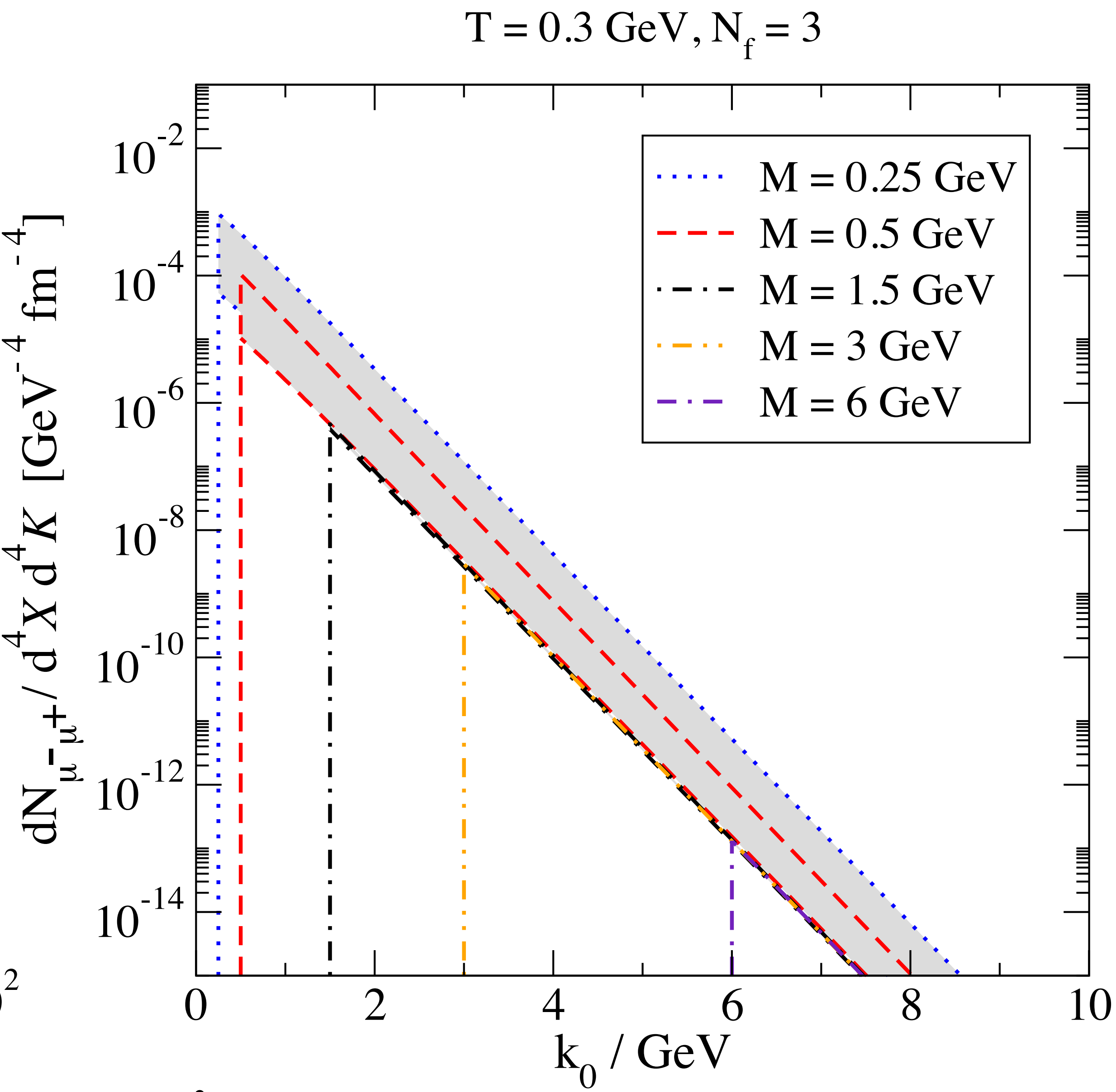
- If $K^2 \ll T^2$ LPM and / or HTL resummations become again necessary

Braaten Pisarski Yuan **PRL64** (1990), Aurenche Gelis Moore Zaraket **JHEP0212** (2002)

pQCD dileptons

- Recently, complete thermal NLO (Drell-Yan + loop correction) rate for *$k>0$, $M^2=K^2 \sim T^2$* Laine **1310.0164**





- NLO rate for *$k>0$, $K^2 \ll T^2$* coming soon

And the lattice?

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- What is measured directly is the Euclidean correlator

$$G_E(\tau, k) = \int d^3x J_\mu(\tau, \mathbf{x}) J_\mu(0,0) e^{i\mathbf{k}\cdot\mathbf{x}}$$

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$$G_E(\tau, k) = \int_0^\infty \frac{dk^0}{2\pi} \rho_V(k^0, k) \frac{\cosh\left(k^0(\tau - 1/2T)\right)}{\sinh\left(\frac{k^0}{2T}\right)}$$

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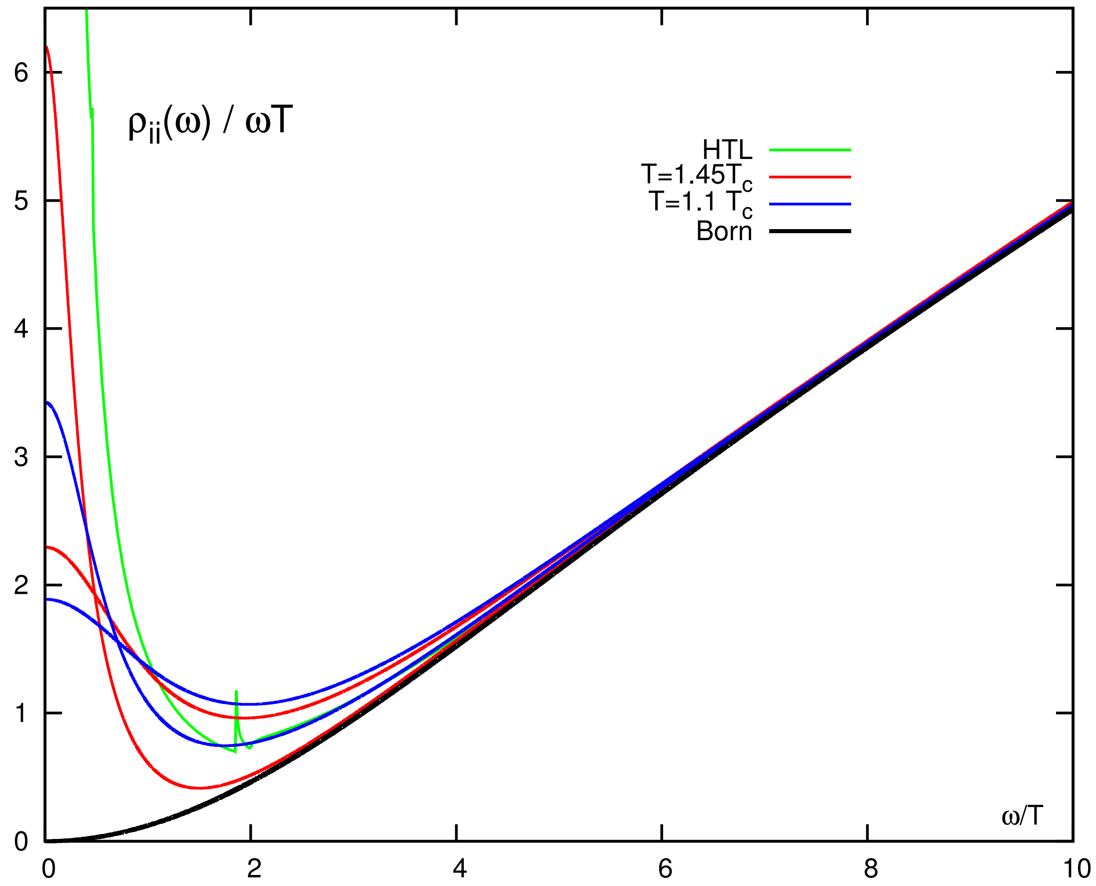
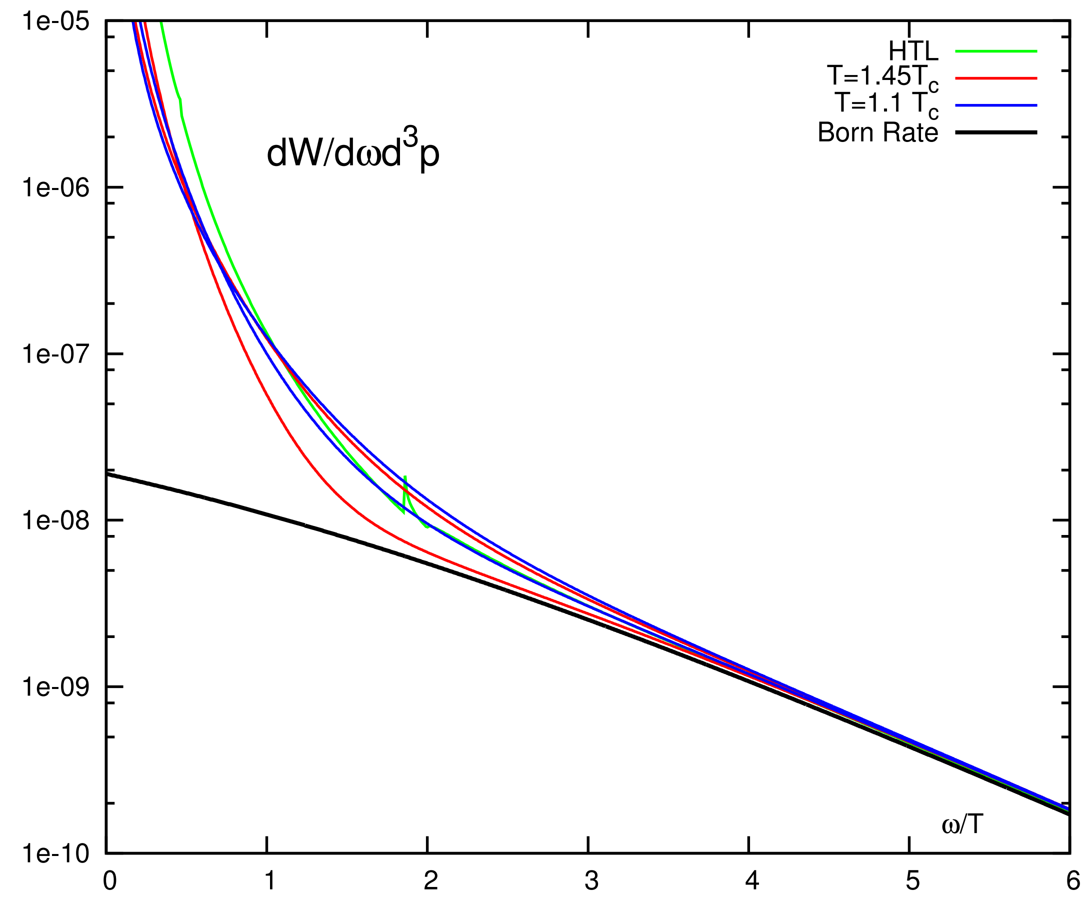
- It contains much more info (full spectral function), but hidden in the convolution. Inversion tricky, discrete dataset with errors

Zero-momentum dileptons

- if k=0: spectral function encodes physics of dileptons and electrical conductivity, easier on the lattice

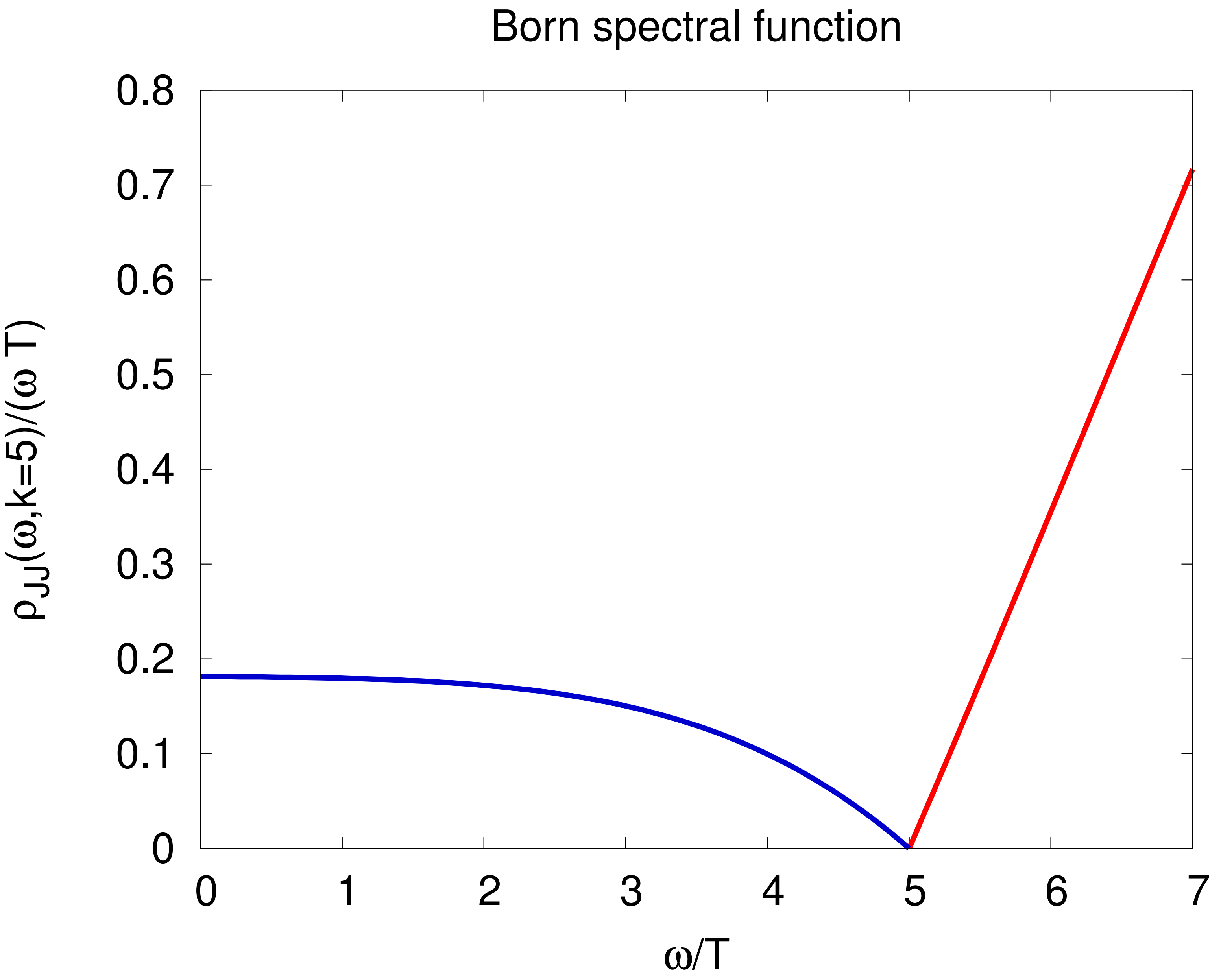




Ding Francis Kaczmarek Karsch Laermann Mukherjee Müller Söldner **1301.7436**

Finite-momentum

- If k>0 spf describes DIS ($k^0<k$), photons ($k^0=k$) and dileptons ($k^0>k$). Finite k tricky on the lattice



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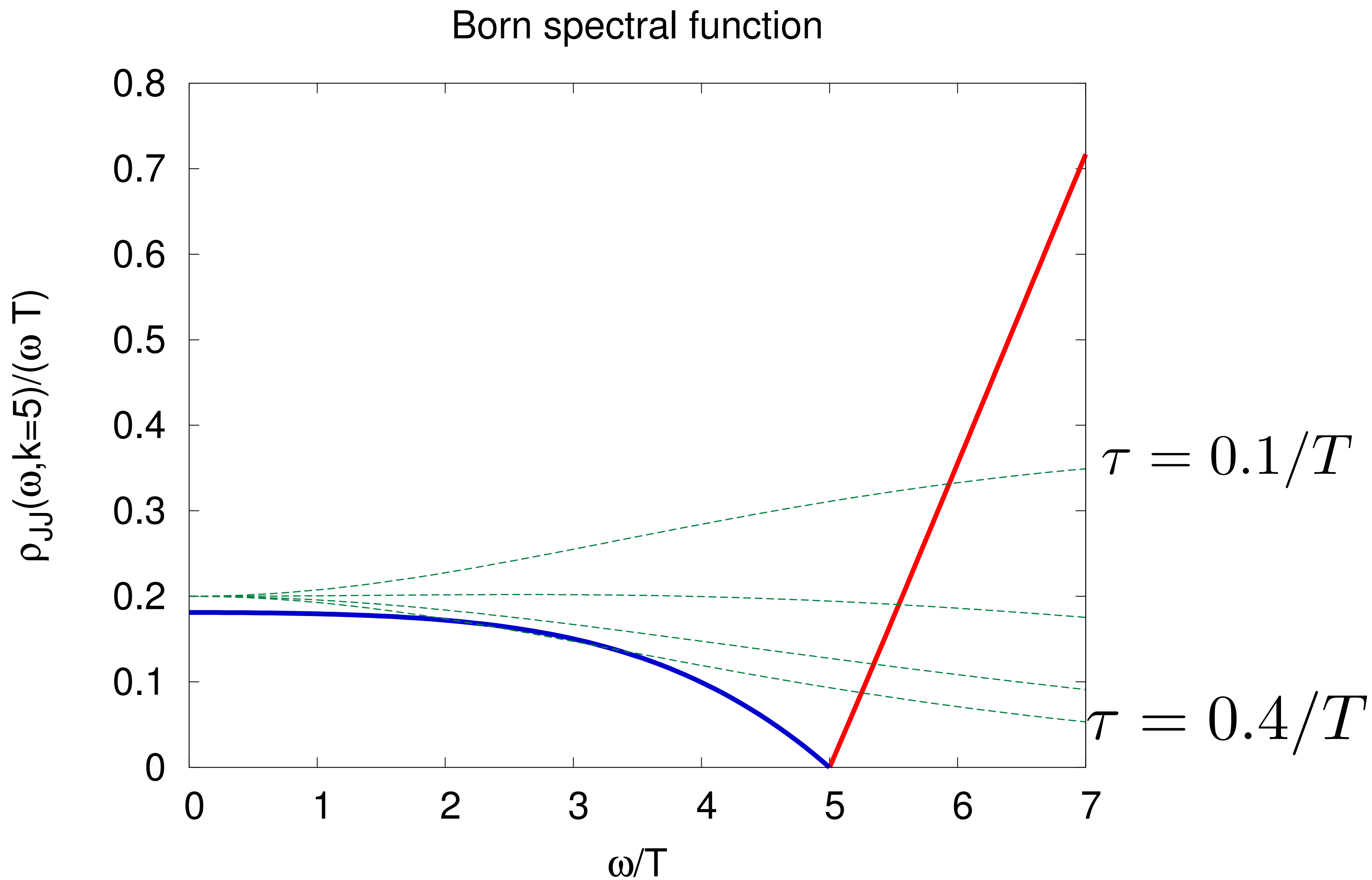
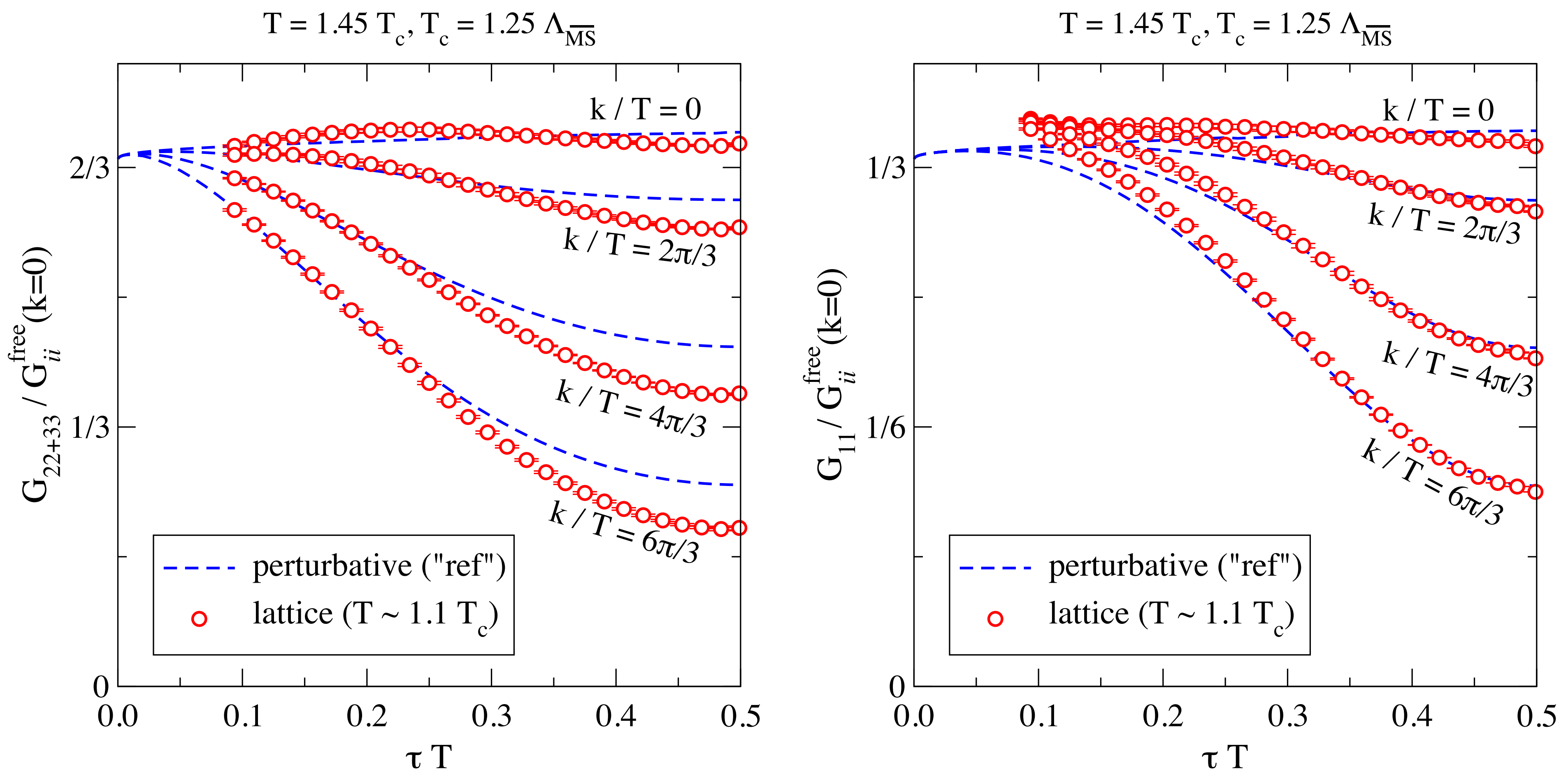


A first comparison

- First comparison at the correlator level, *with several caveats*



- Plot and perturbative calculation Laine **1310.0164**
  Lattice Ding Francis Kaczmarek Karsch Laermann Mukherjee Müller Söldner **1301.7436**

Conclusions

- Knowledge of the rates *and the related uncertainties* important for phenomenology
- NLO calculations for photons and large-mass dileptons are an important step in that direction and bring associated technical goodies to be employed elsewhere (jets, qhat)
- The lattice is already providing partial results for the spectral functions/rates and non-perturbative ingredients to perturbative calculations (qhat, transverse splitting kernels). Possible interplays in the future? Also, comparisons!

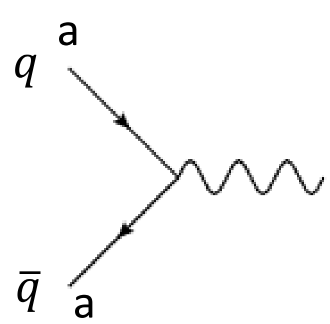
Backup

Towards T_c

- Matrix model approach to distribution

$$f(E) \to \frac{1}{N}\sum_{a=1}^{N}\frac{1}{e^{(E-iQ_a)/T}+1} \sim \ell\,\frac{1}{e^{E/T}+1} \qquad \ell \le 1$$

- Small enhancement for dileptons



Boltzmann approximation

Dilepton rate ~ $e^2 \sum_{a=1}^{N} e^{-(E/2-iQ^a)/T} e^{-(E/2+iQ^a)/T} |\mathcal{M}_{l\bar{l}}|^2$

Effect of Q cancel out!

- Large suppressions for photons, both 2↔2 and collinear

$$e^2 g^2 \sum_{a,b} e^{-(E_1-iQ^a)/T} e^{-(E_2+iQ^b)/T} |\mathcal{M}_\gamma^{ab}|^2 \qquad e^2 g^2 \sum_{a,b} e^{-(E_1-iQ^a)/T} e^{-(E_2+iQ^a-iQ^b)/T} |\mathcal{M}_\gamma^{ab}|^2$$

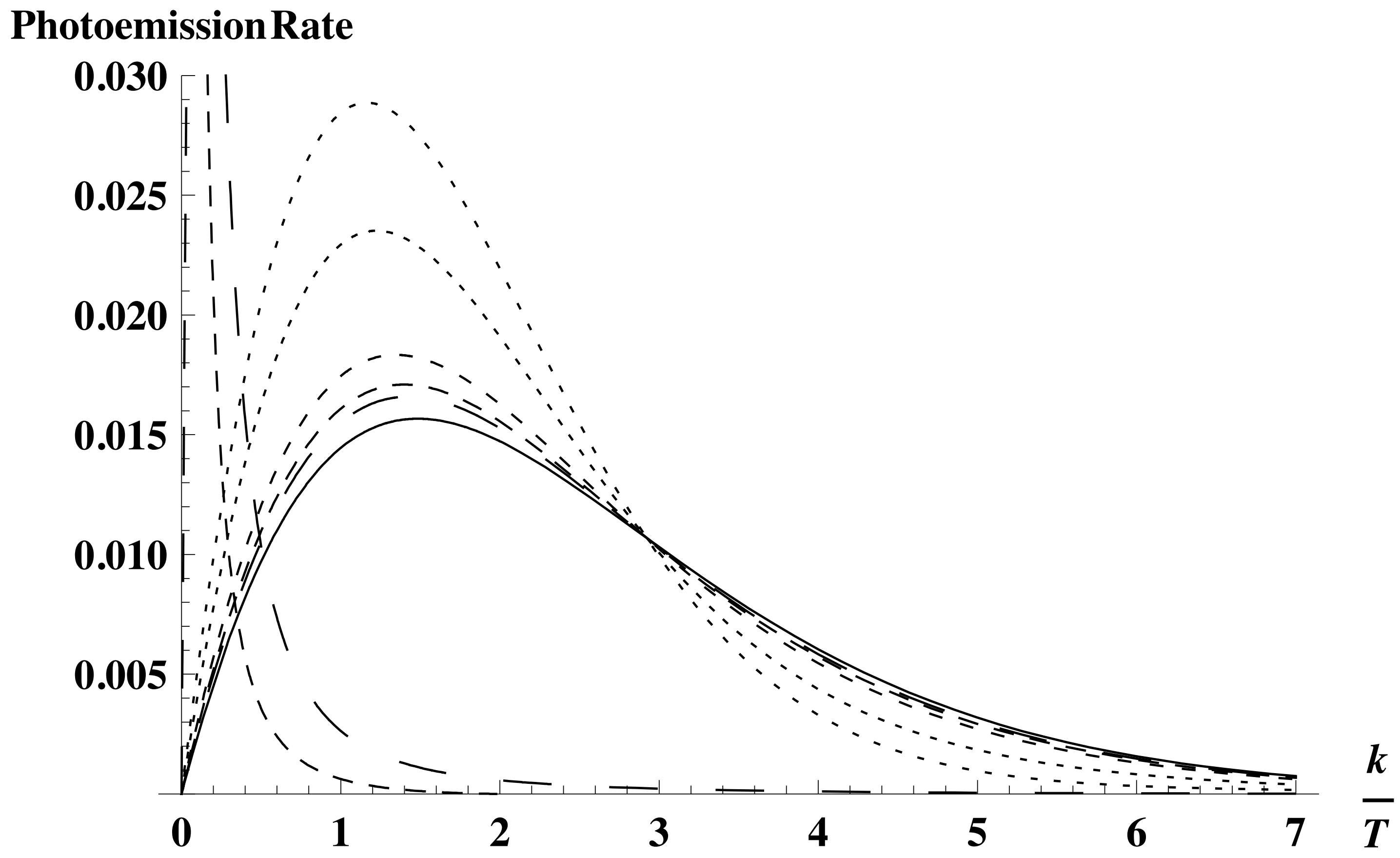
No cancellation! Partial cancellation

Talk by Shu Lin, Monday

- It would be interesting to see for which dilepton mass the enhancement becomes a suppression

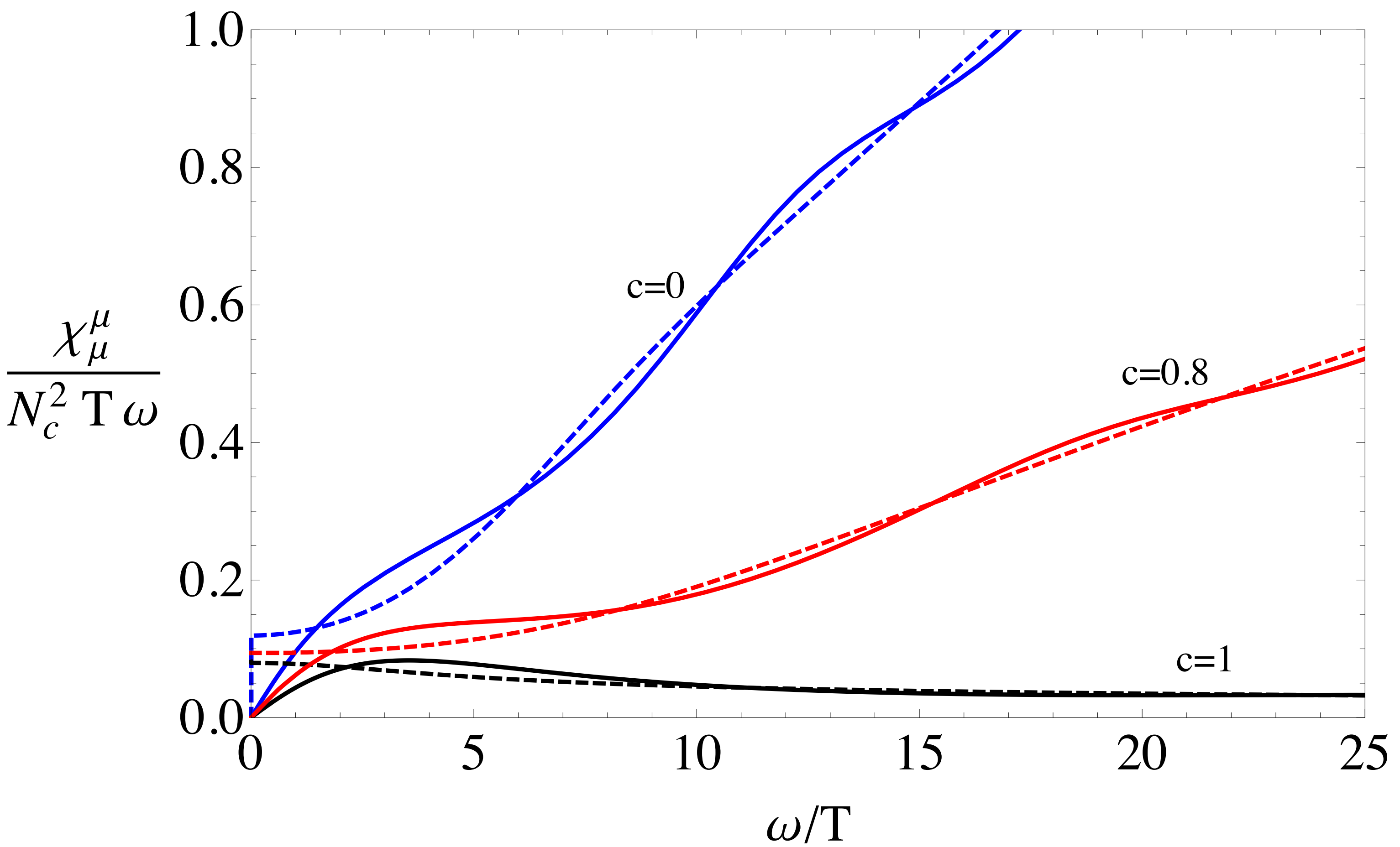
AdS/CFT approaches

- Gauge a U(1) subgroup of $\mathcal{N} = 4$: that's your photon
- LO at weak coupling, $\lambda \to \infty$ at strong coupling in equilibrium
  Caron-Huot Kovtun Moore Starinets Yaffe **JHEP06012** (2006)
- $1/\lambda$ corrections Hassanain Schvellinger **JHEP1212** (2012)
- Holographic thermalizations (out of equilibrium) Baier Stricker Taanila Vuorinen (2012), Steineder Stricker Vuorinen (2013)



- Hassanain Schvellinger strong coupling for decreasing lambda (finer dashing) compared with LO weak coupling (leftmost curves)

- Steineder *et al* strong coupling e.m. spectral function at equilibrium (dashed) and in the thermalizing metric (cont.). c=k/ω



$Backup^2$

LPM resummation

- Quark statistical functions × DGLAP splitting × transverse evolution

$$\frac{d\Gamma}{d^3k} = \frac{\alpha}{\pi^2 k}\int \frac{dp^+}{2\pi} n_{\rm F}(k+p^+)[1-n_{\rm F}(p^+)]\frac{(p^+)^2+(p^++k)^2}{2(p^+(p^++k))^2}\lim_{\mathbf{x}_\perp\to 0} 2{\rm Re}\boldsymbol{\nabla}_{\mathbf{x}_\perp}\mathbf{f}(x_\perp)$$

$$\begin{array}{ccccc} x^+ & \gg & x_\perp & \gg & x^- \\ 1/g^2T & \gg & 1/gT & \gg & 1/T \end{array}$$

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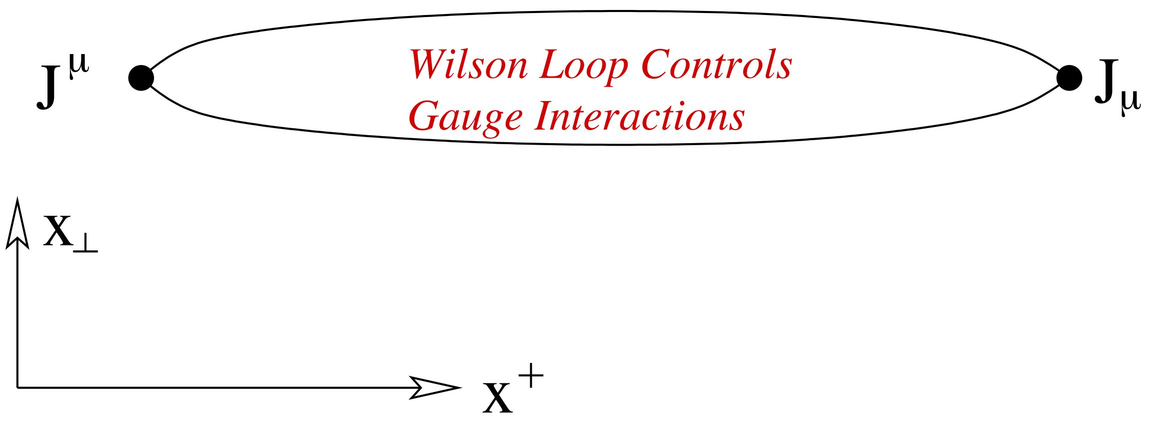
- Transverse diffusion and Wilson-loop correlators evolve the transverse density **f** *along the spacetime light-cone*

$$-2i\nabla\delta^2(\mathbf{x}_\perp) = \left[\frac{ik}{2p^+(k+p^+)}\left(m_\infty^2 - \nabla^2_{\mathbf{x}_\perp}\right) + \mathcal{C}(x_\perp)\right]\mathbf{f}(\mathbf{x}_\perp)$$

$$\begin{array}{ccccc} x^+ & \gg & x_\perp & \gg & x^- \\ 1/g^2T & \gg & 1/gT & \gg & 1/T \end{array}$$

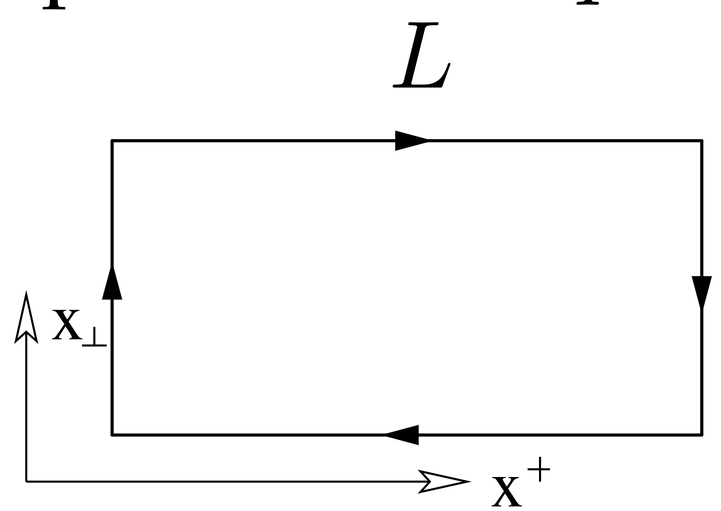


Zakharov 1996-98 AMY 2001-02

LPM resummation: two inputs

- Asymptotic mass $m_\infty^2 = 2g^2 C_R \left(\int \frac{d^3p}{(2\pi)^3} \frac{n_{\rm B}(p)}{p} + \int \frac{d^3p}{(2\pi)^3} \frac{n_{\rm F}(p)}{p} \right)$
- Light-cone Wilson loop, related to $\hat{q}$

$$\hat{q} \equiv \int_0^{q_{\rm max}} \frac{d^2 q_\perp}{(2\pi)^2} q_\perp^2 C(q_\perp)$$



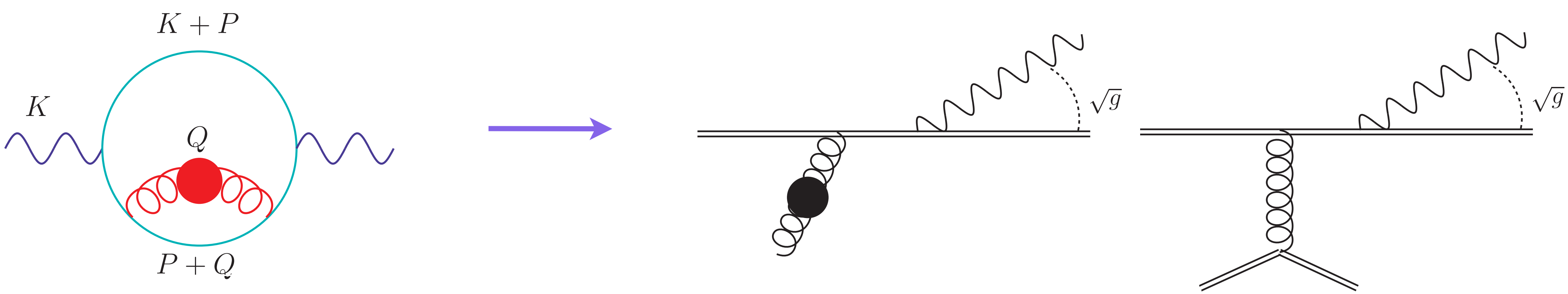
$$\propto e^{\mathcal{C}(x_\perp)L}$$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008), can be "easily" computed in perturbation theory
  Possible lattice measurements Laine Rothkopf **JHEP1307** (2013) Panero Rummukainen Schäfer **1307.5850** talk by Panero

The semi-collinear region

- Seemingly different processes boiling down to wider-angle radiation



P semi-collinear
Q soft

Q soft plasmon, timelike

Q soft cut, spacelike

Evaluation: introduce "*modified* $\hat{q}$" that keep tracks of the changes in the small light-cone component p^- of the quarks

"standard" $$\frac{\hat{q}}{g^2C_R} \equiv \frac{1}{g^2C_R}\int \frac{d^2q_\perp}{(2\pi)^2} q_\perp^2 \mathcal{C}(q_\perp) \propto \int d^4Q \langle F^{+\mu}(Q)F^{+}{}_{\mu}(-Q)\rangle_{q^-=0}$$

"modified" $$\frac{\hat{q}(\delta E)}{g^2C_R} \propto \int d^4Q \langle F^{+\mu}(Q)F^{+}{}_{\mu}(-Q)\rangle_{q^-=\delta E}$$

The "*modified* $\hat{q}$" can also be evaluated in EQCD

Euclideanization of light-cone soft physics

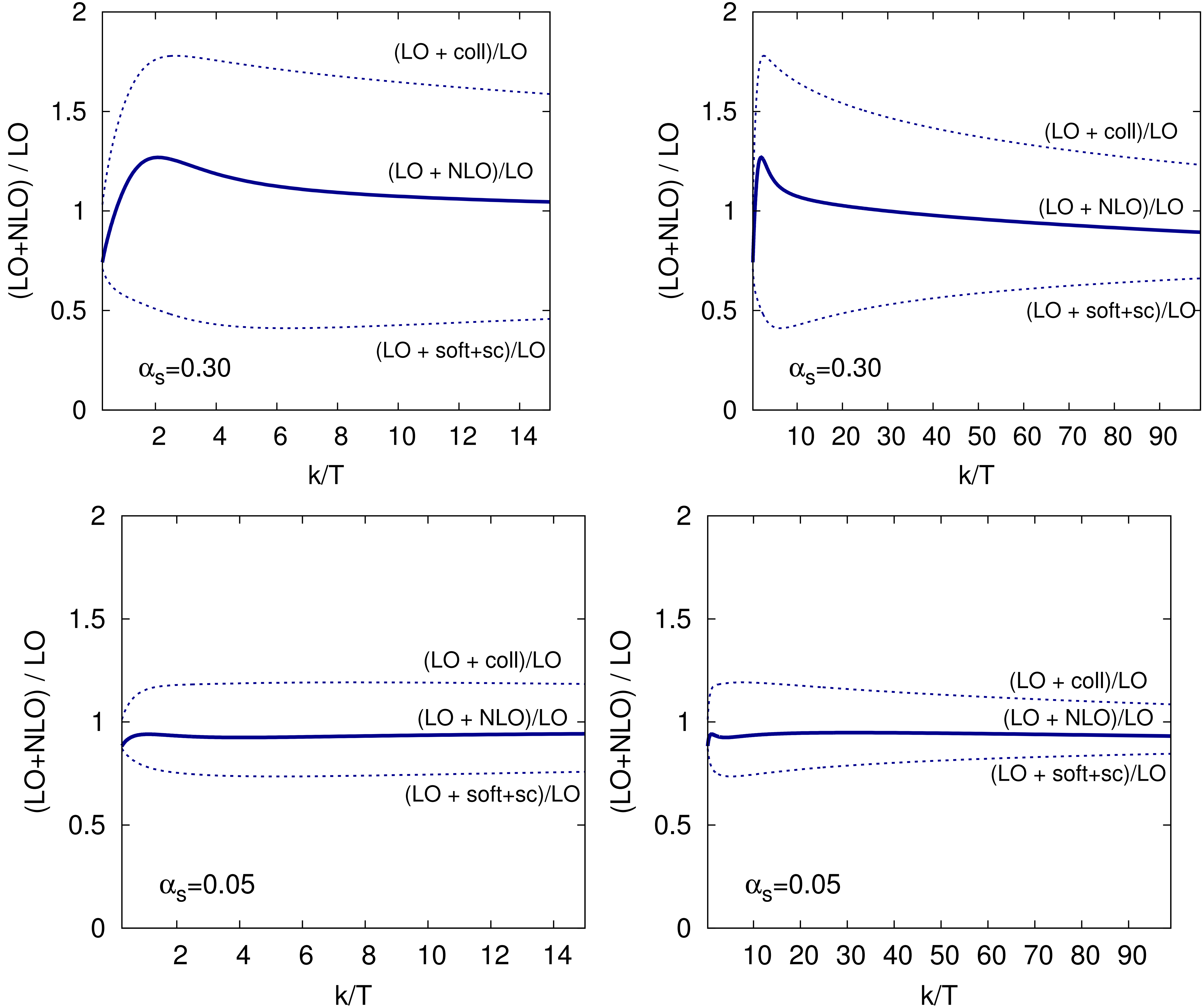
For $v=x_z/t=\infty$ correlators (such as propagators) are the equal time Euclidean correlators.

$$G^>(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n,p)e^{i\mathbf{p}\cdot\mathbf{x}}$$

- Causality: retarded functions analytic for positive imaginary parts of all *timelike* and *lightlike* variables: the above result can be extended to the lightcone

$$G^>(t=x_z,\mathbf{x}_\perp) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n,p_\perp,p_z+i\omega_n)e^{i(\mathbf{p}_\perp\cdot\mathbf{x}_\perp+p_z x_z)}$$

- The sums are dominated by the zero mode for soft physics=>EQCD!
- Equivalent to sum rules   Caron-Huot **PRD79** (2009)


(LO + coll)/LO
(LO + NLO)/LO
(LO + soft+sc)/LO
αs=0.30
(LO+NLO) / LO
k/T
(LO + coll)/LO
(LO + NLO)/LO
(LO + soft+sc)/LO
αs=0.30
(LO+NLO) / LO
k/T
(LO + coll)/LO
(LO + NLO)/LO
(LO + soft+sc)/LO
αs=0.05
(LO+NLO) / LO
k/T
(LO + coll)/LO
(LO + NLO)/LO
(LO + soft+sc)/LO
αs=0.05
(LO+NLO) / LO
k/T

NLO transport coefficients

- The only transport coefficient known so far at NLO is the *heavy quark momentum diffusion coefficient*, which is defined through the noise-noise correlator in a Langevin formalism. In field theory it can be written as

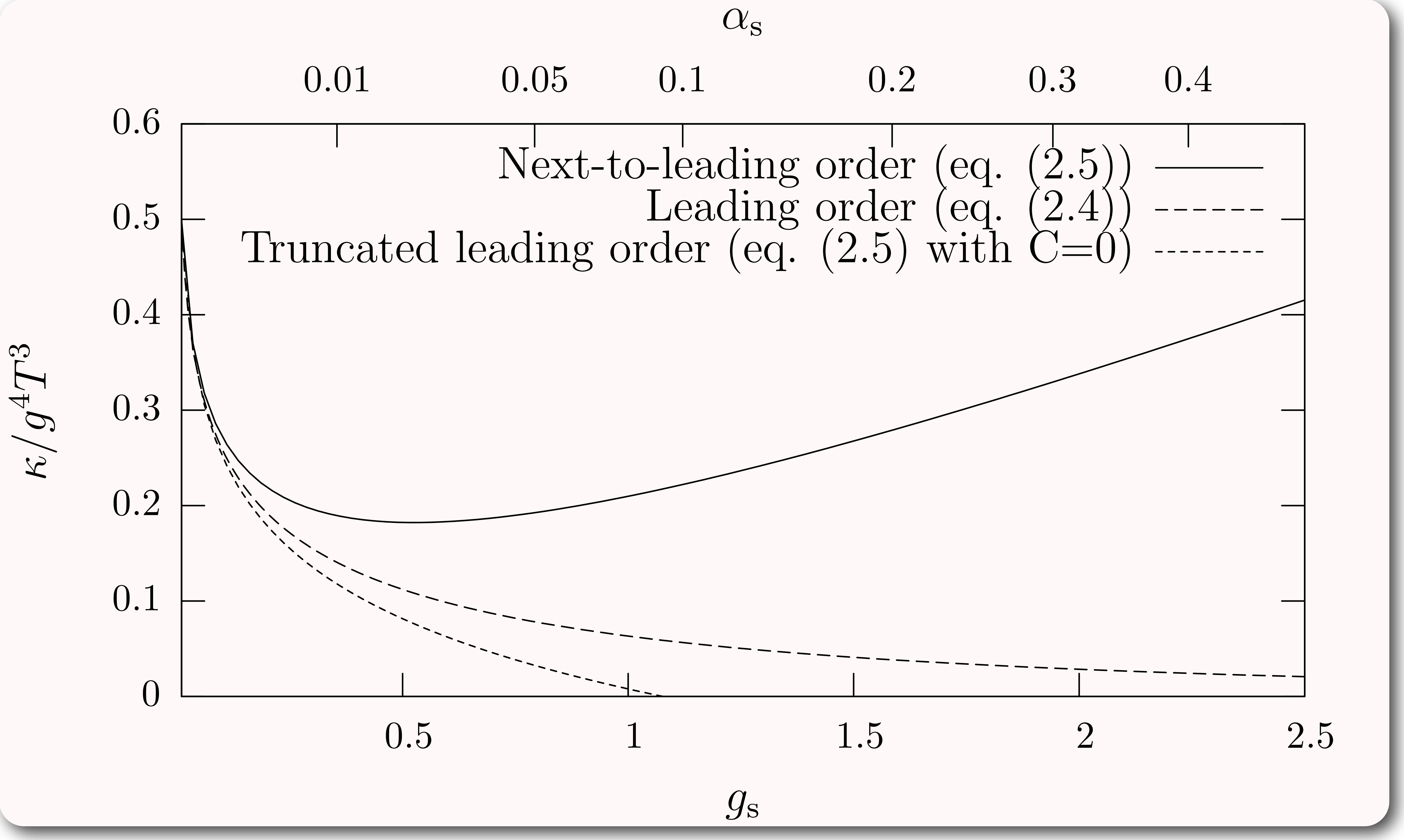
$$\kappa = \frac{g^2}{3N_c}\int_{-\infty}^{+\infty} dt \mathrm{Tr}\langle U(t,-\infty)^\dagger E_i(t) U(t,0) E_i(0) U(0,-\infty)\rangle$$

- The NLO computation factors in the coefficient C, which turns out to be sizeable

$$\kappa = \frac{C_H g^4 T^3}{18\pi}\left(\left[N_\mathrm{c}+\frac{N_\mathrm{f}}{2}\right]\left[\ln\frac{2T}{m_D}+\xi\right]+\frac{N_\mathrm{f}\ln 2}{2}+\frac{N_\mathrm{c} m_D}{T}C+\mathcal{O}(g^2)\right) \qquad \xi = \frac{1}{2}-\gamma_E+\frac{\zeta'(2)}{\zeta(2)}$$

Caron-Huot Moore **PRL100, JHEP0802** (2008)

NLO transport coefficients



Caron-Huot Moore **PRL100, JHEP0802** (2008)