Gravitational wave background from Standard Model physics

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In collaboration with Mikko Laine

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GWs from equilibrium sources

- GWs can be produced from eq. too. Well known
Weinberg
- In thermal eq. particle scatter $\Rightarrow$ GW production
- Naive power counting: for momentum k>T

$$\Gamma \sim \alpha \frac{T^3}{m_{\mathrm{Pl}}^2} e^{-k/T}$$

with some internal plasma coupling, the gravitational coupling and a Boltzmann suppression
- Since k~$3T$ Γ must be small. Is that always true?

In this talk

- Go beyond this usual assumption
- Use a modern, consistent Thermal Field Theory framework
- Try to give reliable estimates for T>160 GeV
- In particular, concentrate on the IR: for $k \ll T$ collective phenomena enter the rate and change the previous estimate

JG Laine **JCAP1507** (2015)

Outline

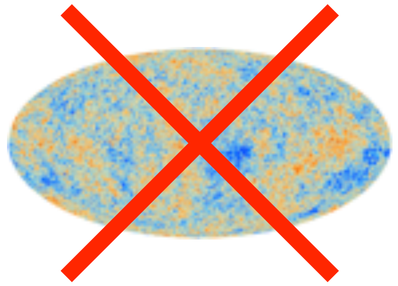
- ✓ Introduction and motivation
- Overview and formalism
- The IR rate and the viscosity of plasmas
- The $k \sim T$ rate
- Embedding the rates in cosmology: limits and prospects for detection

Overview

Equilibrium production

- GWs produced from an equilibrium plasma, but *not in equilibrium with it.*

- Similar to photon production from the QCD plasma in heavy-ion collisions. Common aspect: reinteraction (rescatterings/absorptions) and backreaction (cooling) negligible. In the photon case because $\alpha_{\mathrm{EM}} \ll \alpha_{\mathrm{s}}$



- There $\langle n_{\mathrm{em}} \rangle = 0$ and $\langle \mathbf{J}_{\mathrm{em}} \rangle = 0$, but thermal fluctuations $\Rightarrow$ charge and current fluctuations $\Rightarrow$ photons

$$\frac{\mathrm{d}\Gamma_\gamma(\mathbf{k})}{\mathrm{d}^3\mathbf{k}} = \frac{1}{(2\pi)^3 2k} \sum_\lambda \epsilon^{(\lambda)}_{\mu,\mathbf{k}} \epsilon^{(\lambda)*}_{\nu,\mathbf{k}} \int_{\mathcal{X}} e^{i\mathcal{K}\cdot\mathcal{X}} \langle J^\mu_{\mathrm{em}}(0) J^\nu_{\mathrm{em}}(\mathcal{X}) \rangle$$

Photon production

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- For $k \gtrsim T$ things go as you'd expect (but a very tricky calculation to get the actual numbers)

$$\frac{\mathrm{d}\Gamma_\gamma(\mathbf{k})}{\mathrm{d}^3\mathbf{k}} \overset{k \gtrsim T}{\approx} \frac{\alpha_{\mathrm{EM}}\alpha_{\mathrm{s}} \ln(1/\alpha_{\mathrm{s}}) T^2 e^{-k/T}}{(2\pi)^3 k}$$

LO Arnold Moore Yaffe (AMY) **JHEP0111** **JHEP0112** (2001)
NLO JG Hong Lu Kurkela Moore Teaney **JHEP1305** (2013)

- For $k \ll T$ the amplitude of J fluctuations is related to the conductivity of the plasma, a collective phenomenon

$$\frac{\mathrm{d}\Gamma_\gamma(\mathbf{k})}{\mathrm{d}^3\mathbf{k}} \overset{k \lesssim \alpha_{\mathrm{s}}^2 T}{\approx} \frac{2T\sigma}{(2\pi)^3 k} \sim \frac{\alpha_{\mathrm{EM}} T^2}{(2\pi)^3 k \alpha_{\mathrm{s}}^2 \ln(1/\alpha_{\mathrm{s}})}$$

AMY **JHEP0011** (2000), **JHEP0305** (2003)

Graviton production

- Start from textbooks: TT gauge, Minkowski background (cosmological expansion later)

$$\ddot{h}_{ij}^{\mathrm{TT}} - \nabla^2 h_{ij}^{\mathrm{TT}} = 16\pi G\, T_{ij}^{\mathrm{TT}} \qquad E_{\mathrm{GW}} = \frac{1}{32\pi G} \int_{\mathbf{x}\in V} \left[\dot{h}_{ij}^{\mathrm{TT}}(t,\mathbf{x})\right]^2$$

- h^{TT} superposition of forward and backward propagating GWs. By taking average over oscillations rewrite E as

$$\langle\!\langle E_{\mathrm{GW}} \rangle\!\rangle = \frac{1}{64\pi G} \int_{\mathbf{x}\in V} \left\{ \left[\dot{h}_{ij}^{\mathrm{TT}}(t,\mathbf{x})\right]^2 + \left|\nabla h_{ij}^{\mathrm{TT}}(t,\mathbf{x})\right|^2 \right\}$$

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$$H \equiv \langle\!\langle E_{\mathrm{GW}} \rangle\!\rangle = \frac{1}{64\pi G}\int_{\mathbf{x}\in V}\left\{\left[\dot{h}_{ij}^{\mathrm{TT}}(t,\mathbf{x})\right]^2 + \left|\nabla h_{ij}^{\mathrm{TT}}(t,\mathbf{x})\right|^2\right\}$$

- Canonical form! (Up to trivial normalization) Then do the same as for photons (but taking ϱ_{GW} rather than Γ_γ)

$$\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}t\,\mathrm{d}^3\mathbf{k}} = \frac{4\pi G}{(2\pi)^3}\sum_\lambda \epsilon_{ij,\mathbf{k}}^{\mathrm{TT}(\lambda)}\epsilon_{mn,\mathbf{k}}^{\mathrm{TT}(\lambda)*}\int_{\mathcal{X}} e^{i\mathcal{K}\cdot\mathcal{X}}\left\langle\, T^{ij}(0)\, T^{mn}(\mathcal{X})\,\right\rangle$$

Graviton production

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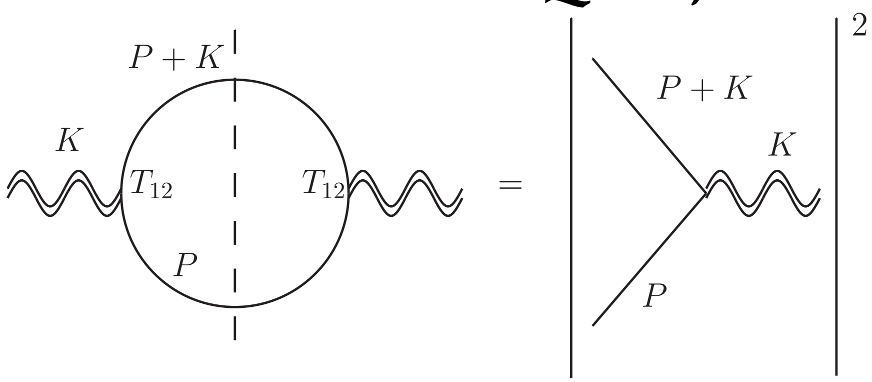
- Take a sum over the polarizations with $k \| z$
- Rotate T^{11}-T^{22} into T^{12}

$$\boxed{\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}t\,\mathrm{d}\ln k} = \frac{8k^3}{\pi m_{\mathrm{Pl}}^2}\int_{\mathcal{X}} e^{ik(t-z)}\langle\, T_{12}(0)\, T_{12}(\mathcal{X})\,\rangle}$$

- The same results can be obtained with a purely classical derivation

$$\boxed{\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}t\,\mathrm{d}\ln k} = \frac{8k^3}{\pi m_{\mathrm{Pl}}^2}\int_{\mathcal{X}} e^{ik(t-z)}\langle\, T_{12}(0)\, T_{12}(\mathcal{X})\,\rangle}$$

- $\langle .....\rangle$ thermal average of the Wightman function of the shear component of T at lightlike momentum
- What it means: cut two-point function with thermal propagators. A naive example: at LO T is bilinear in the fields of the QFT, so

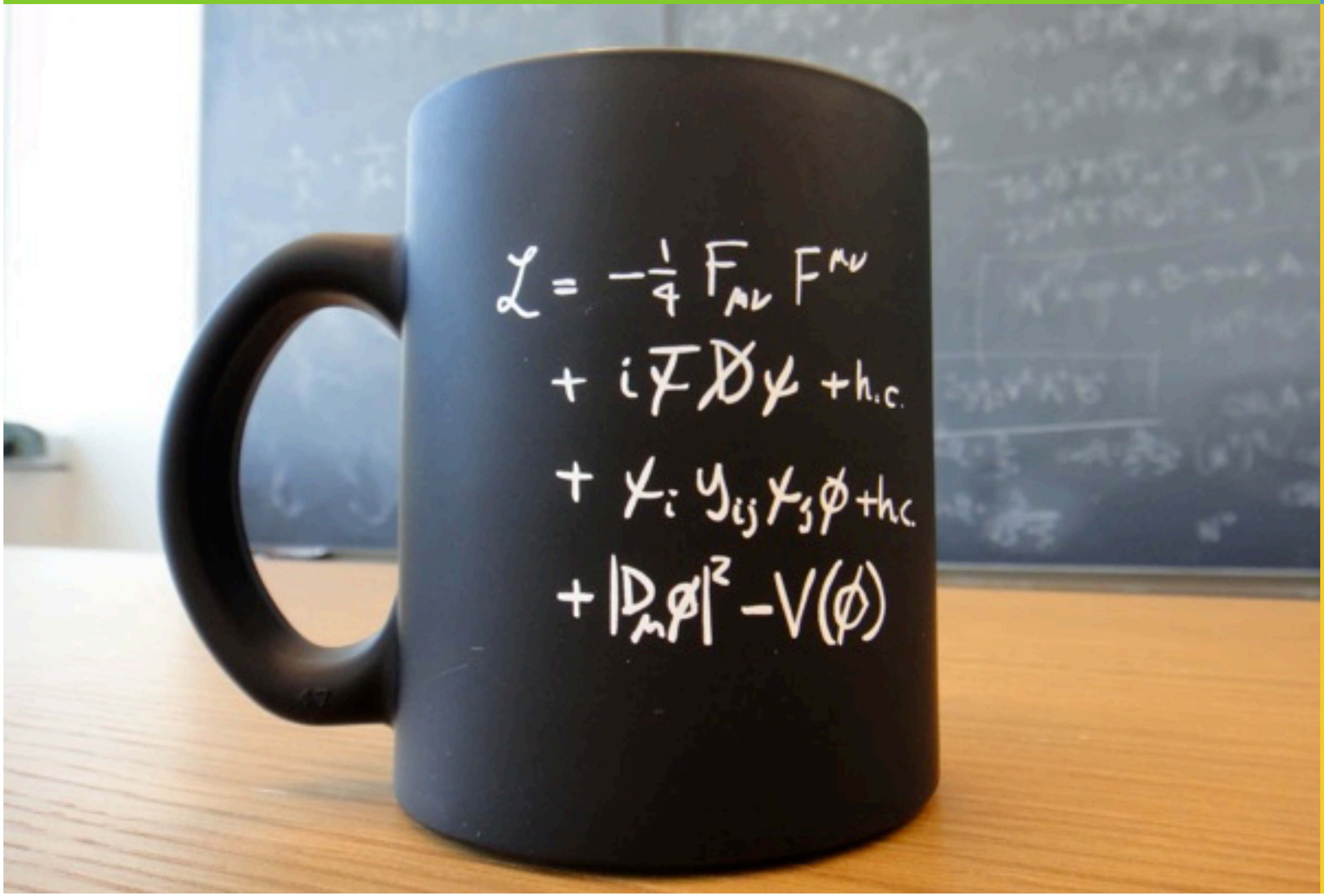


$$\sim \int d^4P\, n(p^0)n(p^0+k^0)\,|\mathcal{M}|^2\,\delta(P^2)\delta((P+K)^2)$$

thermal distribution functions x matrix element x on-shell kinematics

- Kinematically forbidden: need extra scatterings

GW rate for $k \ll T$ at $T > 160\text{GeV}$



Hydrodynamic limit

- Field theories admit a long-wavelength hydrodynamical limit. Hydrodynamics: Effective Theory based on a gradient expansion of the flow velocity

- For hydro fluctuations with local flow velocity $\mathbf{v}$ around an equilibrium state (with temp. T), at first order in the gradients and in $\mathbf{v}$

$$T^{00} = e, \qquad T^{0i} = (e+p)v^i$$

$$T^{ij} = (p - \zeta \nabla \cdot \mathbf{v})\delta^{ij} - \eta \left(\partial_i v^j + \partial_j v^i - \frac{2}{3}\delta^{ij} \nabla \cdot \mathbf{v} \right)$$

Navier-Stokes hydro, two *transport coefficients*: bulk and shear viscosity

Hydrodynamic limit

- Fluctuation-dissipation theorem: the same microscopic mechanism that is responsible for dissipation of a non-equilibrium shear stress will also cause fluctuations around equilibrium $\Rightarrow$ the processes responsible for the shear viscosity will also be responsible for the emission of IR gravitational waves

- Dates back to the Einstein relation in Brownian motion (1905, in Bern): microscopic processes responsible for it will also cause drag

- Connection between the small k rate and η can be made quantitative

Hydrodynamic limit

- 4-momentum conservation for a perturbation along z: decoupling of v^1 and v^2

$$\partial_0 T^{0j} + \partial_i T^{ij} = 0 \quad \Rightarrow \mathbf{v}_\perp(t,\mathbf{k}) = \mathbf{v}_\perp(0,\mathbf{k})\, e^{-\eta k^2 t/(e+p)}$$

- Now look at the T^{0i} correlator (operator ordering irrelevant in the soft limit), $i',j'=\{1,2\}$

$$\frac{1}{V}\int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t}\left\langle \frac{1}{2}\left\{T^{0i'}(t,\mathbf{k}), T^{0j'}(0,-\mathbf{k})\right\}\right\rangle$$

$$= \quad \frac{\frac{2\eta k^2}{e+p}}{\omega^2 + \frac{\eta^2 k^4}{(e+p)^2}} \int_{\mathbf{x}\in V} e^{-i\mathbf{k}\cdot\mathbf{x}}\left\langle T^{0i'}(0,\mathbf{x})\, T^{0j'}(0,\mathbf{0})\right\rangle$$

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$$= \frac{\frac{2\eta k^2}{e+p}}{\omega^2 + \frac{\eta^2 k^4}{(e+p)^2}} \left(T \chi_{\mathbf{p}} \delta^{ij} + \mathcal{O}(k^2)\right)$$

- For small k this becomes the momentum susceptibility

$$\chi_{\mathbf{p}} = e + p$$

Hydrodynamic limit

- Use a Ward identity to go from $T^{0i'}$ to $T^{3i'}$

$$\int_{\mathcal{X}} e^{i(\omega t - kz)} \left\langle \frac{1}{2} \left\{ T^{3i'}(\mathcal{X}), T^{3j'}(0) \right\} \right\rangle \overset{\omega,k \lesssim \alpha^2 T}{=} \frac{2\eta T \omega^2 \delta^{i'j'}}{\omega^2 + \frac{\eta^2 k^4}{(e+p)^2}}$$

- What we want (T^{12}) is different but related. Since v^1 and v^2 are uncoupled from the EOMS, their fluctuations are uncorrelated in spacetime ⇒ ω,k independent

$$\left\langle \frac{1}{2} \left\{ T^{\rm TT}_{i'j'}(t_1, \mathbf{x}_1), T^{\rm TT}_{k'l'}(t_2, \mathbf{x}_2) \right\} \right\rangle \; = \; \Phi_{i'j'k'l'} \, \delta(t_1 - t_2) \, \delta^{(3)}(\mathbf{x}_1 - \mathbf{x}_2)$$

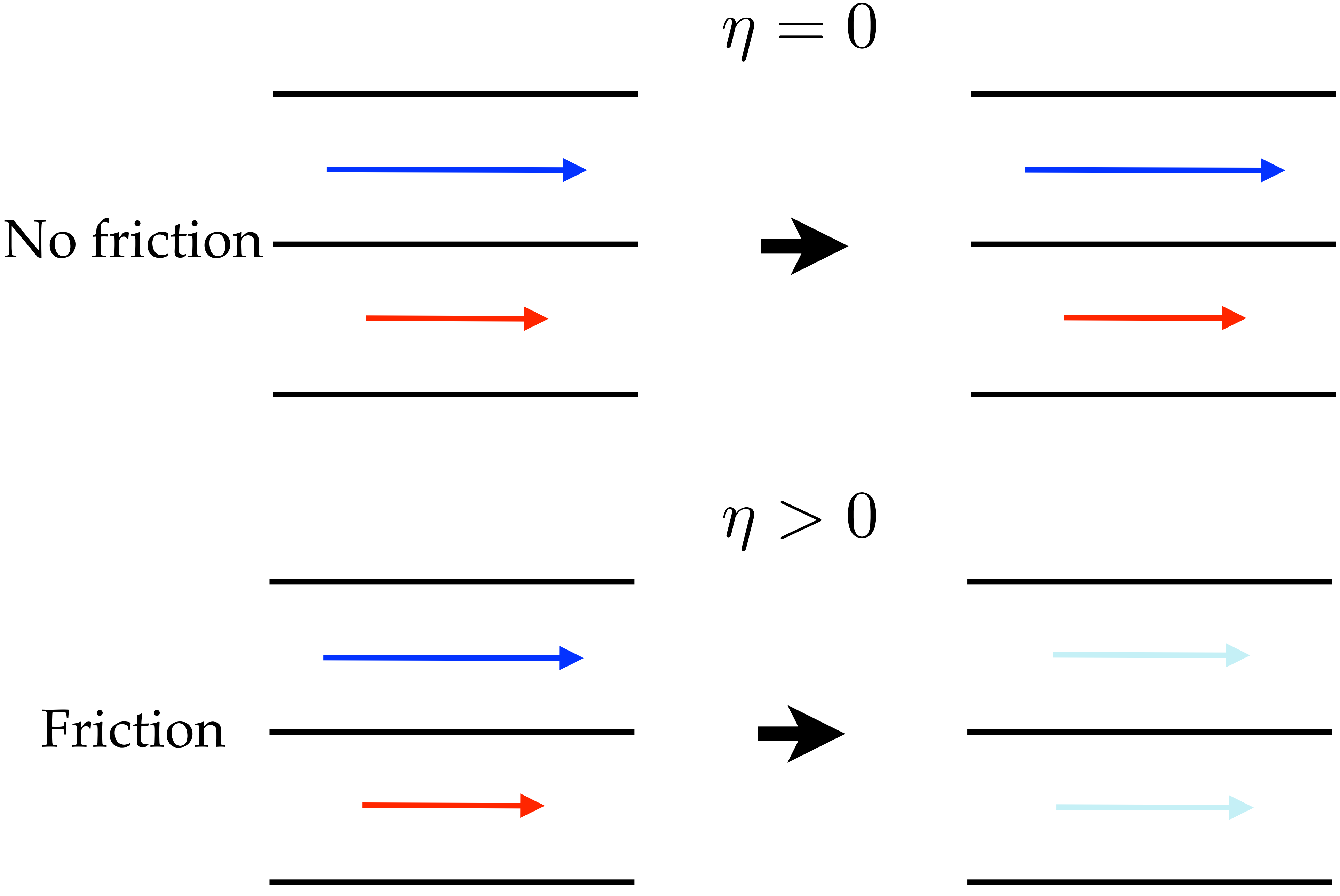
- Setting ω=k and sending k⟶0 restores 3D symmetry, so that by comparing

$$\lim_{k\to 0} \int_{\mathcal{X}} e^{ik(t-z)} \left\langle \frac{1}{2} \left\{ T_{12}(\mathcal{X}), T_{12}(0) \right\} \right\rangle = 2\,\eta\, T$$

$$\frac{\mathrm{d}\rho_{\rm GW}}{\mathrm{d}t\, \mathrm{d}\ln k} \overset{k \lesssim \alpha^2 T}{=} \frac{16 k^3 \eta T}{\pi m_{\rm Pl}^2}$$

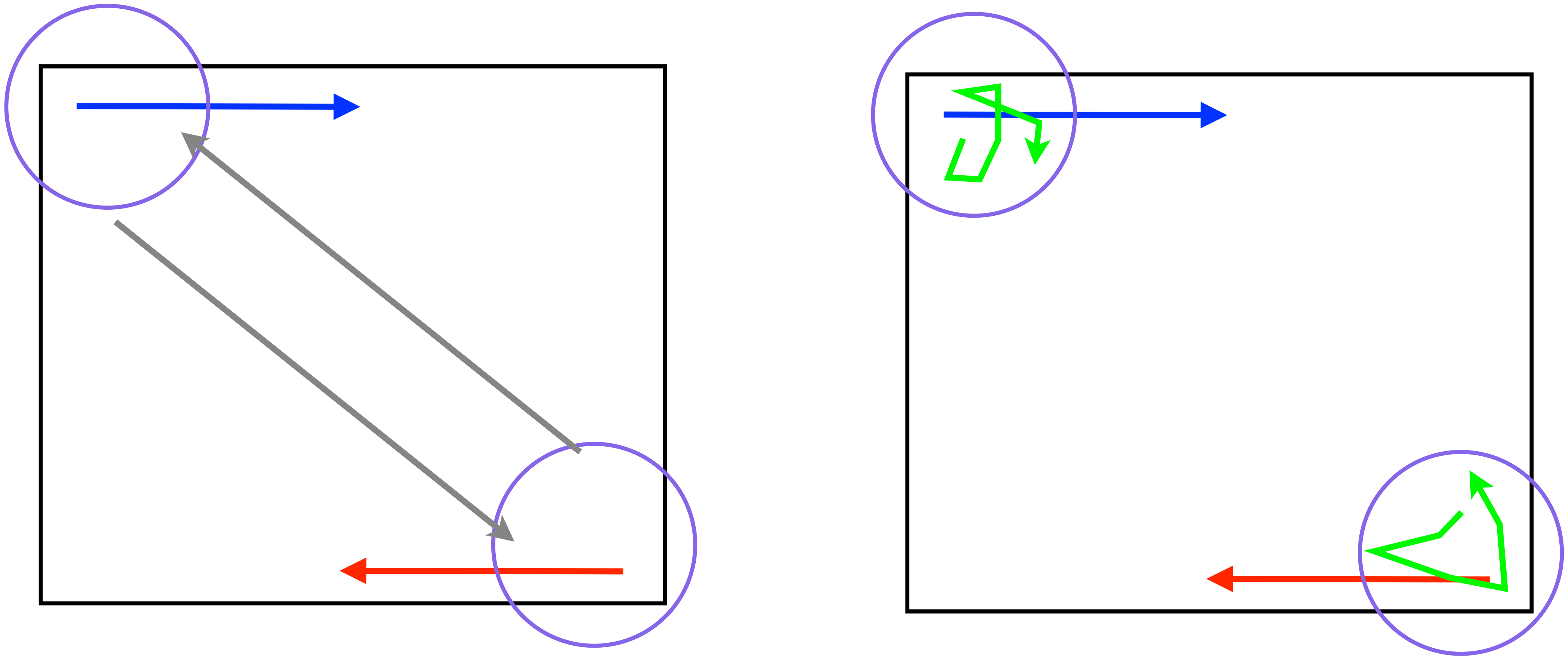
Obtainable formally by linear response Hong Teaney (2010)

The shear viscosity



- Finite shear viscosity smears out flow differences (diffusion)

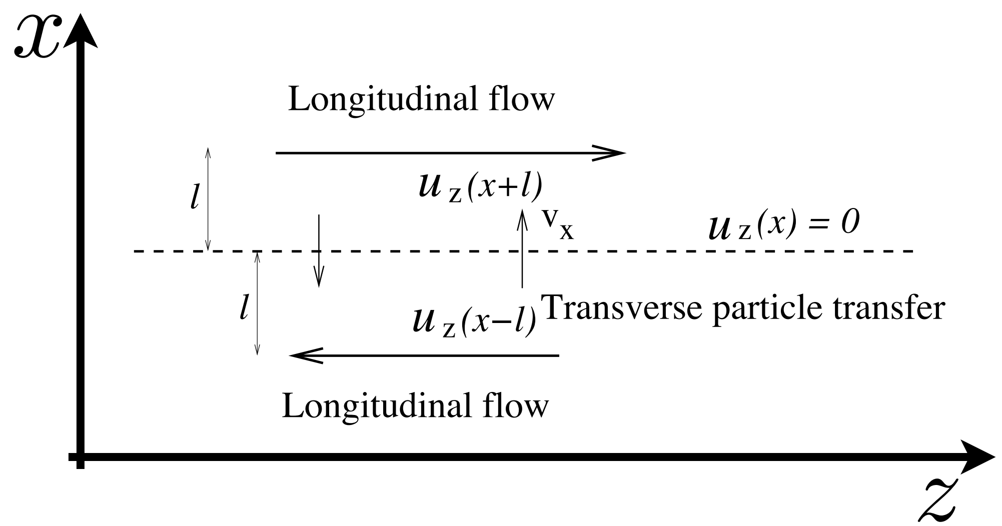
Estimating η: counterintuitive?



- Weak coupling: long distances between collisions, easy diffusion. **Large** η
- Strong coupling: short distances between collisions, little diffusion. **Small** η

Estimating η

- u flow velocity, v_x microscopical velocity of particles



- T^{0z}=$(e+P)u^0u^z$ diffuses along x with v^x=u^x/u^0 . Net change

$$(e+p)v^x u^0 (u^z(x - l_{\rm mfp}) - u^z(x + l_{\rm mfp}) \approx -2(e+p)v^x u^0 l_{\rm mfp} \partial_x u^z(x) \sim -\eta u^0 \partial_x u^z(x)$$

- Using $e + p = sT$ and in the high-T limit (v^x~1)

$$\boxed{\frac{\eta}{s} \sim T l_{\rm mfp}}$$

Estimating η

- (Mean free path)$^{-1}$~ cross section x density

$$\frac{\eta}{s} \sim T l_{\rm mfp} \sim \frac{T}{n\sigma} \sim \frac{1}{T^2\sigma}$$

- Cross section in a perturbative gauge theory (T only scale*)

$$\sigma \sim \frac{g^4}{T^2} \qquad \frac{\eta}{s} \sim \frac{1}{g^4}$$

* Coulomb divergences and screening scales (m_D~gT) in gauge theories

$$\sigma \sim \frac{g^4}{T^2}\ln(1/g) \qquad \frac{\eta}{s} \sim \frac{1}{g^4\ln(1/g)}$$

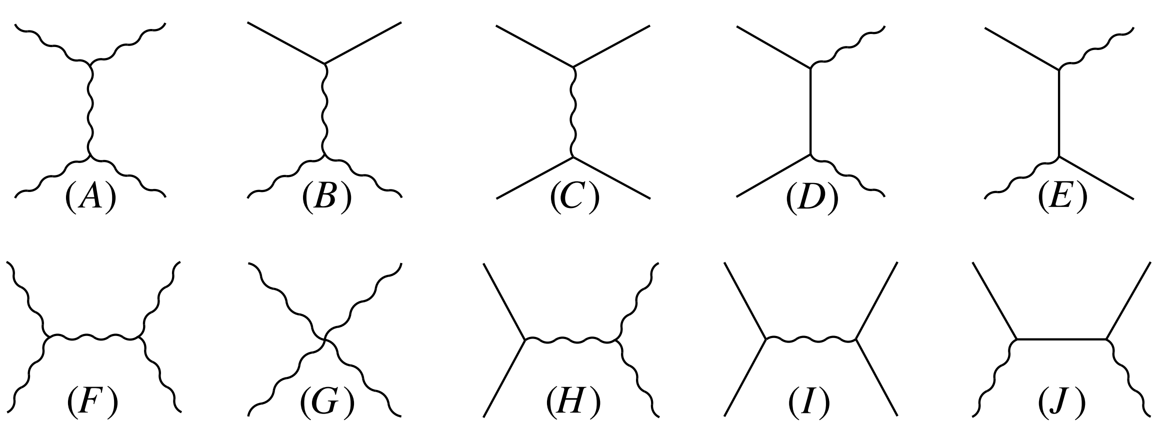
Computing η

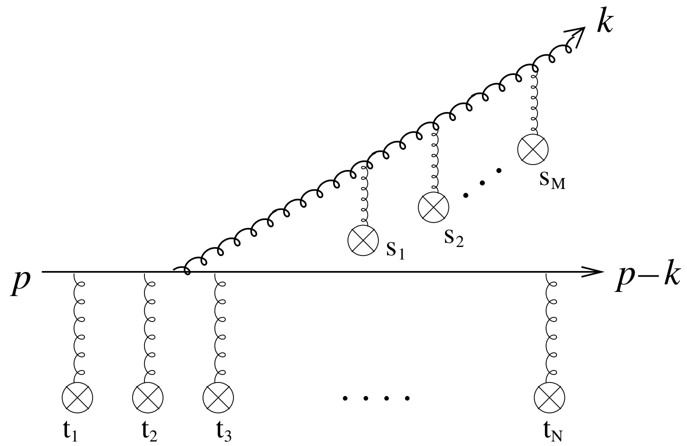
- Kubo formula from *linear response theory* (S TT part of T)

$$\eta = \frac{1}{20}\lim_{\omega\to 0}\frac{1}{\omega}\int d^4x\, \mathrm{e}^{i\omega t}\left\langle\left[\mathcal{S}^{ij}(t,\mathbf{x}),\, \mathcal{S}^{ij}(0,\mathbf{0})\right]\right\rangle\theta(t)$$

- Not practical at weak coupling: use effective kinetic theory with 2↔2 and 1↔2 processes AMY (2000-2003)

$$(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}})f(t,\mathbf{x},\mathbf{p}) = C^{2\leftrightarrow 2}[f] + C^{1\leftrightarrow 2}[f]$$





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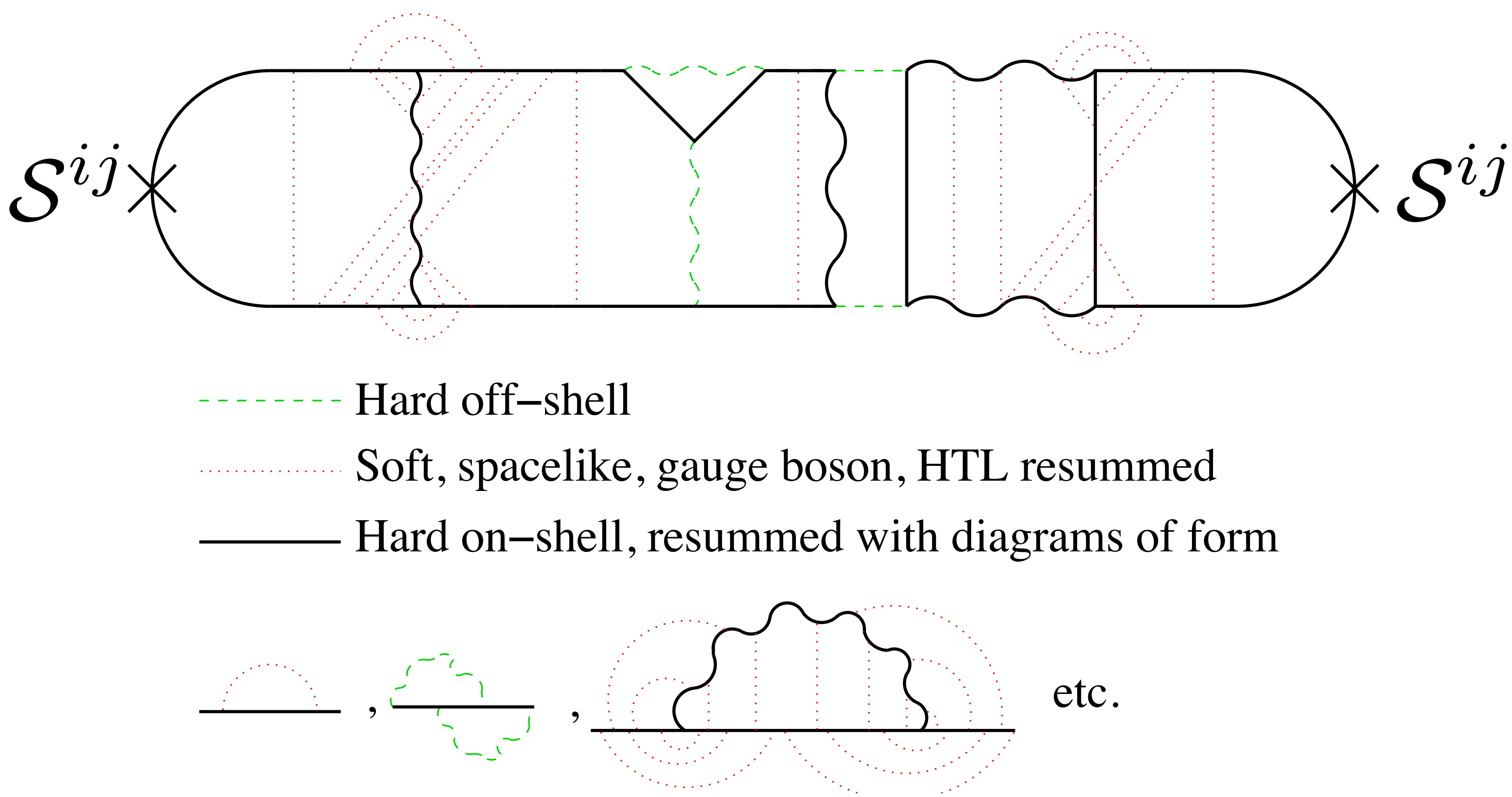

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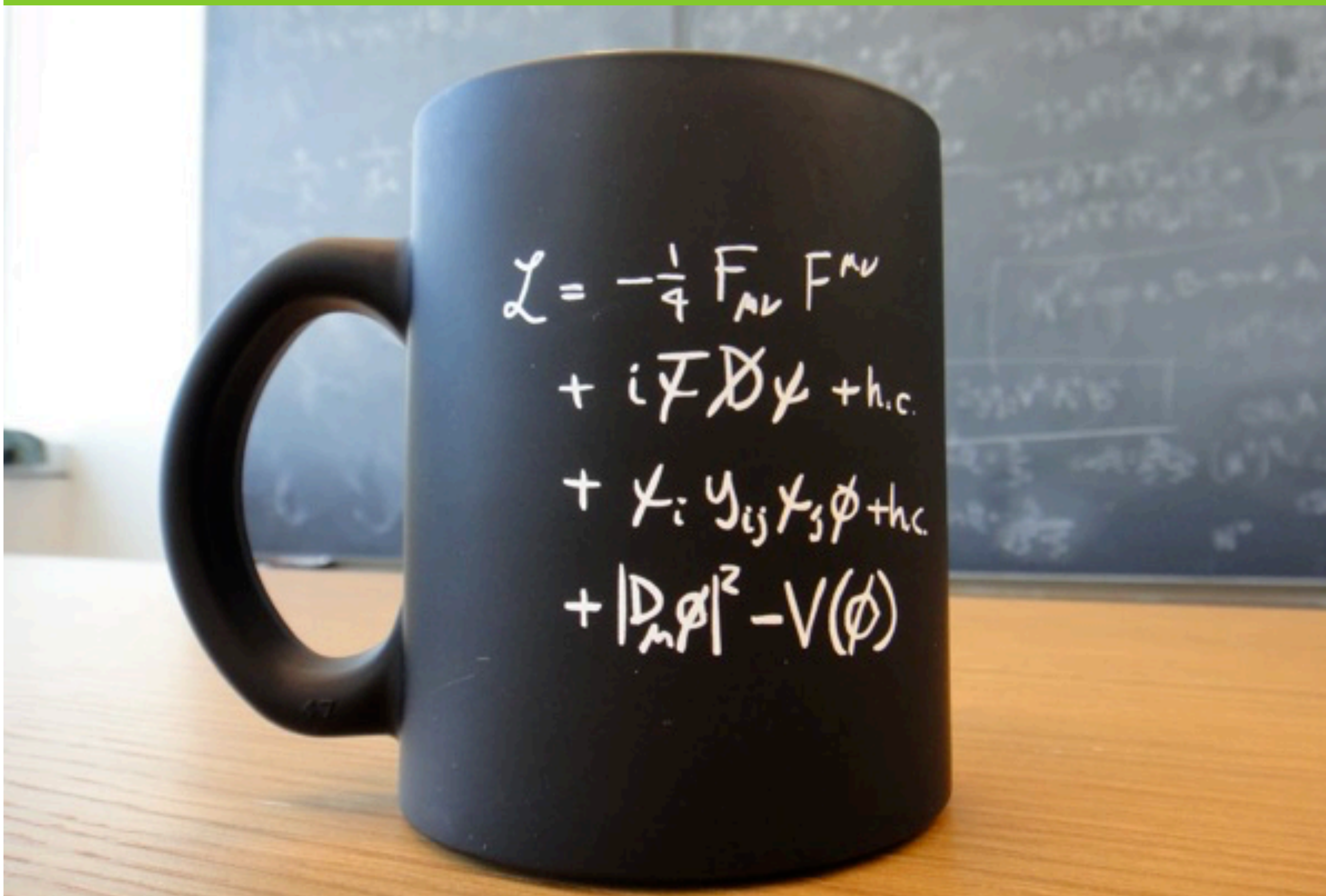
- For the SM at T>160 GeV η is dominated by the slowest processes, those involving right-handed leptons only

$$\eta \simeq \frac{16T^3}{g_1^4\ln(5T/m_{\mathrm{D1}})} \quad \rightarrow \quad \eta \simeq 400\,T^3$$

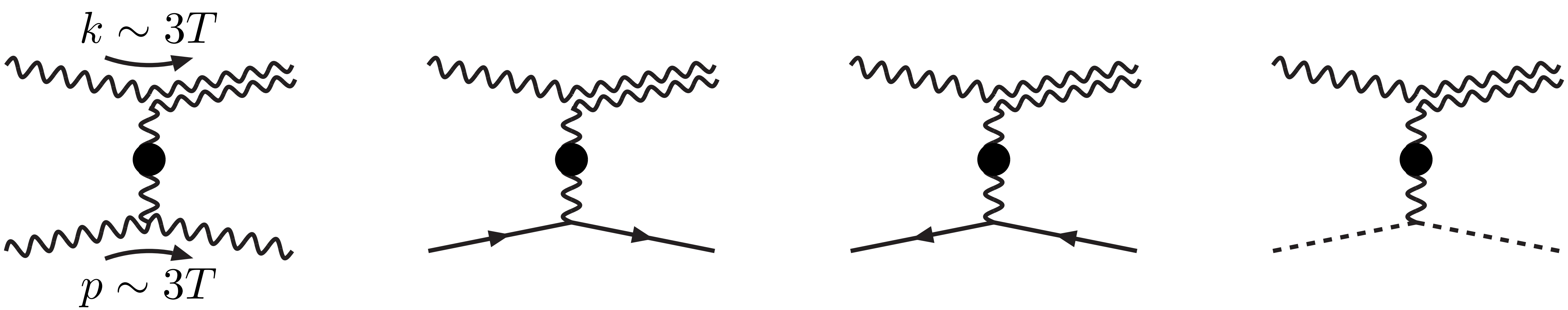
g_1 hypercharge coupling with screening mass $m_{\mathrm{D1}} = \sqrt{11/6}\,g_1 T$
Only a leading-log estimate, no complete LO for T>160 GeV
AMY (2000-2003)

GW rate for k~T at T>160GeV



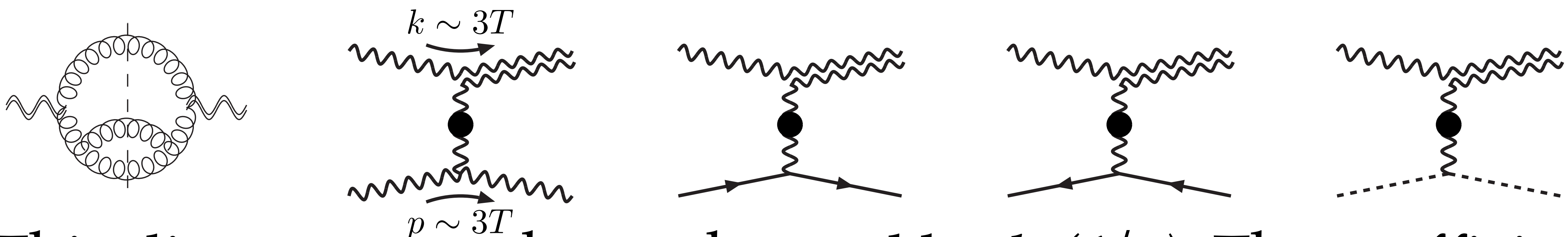
A leading-log estimate

- A complete leading-order calculation for $k{\sim}T$ is not easy: requires all 2↔2 scatterings between SM particles
- However, scatterings with intermediate gauge bosons are IR sensitive: logarithmic divergence with bare propagators



- The cure is Hard Thermal Loop resummation (collective physics: screening, plasma oscillations and Landau damping)

A leading-log estimate



- This diagrams are then enhanced by *ln(1/g)*. The coefficient of this term is easy to determine: a *leading-log calculation*

$$\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}t\,\mathrm{d}\ln k} = \frac{2k^4 T n_B(k)}{\pi^2 m_{\mathrm{Pl}}^2}\left\{\sum_{i=1}^{3} d_i\, m_{\mathrm{D}i}^2 \ln\frac{5T}{m_{\mathrm{D}i}} + \mathcal{O}\left(g^2T^2\chi\left(\frac{k}{T}\right)\right)\right\}$$

- d_i multiplicities of the gauge groups (d_1=1, d_2=3, d_3=8), m_{Di} Debye masses $m_{\mathrm{D1}}^2 = 11g_1^2T^2/6,\ m_{\mathrm{D2}}^2 = 11g_2^2T^2/6,\ m_{\mathrm{D3}}^2 = 2g_3^2T^2$ Non-logarithmic unknown part. QCD leading contribution

- Bonus: all fundamental[1] forces in one equation

[1]As of 24/9/2015

Cosmological implications

Summary

- Our computations can be summarized as

$$\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}t\,\mathrm{d}\ln k} = \frac{16k^3\eta T}{\pi m_{\mathrm{Pl}}^2}\,\phi\Big(\frac{k}{T}\Big)$$

with

$$\phi\Big(\frac{k}{T}\Big) \simeq \begin{cases} 1 & , \quad k \lesssim \alpha^2 T \\ \dfrac{k\,f_{\mathrm{B}}(k)}{8\pi\eta}\displaystyle\sum_{i=1}^{3} d_i\, m_{\mathrm{D}i}^2\Big(\ln\frac{5T}{m_{\mathrm{D}i}} + \mathcal{O}(1)\Big) & , \quad k \gtrsim 3T \end{cases}$$

Embedding in Hubble expansion

- Take as reference temperature $T_0 \equiv 160$ GeV (EW crossover)
- Since $\rho_{\rm GW}(t) = \int_{\mathbf{k}} k\, f(t,k)$, with f GW phase space distribution

$$(\partial_t + 4H)\rho_{\rm GW}(t) = \int_{\mathbf{k}} R(T,k) = \frac{32\pi}{m_{\rm Pl}^2}\int_{\mathbf{k}} 32\pi\eta T\phi(k/T)$$

- Normalize by $s^{4/3}$ to get rid of H and integrate

$$\frac{\rho_{\rm GW}(t_0)}{s^{4/3}(t_0)} = \int_{t_{\rm min}}^{t_0} \mathrm{d}t \int_{\mathbf{k}} \frac{R(T,k)}{s^{4/3}(t)} = \int_{T_0}^{T_{\rm max}} \mathrm{d}T \int_{\mathbf{k}} \frac{R(T,k)}{TH(T)3c_s^2(T)s^{4/3}(T)}$$

at initial time $t_{\rm min}$ (maximum temperature $T_{\rm max}$) no thermally produced GWs present

Embedding in Hubble expansion

- Finally, redshift momenta to reference k_0 at T_0=160 GeV

$$\Omega_{\rm GW}(k_0) \equiv \frac{1}{e(T_0)}\frac{\mathrm{d}\rho_{\rm GW}}{\mathrm{d}\ln k_0}$$
$$= \frac{8k_0^3 s^{1/3}(T_0)}{m_{\rm Pl}\sqrt{6\pi^3}e(T_0)}\int_{T_0}^{T_{\rm max}}\mathrm{d}T\,\frac{\eta(T)}{c_s^2(T)s^{1/3}(T)e^{1/2}(T)}\,\phi\Big(\frac{k_0}{T}\Big[\frac{s(T)}{s(T_0)}\Big]^{\frac{1}{3}}\Big)$$

- Approximate form for $c_s^2 = 1/3$ and dimensional scaling $s = \hat{s}T^3, \eta = \hat{\eta}T^3, e = \hat{e}T^4$

$$\boxed{\frac{\mathrm{d}\,\Omega_{\rm GW}(k_0)}{\mathrm{d}T} \simeq \frac{24\hat{\eta}}{\sqrt{6\pi^3\hat{e}^3}}\,\frac{1}{m_{\rm Pl}}\,\frac{k_0^3}{T_0^3}\,\phi\Big(\frac{k_0}{T_0}\Big)}$$

- Differential form T-independent, so integrated form ($T_{\rm max} \gg T_0$)

$$\boxed{\Omega_{\rm GW}(k_0) \simeq \frac{24\hat{\eta}}{\sqrt{6\pi^3\hat{e}^3}}\,\frac{T_{\rm max}}{m_{\rm Pl}}\,\frac{k_0^3}{T_0^3}\,\phi\Big(\frac{k_0}{T_0}\Big)}$$

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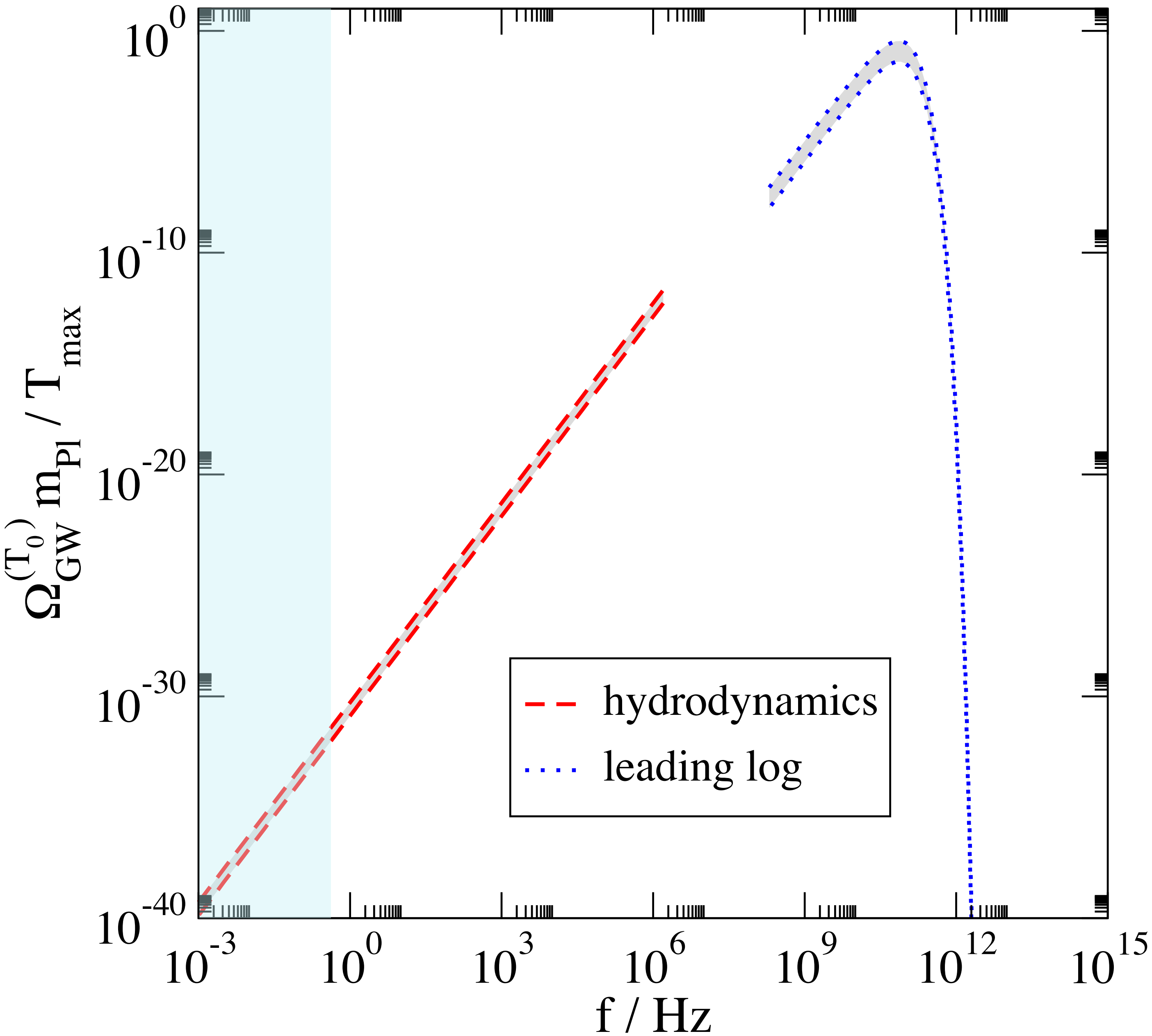
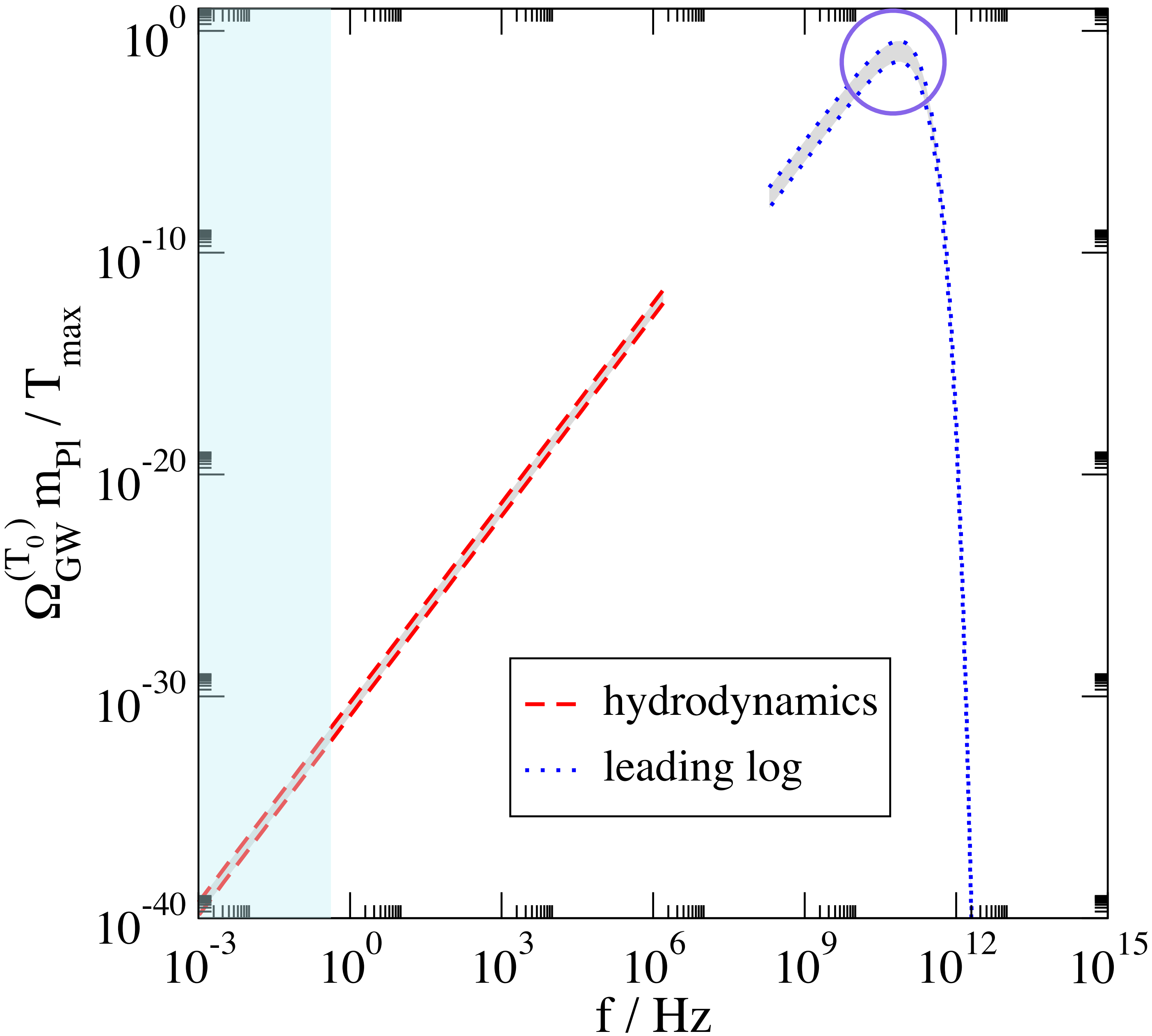
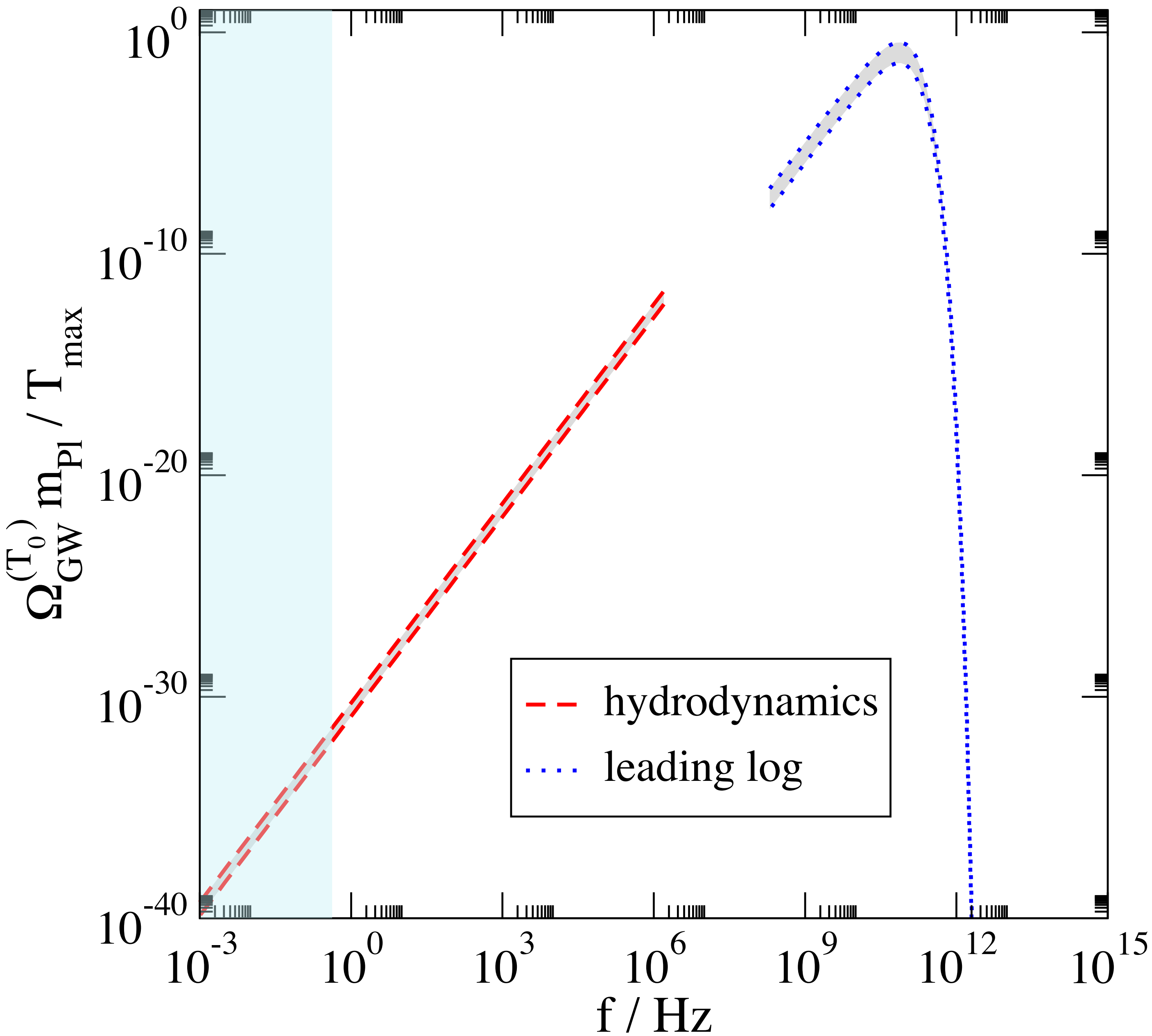


The bands for the hydrodynamic and leading-log results correspond to varying η=100...400 and to varying the constant $O(1)$ within the range 0...10. The couplings were fixed at a scale $\mu = \pi T$ with $T \simeq 10^6$ GeV. For obtaining the current day energy fraction the result needs to be multiplied by $\Omega_{\rm rad} \sim 5 \times 10^{-5}$

$$\Omega_{\rm GW}(k_0) \equiv \frac{1}{e(T_0)}\frac{{\rm d}\rho_{\rm GW}}{{\rm d}\ln k_0} \simeq \frac{24\hat\eta}{\sqrt{6\pi^3\hat e^3}}\frac{T_{\max}}{m_{\rm Pl}}\frac{k_0^3}{T_0^3}\,\phi\Big(\frac{k_0}{T_0}\Big)$$



- The peak is for $k \approx 3.92T$ and redshifts at decoupling to $k_{\rm dec} \approx 3.92 T_{\rm dec} (3.9/106.75)^{1/3} \sim T_{\rm dec}$. Today f≈464 GHz, in the μ-wave range. Amplitude determined by $T_{\max}$.



- This microwave peak could be relevant for future high-frequency experiments
  Chongqing HFGW

- In other words the thermal background continues to grow with k for 10+ decades after the peak eLISA frequency

- If compared to EWPT sources, this peak will eventually overtake their rapidly falling spectra

The total energy

- The peak at $k\sim 4T_{\rm max}$ also implies that the total energy might not be so negligible

$$\int \mathrm{d}\ln k_0\, \Omega_{\rm GW}(k_0) \simeq \frac{24\hat\eta}{\pi\sqrt{6\pi\hat e^3}}\frac{T_{\rm max}}{m_{\rm Pl}T_0^3}\int_0^\infty \mathrm{d}k_0\, k_0^2\, \phi\left(\frac{k_0}{T_0}\right) \simeq \frac{24}{\pi\sqrt{6\pi\hat e^3}}\left(8\ldots\frac{\hat\eta}{3}\right)\frac{T_{\rm max}}{m_{\rm Pl}}$$

- Parametrizing our ignorance of $\phi\left(\frac{k_0}{T_0}\right)$ with two limits (lead-log...hydro)

- GWs are constrained not to carry as much energy as one relativistic d.o.f.
  Smith Pierpaoli Kamionkowski **PRL97** (2006) Henrot-Versille *et al* Class. Quant. Grav. **32** (2015)

The total energy

- At T_0~160 GeV we must then require

$$\boxed{\frac{24}{\pi\sqrt{6\pi\hat{e}^3}}\left(8\ldots\frac{\hat{\eta}}{3}\right)\frac{T_{\rm max}}{m_{\rm Pl}} \ll \frac{1}{100}}$$

- This can be used to constrain T_{max}. For $\hat{e} \sim 35, \ \hat{\eta} \sim 400$ we have $T_{max} \lesssim 10^{17} \ldots 10^{18}$ GeV

- Not a stringent constraint (reheating temperatures above 10^{16} GeV excluded in standard inflation, temperature close to Planck mass), but could be sharpened by knowing more about $\phi\left(\frac{k_0}{T_0}\right)$

Conclusions

- We have shown how to set up the determination of the equilibrium contribution to gravitational waves
- We have determined it at leading order in the infrared: it is related to the shear viscosity of the EW plasma, which is not small
- We have obtained a leading-log estimate for k~T, coming from scatterings of thermal plasma constituents

Conclusions

- The resulting Ω_{GW} is tiny in the eLISA window, but it peaks in the GHz range, where it would overtake non-equilibrium EWPT sources
- The best observational prospect is in future hi-freq. exps
- This thermal background is however guaranteed to be present and, since its production spans many decades, the associated total energy is not small
- This energy can be used to (weakly) constrain the highest temperature of the radiation epoch
- Estimates could be sharpened with a full leading-order calculation