Sterile Neutrino Production in the Early Universe

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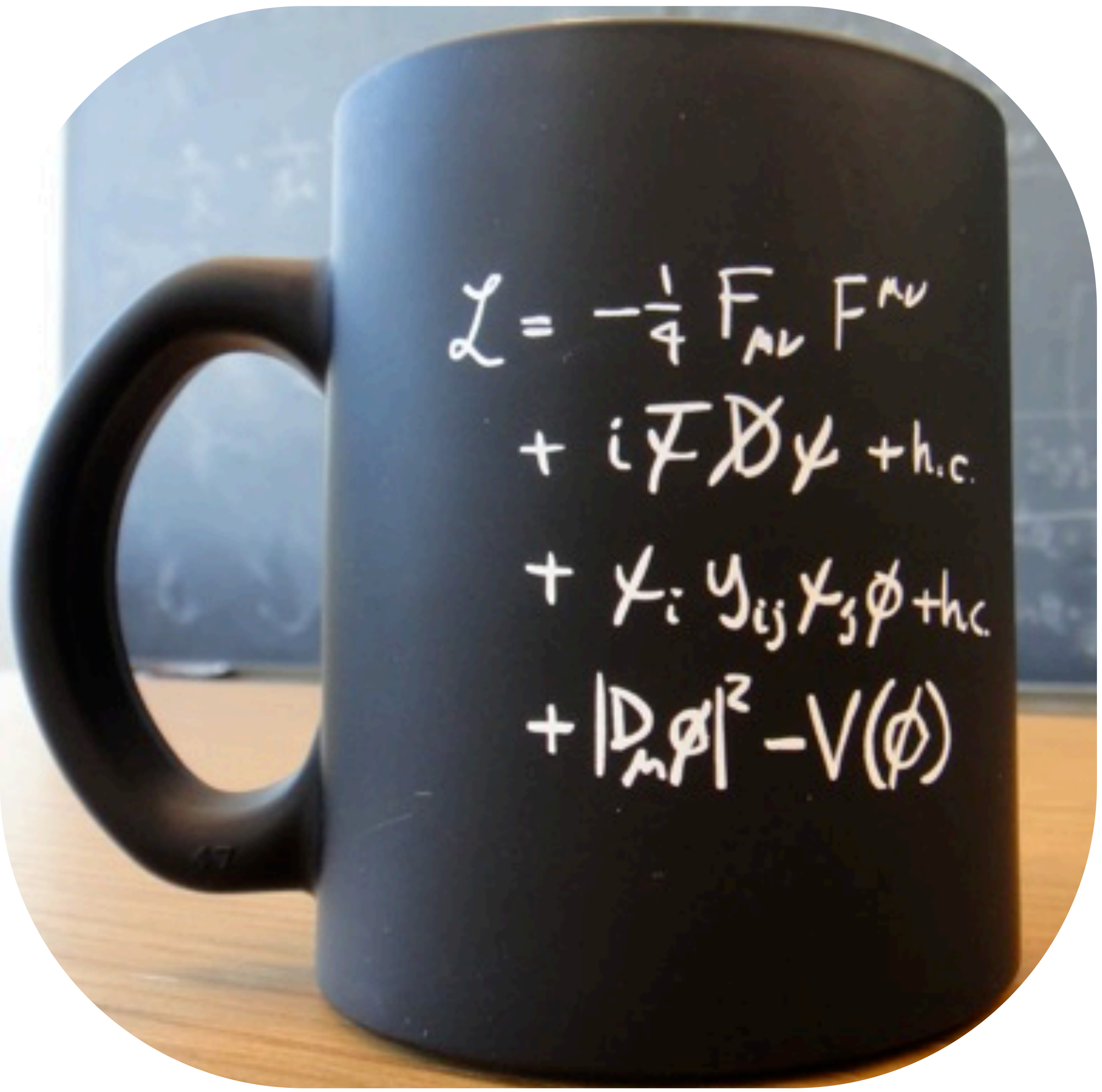
Teilchentee, ITP Heidelberg, 06.07.2017

Outline

- Introduction to right-handed sterile neutrinos
- Theory overview
- Production and washout rates for ultrarelativistic neutrinos
- Resonant production of keV-scale sterile neutrinos
- Conclusions

The SM: (n-1)/n full or 1/n empty?



- The SM seems to do quite well in collider experiments, no smoking mugs there yet
- However
  - Neutrino oscillations (and masses) are unexplained in vanilla SM
  - No mechanism for baryogenesis (more later)
  - No candidate for dark matter (5x more abundant than baryonic matter)

Right-handed neutrinos

- Minimal model: add n sterile (SM gauge singlet), Majorana neutrinos coupling to the three active lepton flavours and the (conjugate) Higgs field

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\sum_I \bar{N}_I (i\gamma^\mu \partial_\mu - M_I) N_I - \sum_{I,a} \left(\bar{N}_I \, h_{Ia} \tilde{\phi}^\dagger a_L \, l_a + \bar{l}_a \, a_R \, \tilde{\phi} \, h^*_{Ia} N_I \right)$$

- h_{Ia} (minimal) Yukawa coupling

- At $T \ll T_{\rm EW}$=160 GeV: EW symmetry breaks $\tilde{\phi} \simeq (v\,0)^T/\sqrt{2}$

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\sum_I \bar{N}_I (i\gamma^\mu \partial_\mu - M_I) N_I - \sum_{I,a} \left(\bar{N}_I \, M^\dagger_{D\,Ia} a_L \, \nu_a + \bar{\nu}_a \, a_R \, M_{D\,ai} N_I \right)$$

$M_{D\,ai} = h^\dagger_{aI} v/\sqrt{2}$: Dirac mass connects left- and right-handed spinors (European color coding, sorry USA friends)

Seesaw

- Seesaw: when $M_D \ll M_I$ diagonalization yields
  - n almost purely sterile states with masses $\sim M_I$
  - 3 almost purely active states with masses given by the roots of the eigenvalues of $M_D(1/M_I)M_D^T$
- Gauge-invariant generation of a mass term for the left-handed neutrinos Minkowski Gell-Mann Ramond Slansky Yanagida Glashow Mohapatra Senjanovic
  Possible also through scalar exchange Magg Wetterich Lazarides Shafi Mohapatra Senjanovic Schecter Valle
- In general mass and flavor bases do not coincide ⇒ oscillations

Baryogenesis

- Need to satisfy Sakharov's conditions
  - B violation
  - C and CP violation
  - Deviations from thermal equilibrium

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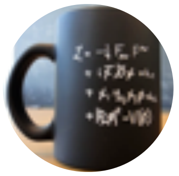


Feynman rules always conserve B, but sphaleron processes violate B (and conserve B-L)

- Non-perturbative solutions, in equilibrium at $T > T_{\mathrm{EW}}$, exponentially suppressed below. Decouple at $T \sim 130$ GeV
  D'Onofrio Rummukainen Tranberg **PRL113** (2014)

Baryogenesis

- Need to satisfy Sakharov's conditions
  - B violation
  - C and CP violation
  - Deviations from thermal equilibrium

The CKM phase violates CP

No mechanism for a deviation from equilibrium. For m_{H}=125 GeV the electroweak transition is a crossover

- Electroweak baryogenesis not possible in vanilla SM

Leptogenesis

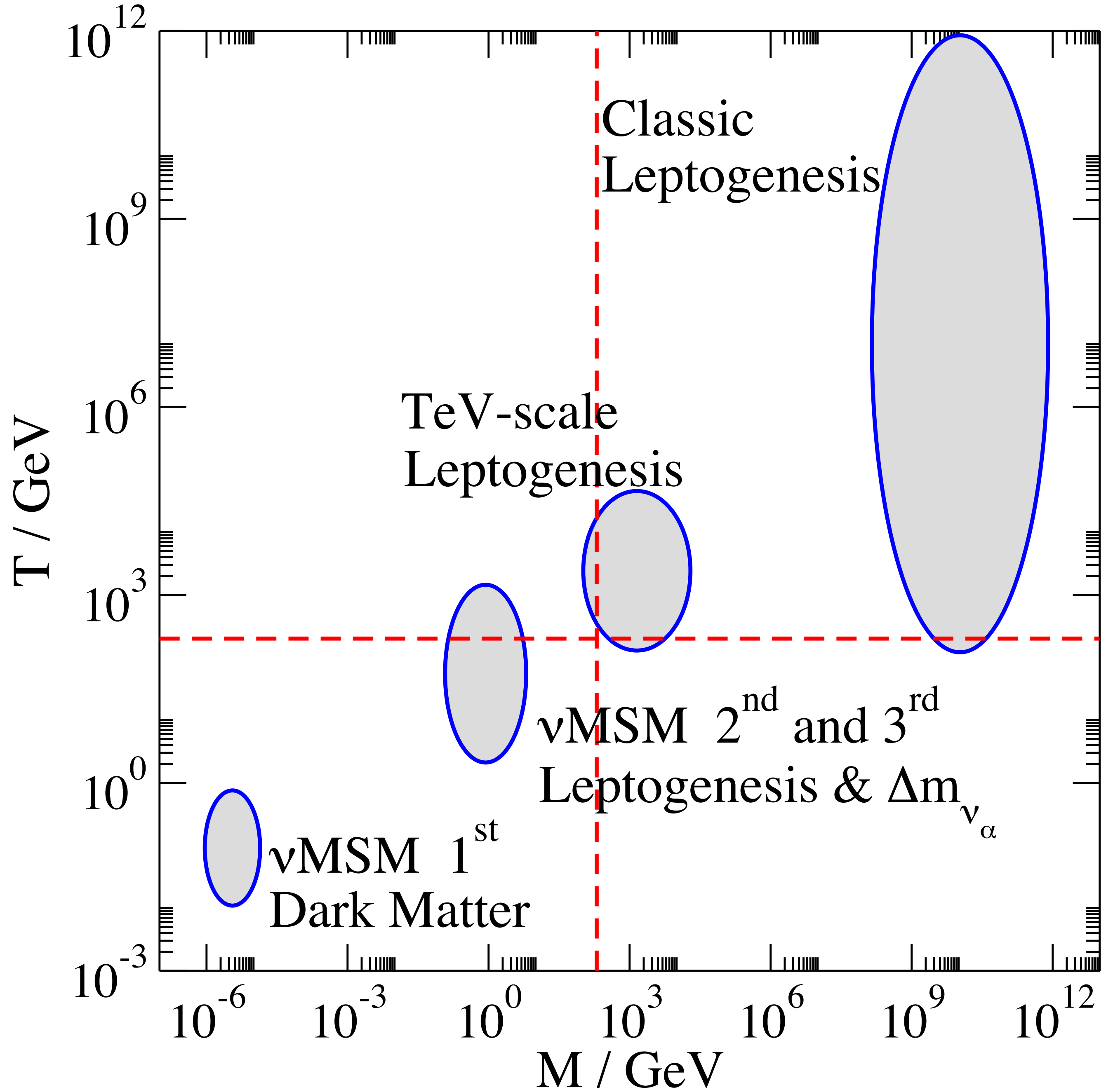
- Main idea: generate L first (BSM) and then let sphalerons turn it into B
- Sphalerons provide ~~B~~
- Lepton-neutrino Yukawas provide ~~CP~~
- Model-dependent mechanisms for ~~equilibrium~~
  - "Classic leptogenesis": massive ($M \gg T_{\mathrm{EW}}$) RHN
    1) produced thermally $T \gtrsim M$ ($l\phi \to N$)
    2) decay out of equilibrium ($N \to l\phi$) when $T \ll M$ (no inverse process) with CP violating phases, thus generating lepton imbalance
    Fukugita Yanagida **PLB174** (1986)

Leptogenesis

- Main idea: generate L first (BSM) and then let sphalerons turn it into B
- Sphalerons provide ~~B~~
- Lepton-neutrino Yukawas provide ~~CP~~
- Model-dependent mechanisms for ~~equilibrium~~
    - "ARS leptogenesis": GeV scale RHNs
      1) produced thermally at $T > T_{EW} \gg M$ conserving CP
      2) oscillations of N and their CP violating mixings create L_I for the I flavors, which can then be transformed into B in certain conditions
      Akhmedov Rubakov Smirnov **PRL81** (1998)

Leptogenesis

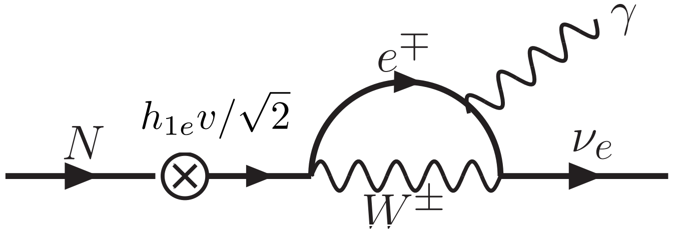
- Other scenario: νMSM. Two GeV RHNs for ARS leptogenesis, a keV one for dark matter Asaka Blanchet Shaposhnikov **PLB620, PLB631** (2005)



Plot by Mikko Laine

Dark matter

- A sterile neutrino can be a good DM candidate. No gauge interactions, sufficiently long lived.
- Why keV?
  - Fermionic DM cannot be arbitrarily packed together. Inferred DM density cannot exceed degenerate Fermi gas phase space density ($\propto M^4$) $\Rightarrow$ lower bound on the mass
    Tremaine Gunn **PRL42** (1979)
  - Radiative decay N→νγ creates a monocromatic (X-ray) line. Decay width $\propto M^5$. Non-observation yields upper bound on the mass. Recent disputed hints of a 3.55 keV line observation



Dark matter

- keV-scale RHN DM would not be Cold Dark Matter. If spectrum is thermal it would be Warm Dark Matter, if not more complicated spectra. Might solve some CDM discrepancies Lovell *et al* **1605.03179 1611.00005 1611.00010**
- Production would happen in the early universe from the mixing with active neutrinos.
    - In the absence of a lepton asymmetry at production time, thermal production proceeds non-resonantly. Strong tension with observational bounds
      Dodelson Widrow **PRL72** (1994)
    - If a lepton asymmetry is present, MSW-type resonant production Shi Fuller **PRL82** (1999)

Theory overview

General approach

- Many theory approaches in the literature for right handed neutrino dynamics (production, leptogenesis, washout) in the early universe
  - Boltzmann equations Giudice Notari Raidal Riotto Strumia...
  - Closed-time path, Kadanoff-Baym equations Garny Kartavtsev Hohenegger Lindner Garbrecht Beneke Buchmüller Drewes Mendizabal Weniger...
  - Operatorial approach Bödeker Laine Sangel Wormann...
  - ...
- Review to appear soon Biondini *et al*... **1707.xxxxx**

General approach

- Factor the system into "fast" and "slow" modes, and integrate out the former to obtain evolution eqs. for the latter

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- For instance

for 130 GeV $\lesssim T \lesssim 10^5$ GeV, all SM interactions are in thermal equilibrium

O(GeV) RHNs have ~10^{-7} Yukawas: non-eq. ensemble

Lepton (and baryon) densities also evolve slowly

Textbook example: thermal production

- Assume an equilibrated hot bath (QGP, early universe) with its internal coupling g and a particle ϕ, weakly coupled (coupling h) to other d.o.f.s, so that ϕ is not in equilibrium

$$\mathcal{L} = \mathcal{L}_\phi + h\phi^* J + h^* J^* \phi + \mathcal{L}_{\rm bath}$$

  J built of bath operators

- With a simple derivation one obtains that the rate (per unit volume) is proportional to a thermal average of a JJ correlator

$$\frac{d\Gamma_\phi}{d^3k} = \frac{|h|^2}{2E_k}\Pi^<(k) = \frac{|h|^2}{2E_k}\int d^4X e^{iK\cdot X}{\rm Tr}\,\rho_{\rm bath}J(0)J(x)$$

- The expression is LO in h but to all orders in g

In this talk

- Computing reliably the lepton asymmetry in a specific scenario is usually challenging (CP violation, oscillations, plasma physics)

- On the other hand, establishing

  - the production rate of RHNs

  - whether an existing asymmetry gets *washed out*

  allows to put constraints (or rule out) scenarios

- In this talk: the production and washout rates for GeV-scale RHNs (ARS leptogenesis) and for keV scale DM RHNs in the resonant case

General structure of the evolution equations

- By applying the slow-fast factorization to this case one can obtain coupled equations for the right-handed phase space distribution and the lepton asymmetry

$$\begin{cases} \dot{f}_{I\mathbf{k}} &= \gamma_{I\mathbf{k}} \left(n_{\rm F}(E_I) - f_{I\mathbf{k}} \right) \\ \dot{n}_a &= -\gamma_{ab} n_b \end{cases}$$

- The equilibration and washout rates are related to the spectral function of the SM current $j_a = \tilde{\phi}^\dagger a_L \, l_a$

- Detailed derivation and structure, accounting for helicity and flavor effects, in JG Laine **JHEP1705** (2017)

GeV-scale production and washout rates

- Three relevant scales: M, T and $T_{\rm EW}$~160 GeV
- For M~GeV $\pi T \gg M$ down to ~5 GeV
- Previous calculations in the symmetric phase for all kinematic ranges
  M$\gg \pi T$: Salvio Lodone Strumia (2011), Laine Schröder (2012), Biondini Brambilla Escobedo Vairo (2012),
  M$\lesssim \pi T$: Garbrecht Glowna Herranen (2013), Laine (2013), Anisimov Besak Bödeker (2010-12), Ghisoiu Laine (2014)
- In this talk $\pi T \gg M$ in the broken phase (new) and in the symmetric phase
  JG Laine **JCAP1607** (2016)

The rates in detail

$$\Pi_{\rm E}(K) \equiv {\rm Tr}\Big\{ i\not{K} \int_0^{1/T} {\rm d}\tau \int_{\mathbf{x}} e^{iK\cdot X} \Big\langle (\tilde{\phi}^\dagger a_L\, l)(X)\, (\bar{l}\, a_R\, \tilde{\phi})(0) \Big\rangle_T \Big\}$$

$$\rho(K) \equiv {\rm Im}\, \Pi_{\rm E}(K)|_{k_n \to -i(k_0 + i\epsilon)}$$

- RHN equilibration rate

$$\dot{f}_{I\mathbf{k}} = \gamma_{I\mathbf{k}} \left(n_{\rm F}(E_I) - f_{I\mathbf{k}}\right) + \mathcal{O}\left[\left(n_{\rm F} - f_{I\mathbf{k}}\right)^2, n_a^2\right]$$

$$\gamma_{I\mathbf{k}} = \sum_a \frac{|h_{Ia}|^2 \rho(K)}{E_I} + \mathcal{O}(h^4)$$

Approach to equilibrium of the RHN phase space distribution (on-shell RHNs, $E_I = (\mathbf{k}^2 + M^2)^{1/2}$)

Bödeker Sangel Wörmann **PRD93** (2015)

The rates in detail

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$$\rho(K) \equiv {\rm Im}\, \Pi_{\rm E}(K)|_{k_n \to -i(k_0 + i\epsilon)}$$

- Washout rate for the lepton number for flavour *a*

$$\dot{n}_a = -\gamma_{ab} n_b + \mathcal{O}[n_a(n_{\rm F} - f_{I\mathbf{k}}), n_a^3]$$

$$\gamma_{ab} = -\sum_I \int \frac{d^3k}{(2\pi)^3} \frac{2n_{\rm F}'(E_I)|h_{Ia}|^2 \rho(K)}{E_I} \Xi^{-1}_{ab} + \mathcal{O}(h^4)$$

Depends on the susceptibility $\Xi_{ab} = \partial n_a / \partial \mu_b|_{\mu_b = 0}$
not diagonal because of charge neutrality constraints

Bödeker Laine **JCAP05** (2014)

Computing ρ

- In the broken phase the Higgs e.v. v>0. We consider the parametric range $T \gtrsim v$, so that thermal masses ($O(gT)$) and Higgs mechanism masses ($O(gv)$) are of the same order. In practice

$$30\,\mathrm{GeV} \lesssim T \lesssim 160\,\mathrm{GeV}$$

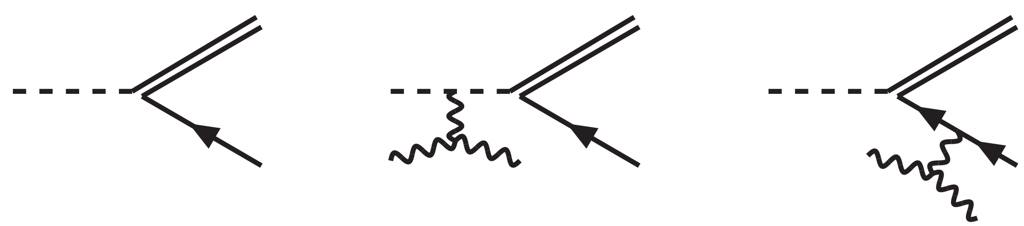
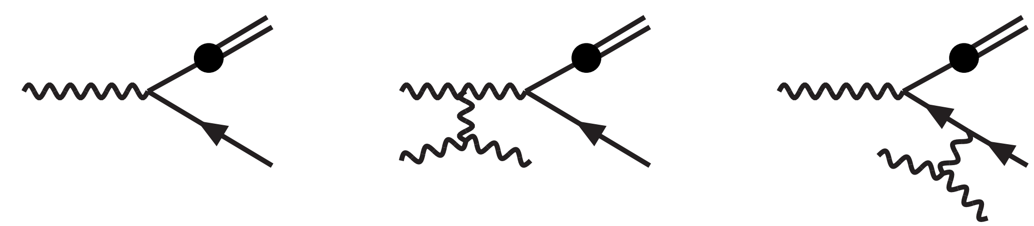
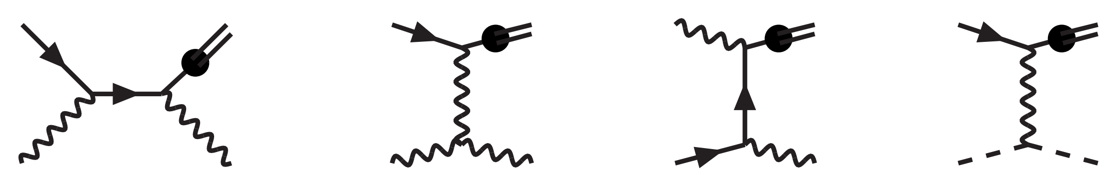
  where $g=(g_1,g_2,h_t,\lambda^{1/2})$ (parametrically equivalent)

  - In this region $M_I \lesssim gT$

- We also consider $m_W \gtrsim \pi T$ to cover the low-temperature region down to 5 GeV

$$\Pi_{\mathrm{E}}(K) \equiv \mathrm{Tr}\Big\{ i\not{K} \int_0^{1/T} \mathrm{d}\tau \int_{\mathbf{x}} e^{iK\cdot X} \Big\langle (\tilde{\phi}^\dagger a_L\, l)(X)\, (\bar{l}\, a_R\, \tilde{\phi})(0) \Big\rangle_T \Big\}$$

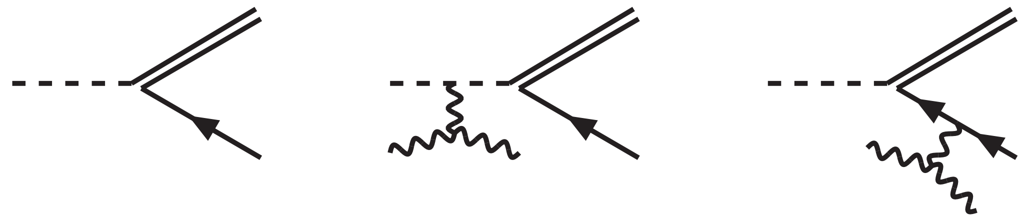
- The Higgs doublet can be a propagating d.o.f. (Higgs or Goldstone) or an expectation value insertion. Distinction into direct and indirect processes

	Direct	Indirect
1↔2	 and others, and crossings	 and others, and crossings
2↔2	 and others, and crossings	 and others, and crossings

- Only the sum is gauge invariant. Feynman R_ξ gauge simplest

- Direct processes give $\varrho \sim g^2 T^2$. Indirect processes can have a near-resonant enhancement (hold on)

Direct 1↔2 processes



- Since all masses are $O(gT)$, tree level processes (if possible) are $\sim m^2 \sim g^2T^2$ and collinear
- Long formation times $O(1/g^2T)$) imply that soft scatterings, at rate g^2T, need to be resummed to all orders ⇒ Landau-Pomeranchuk-Migdal (LPM) effect
  Long QCD history (BDMPS, AMY). Introduced for RHNs in the *symmetric phase* in Anisimov Besak Bödeker **JCAP03** (2011), Besak Bödeker **JCAP03** (2012), Ghisoiu Laine **JCAP12** (2014)

Symmetric phase LPM

- In the **symmetric phase**

$$\rho(K)^{\rm LPM} = \frac{1}{16\pi}\int_{-\infty}^{\infty} d\omega \left[1 - n_{\rm F}(\omega) + n_{\rm B}(k_0-\omega)\right] \frac{k_0}{k_0-\omega}$$
$$\times \lim_{\mathbf{y}\to\mathbf{0}} \; 4\left\{\frac{M^2}{k_0^2}{\rm Im}\left[g\;(\mathbf{y})\right] + \frac{1}{\omega^2}{\rm Im}\left[\nabla_\perp\cdot\mathbf{f}\;(\mathbf{y})\right]\right\}$$

- The functions **f** and *g* encode the resummed soft interactions through

$$\hat{H} \;\equiv\; -\frac{M^2}{2k_0} + \frac{m_l^2-\nabla_\perp^2}{2\omega} + \frac{m_\phi^2-\nabla_\perp^2}{2(k_0-\omega)} - i\,\Gamma(y)$$

$$(\hat{H}+i0^+)\,g(\mathbf{y}) \;=\; \delta^{(2)}(\mathbf{y})\,, \quad (\hat{H}+i0^+)\,\mathbf{f}(\mathbf{y}) \;=\; -\nabla_\perp\delta^{(2)}(\mathbf{y})$$

where m_l and m_ϕ are the thermal masses of leptons and scalars and the soft interactions are ($m_{{\rm E}i}$ screening masses)

$$\Gamma(y) = \frac{T}{8\pi}\sum_{i=1}^{2} d_i\,g_i^2\left[\ln\left(\frac{m_{{\rm E}i}y}{2}\right) + \gamma_{\rm E} + K_0\left(m_{{\rm E}i}y\right)\right]$$

Symmetric phase LPM

- In **QCD** (photon/dilepton production)

$$\rho(K)^{\rm LPM} = \frac{N_c}{\pi}\int_{-\infty}^{\infty}{\rm d}\omega\big[1-n_{\rm F}(\omega)-n_{\rm F}(k_0-\omega)\big]$$
$$\times \lim_{\mathbf{y}\to\mathbf{0}}\left\{\frac{M^2}{k_0^2}{\rm Im}\left[g(\mathbf{y})\right]+\left(\frac{1}{2\omega^2}+\frac{1}{2(k_0-\omega)^2}\right){\rm Im}\left[\nabla_\perp\cdot\mathbf{f}(\mathbf{y})\right]\right\}$$

- The functions **f** and g encode the resummed soft interactions through

$$\hat{H} \;\equiv\; -\frac{M^2}{2k_0}+\frac{m_q^2-\nabla_\perp^2}{2\omega}+\frac{m_q^2-\nabla_\perp^2}{2(k_0-\omega)}-i\,\Gamma(y)$$

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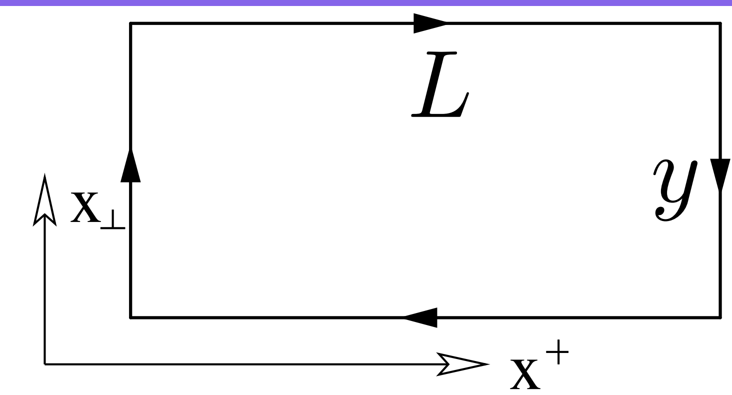
where m_q is the thermal mass of quarks and the soft interactions are ($m_{\rm D}$ SU(3) screening mass)

$$\Gamma(y) = \frac{g^2C_FT}{2\pi}\left[\ln\left(\frac{m_Dy}{2}\right)+\gamma_E+K_0(m_Dy)\right]$$

The soft interactions



$$\propto e^{-\Gamma(y)L}$$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- All points at spacelike or lightlike separation, only preexisting correlations
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008)
    - Can be "easily" computed in perturbation theory
    - Possible lattice QCD measurements Laine Rothkopf **JHEP1307** (2013) Panero Rummukainen Schäfer **PRL112** (2014)

Euclideanization of light-cone soft physics

- For $t/x_z = 0$: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n, p) e^{i\mathbf{p}\cdot\mathbf{x}}$$

Caron-Huot **PRD79** (2009)

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- Change variables to $\tilde{p}^z = p^z - p^0(t/x^z)$

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- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

$$G_{rr}(t,\mathbf{x}) = T\sum_n \int dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)} G_E(\omega_n, p_\perp, p^z + i\omega_n t/x^z)$$

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- Soft physics dominated by *n=0* (and *t*-independent) =>EQCD!

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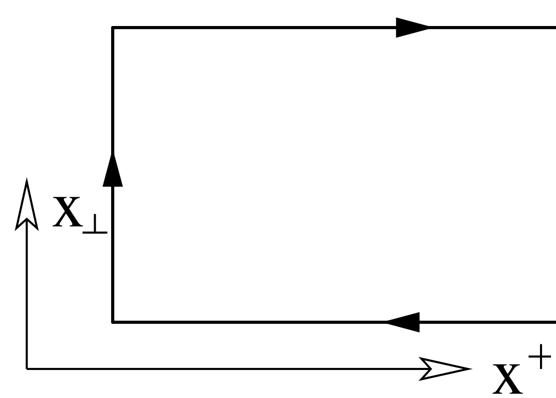
- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

$$G_{rr}(t,\mathbf{x})_{\rm soft} = T\int d^3p\, e^{i\mathbf{p}\cdot\mathbf{x}}\, G_E(\omega_n = 0, \mathbf{p})$$

- Soft physics dominated by *n=0* (and *t*-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics



$$\propto e^{-\Gamma(y)L}$$

- At leading order

$$\Gamma(y) \;\propto T\int \frac{d^2q_\perp}{(2\pi)^2}\left(1-e^{i\mathbf{x}_\perp\cdot\mathbf{q}_\perp}\right)G_E^{++}(\omega_n=0,q_z=0,q_\perp) = T\int \frac{d^2q_\perp}{(2\pi)^2}\left(1-e^{i\mathbf{x}_\perp\cdot\mathbf{q}_\perp}\right)\left(\frac{1}{q_\perp^2}-\frac{1}{q_\perp^2+m_D^2}\right)$$

- Agrees with the earlier sum rule in Aurenche Gelis Zaraket **JHEP0205** (2002)

Symmetric phase LPM

- In the **symmetric phase**

$$\rho(K)^{\mathrm{LPM}} = \frac{1}{16\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega \left[1 - n_{\mathrm{F}}(\omega) + n_{\mathrm{B}}(k_0-\omega)\right] \frac{k_0}{k_0-\omega}$$

$$\times \lim_{\mathbf{y}\to\mathbf{0}} \; 4 \left\{ \frac{M^2}{k_0^2}\mathrm{Im}\left[g\;(\mathbf{y})\right] + \frac{1}{\omega^2}\mathrm{Im}\left[\nabla_\perp \cdot \mathbf{f}\;(\mathbf{y})\right] \right\}$$

- The functions **f** and *g* encode the resummed soft interactions through

$$\hat{H} \;\equiv\; -\frac{M^2}{2k_0} + \frac{m_l^2 - \nabla_\perp^2}{2\omega} + \frac{m_\phi^2 - \nabla_\perp^2}{2(k_0-\omega)} - i\,\Gamma(y)$$

$$(\hat{H} + i0^+)\, g(\mathbf{y}) \;=\; \delta^{(2)}(\mathbf{y})\;, \quad (\hat{H} + i0^+)\, \mathbf{f}(\mathbf{y}) \;=\; -\nabla_\perp \delta^{(2)}(\mathbf{y})$$

where m_l and m_ϕ are the thermal masses of leptons and scalars and the soft interactions are ($m_{\mathrm{E}i}$ screening masses)

$$\Gamma(y) = \frac{T}{8\pi}\sum_{i=1}^{2} d_i\, g_i^2 \left[\ln\left(\frac{m_{\mathrm{E}i}y}{2}\right) + \gamma_{\mathrm{E}} + K_0\left(m_{\mathrm{E}i}y\right)\right]$$

Broken phase LPM

$$\rho(K)^{\mathrm{LPM}} = \frac{1}{16\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega \left[1 - n_{\mathrm{F}}(\omega) + n_{\mathrm{B}}(k_0 - \omega)\right] \frac{k_0}{k_0 - \omega}$$
$$\times \lim_{\mathbf{y}\to\mathbf{0}} \; 4 \left\{ \frac{M^2}{k_0^2}\mathrm{Im}\left[g\;\;(\mathbf{y})\right] + \frac{1}{\omega^2}\mathrm{Im}\left[\nabla_\perp \cdot \mathbf{f}\;\;(\mathbf{y})\right] \right\}$$

- Broken electroweak symmetry implies
  - Broken degeneracy of scalar masses $m_\phi^2 \to \mathrm{diag}(m_{\phi_0}^2, m_{\phi_3}^2, m_{\phi_1}^2)$
  - Soft interactions become sensitive to "vacuum" masses and to the electromagnetic charges

Broken phase LPM

$$\rho(K)^{\mathrm{LPM}} = \frac{1}{16\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega \left[1 - n_{\mathrm{F}}(\omega) + n_{\mathrm{B}}(k_0 - \omega)\right] \frac{k_0}{k_0 - \omega}$$
$$\times \lim_{\mathbf{y}\to\mathbf{0}} \sum_{\mu=0}^{3} \left\{ \frac{M^2}{k_0^2} \mathrm{Im}\left[g_\mu(\mathbf{y})\right] + \frac{1}{\omega^2} \mathrm{Im}\left[\nabla_\perp \cdot \mathbf{f}_\mu(\mathbf{y})\right] \right\}$$

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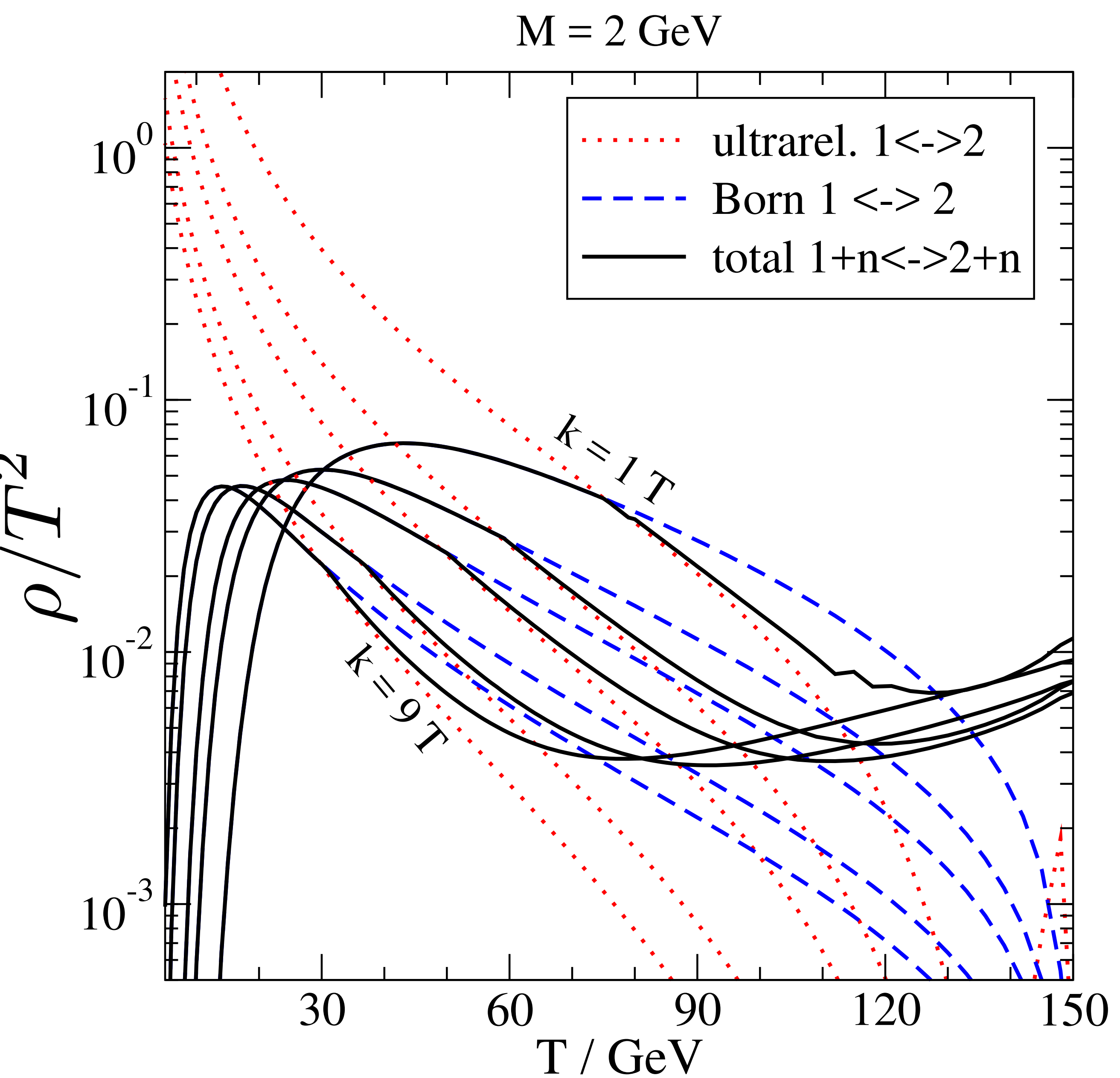
  - Soft interactions become sensitive to "vacuum" masses and to the electromagnetic charges

$\Rightarrow$ Matrix structure between the $\nu\phi_0$, $\nu\phi_3$ and $e\phi_\pm$ states

$$\Gamma_{3\times 3} = \begin{pmatrix} 2\Gamma_W(0) + \Gamma_Z(0) & -\Gamma_Z(y) & -2\Gamma_W(y) \\ -\Gamma_Z(y) & 2\Gamma_W(0) + \Gamma_Z(0) & -2\Gamma_W(y) \\ -\Gamma_W(y) & -\Gamma_W(y) & 2\Gamma_W(0) + \Gamma_{Z'}(0) - \Gamma_{Z'}(y) \end{pmatrix}$$

Direct 1↔2 processes

- Red: tree level processes with collinear ($m \ll T$) approx. Unphysical growth at low T
- Blue: full tree level with m_l=0, proper m_ϕ, accurate at low T
- Black: full solution of the LPM equations at high T, manually switched to blue at low T. **Final 1↔2 result**



Direct 2↔2 processes

- As long as all external state masses are $O(gT)$ or $O(gv)$ they can be neglected at leading order ($O(g^2T^2)$). Hence, no change with respect to the symmetric phase evaluation in Besak Bödeker **JCAP03** (2012)

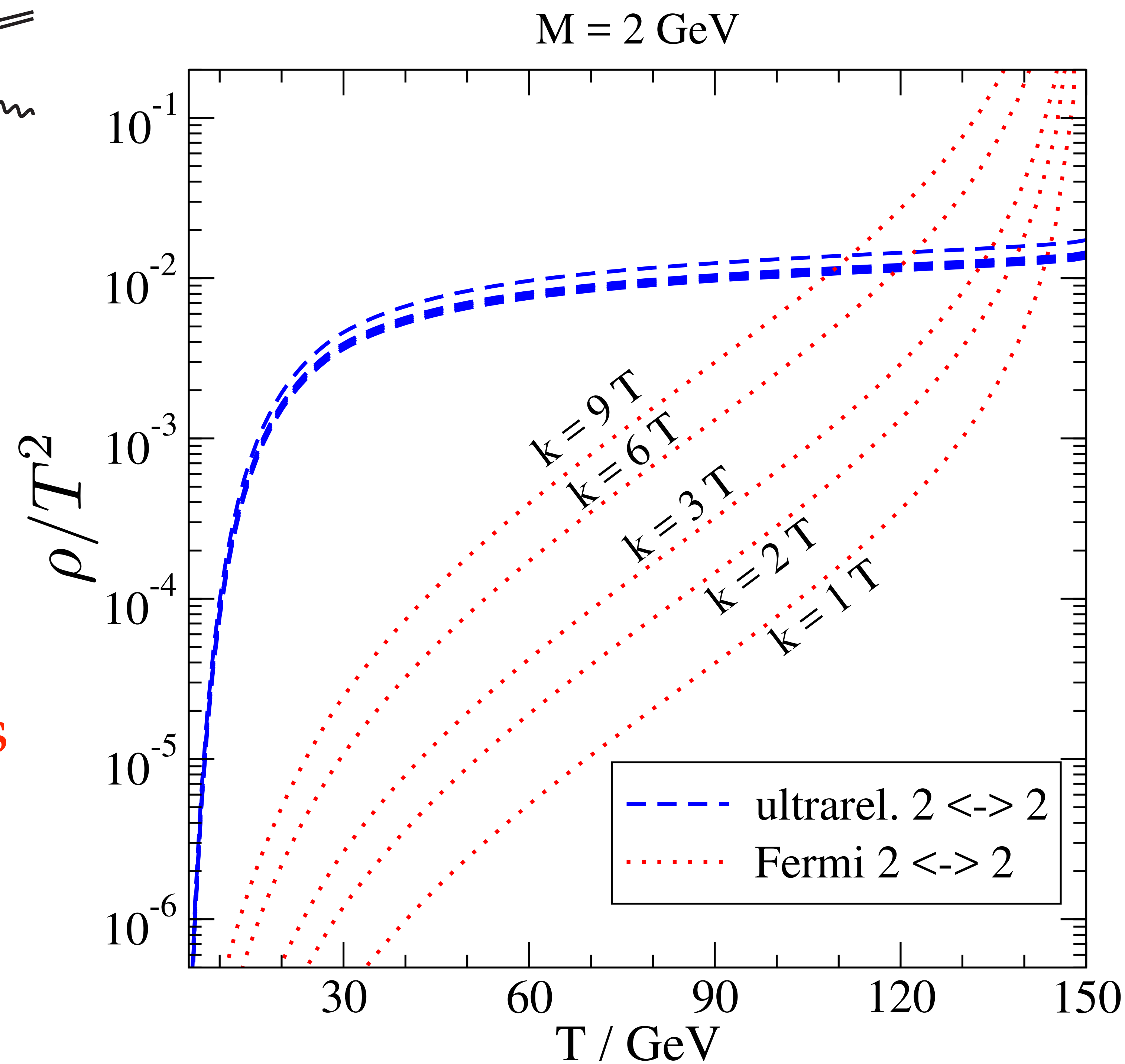
$$\int_{\mathrm{ph.\,space}} f(p)f(p')(1\pm f(k'))|\mathcal{M}|^2\delta^4(P+P'-K-K')$$

- Phase space convolution of statistical functions and matrix elements. HTL resummation needed for soft fermion exchange. Analiticity arguments lead to a simple form for the soft part of the result Besak Bödeker **JCAP03** (2012) JG Hong Lu Kurkela Moore Teaney **JHEP05** (2013)

Direct 2↔2 processes

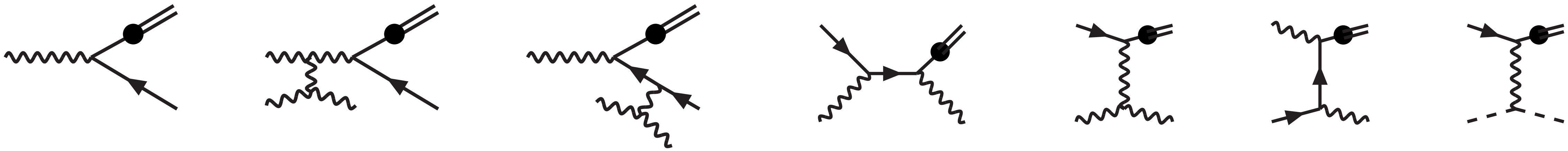
- As long as all external state masses are $O(gT)$ or $O(gv)$ they can be neglected at leading order ($O(g^2T^2)$). Hence, no change with respect to the symmetric phase evaluation in Besak Bödeker **JCAP03** (2012)
- At low $T<m_W$ initial state bosons (scalar or gauge) are very massive. We switch off the rate at low T by multiplying it for the W boson susceptibility
- The formally leading-order contribution at low T is scalar-mediated scatterings off b quarks. We find it is however negligible

Direct 2↔2 processes



- Besak-Bödeker rate times the *W* boson susceptibility
- Scalar-mediated scatterings off *b* quarks in the Fermi limit

Indirect processes

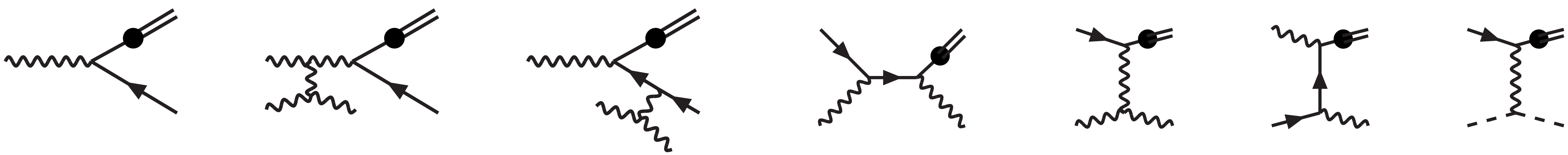


- In the indirect case ρ is directly proportional to the spf of active neutrinos, i.e.

$$\rho(K)^{\text{indir.}} = \frac{v^2}{2} \frac{M^2 \, 2K \cdot \text{Im}\,\Sigma}{(M^2 + 2K \cdot \text{Re}\,\Sigma)^2 + 4(K \cdot \text{Im}\,\Sigma)^2}$$

- Real part of the active neutrino self- energy
  - At high T $\;2K \cdot \text{Re}\,\Sigma = -m_l^2 \sim g^2 T^2$
  - At low T (positive) matter potential
  - (Broad) resonance

Indirect processes



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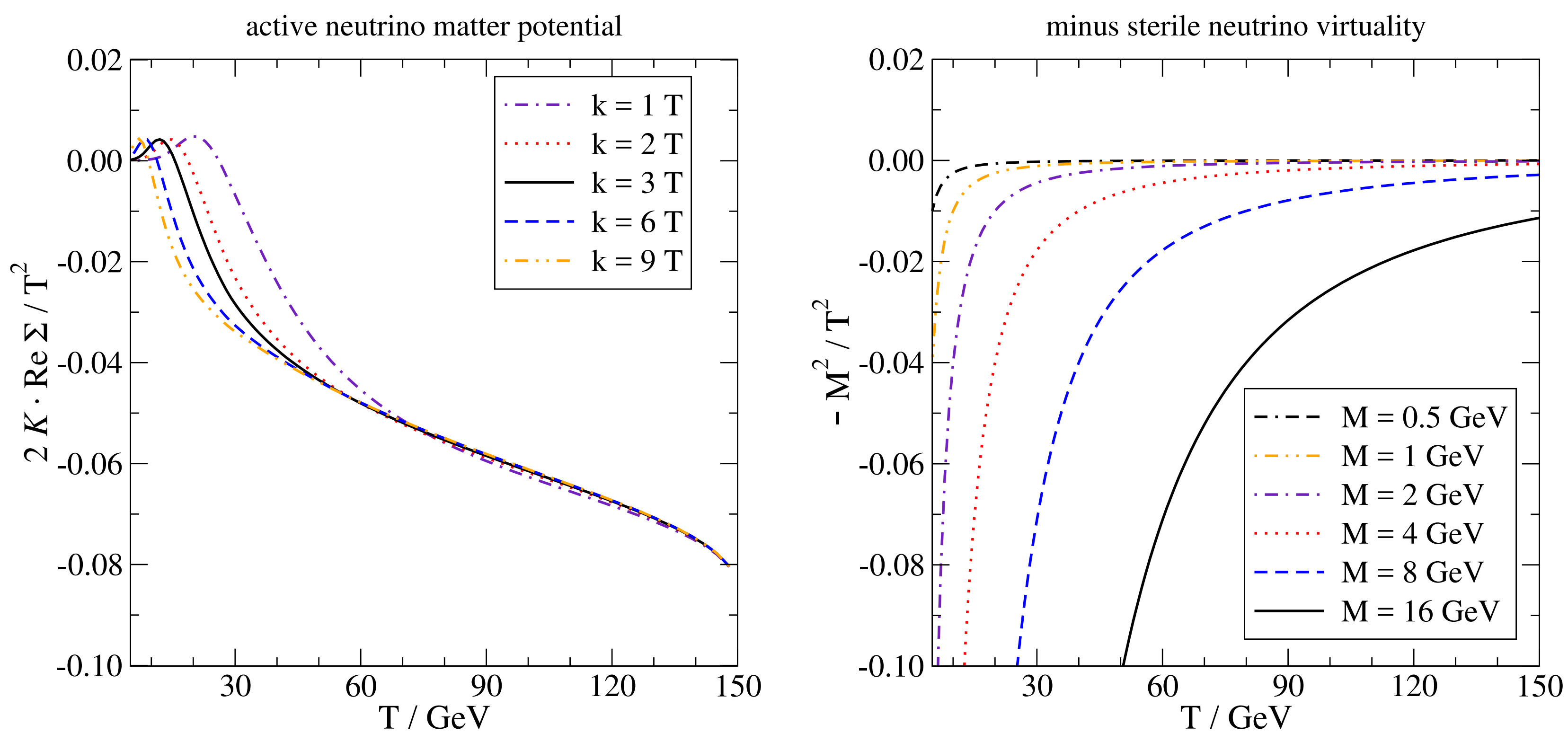
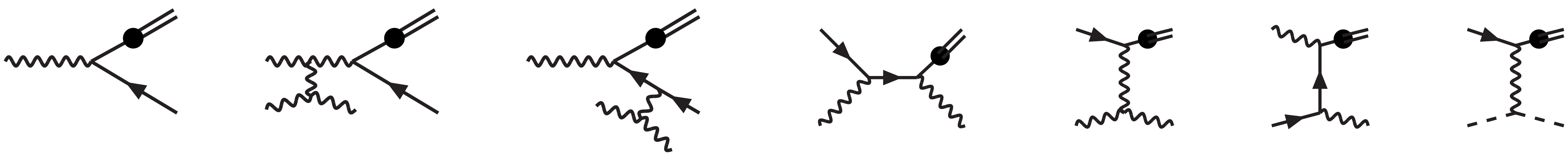


Indirect processes

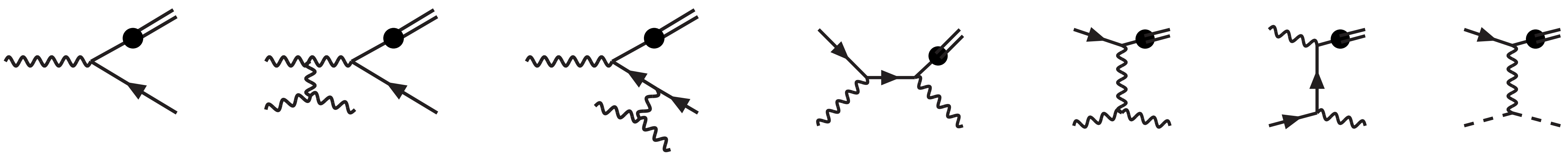


- In the indirect case ρ is directly proportional to the spf of active neutrinos, i.e.

$$\rho(K)^{\text{indir.}} = \frac{v^2}{2} \frac{M^2\, 2K \cdot \mathrm{Im}\,\Sigma}{(M^2 + 2K \cdot \mathrm{Re}\,\Sigma)^2 + 4(K \cdot \mathrm{Im}\,\Sigma)^2}$$

- Imaginay part of the active neutrino self- energy: active neutrino width $2K \cdot \mathrm{Im}\,\Sigma = k_0 \Gamma$
  - At high T dominated by soft 2↔2 scatterings. $\Gamma \sim g^2 T$ and thus (for $M \sim gT$) $\rho \sim v^2$
  - At low T dominated by 1↔2 decays of gauge bosons

Indirect processes



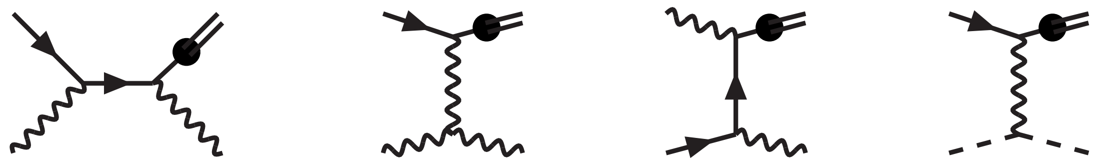
- In the indirect case ρ is directly proportional to the spf of active neutrinos, i.e.

$$\rho(K)^{\text{indir.}} = \frac{v^2}{2} \frac{M^2\, 2K \cdot \mathrm{Im}\,\Sigma}{(M^2 + 2K \cdot \mathrm{Re}\,\Sigma)^2 + 4(K \cdot \mathrm{Im}\,\Sigma)^2}$$

- Real part of the active neutrino self- energy
- Imaginary part of the active self energy
- Medium-modified mixing angle squared

$$\theta^2_{\text{med}} = \frac{v^2}{2} \frac{M^2}{(M^2 + 2K \cdot \mathrm{Re}\,\Sigma)^2 + 4(K \cdot \mathrm{Im}\,\Sigma)^2}$$

Indirect 2↔2 processes

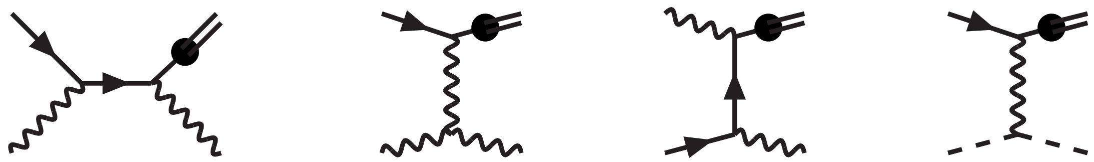


- Naively $\Gamma \sim g^4 T$, but soft ($Q \sim gT$) t-channel gauge boson scatterings have a large enhancement. Need to resum the "vacuum" masses and Hard Thermal Loops

- Euclideanization (Caron-Huot **PRD82** (2008)) still applicable. In the W exchange case
$$\Gamma_W^{\rm soft} = \frac{g_2^2 T}{4\pi}\int_0^\infty dq_\perp\, q_\perp \left[\frac{1}{q_\perp^2 + m_W^2} - \frac{1}{q_\perp^2 + m_W^2 + m_{E2}^2}\right]$$
transverse Euclidean propagator (vacuum mass only) - longitudinal propagator (vacuum and SU(2) screening mass)

- Z exchange more complicated (mixing of $SU(2)_L$ and $U(1)_Y$) but conceptually the same

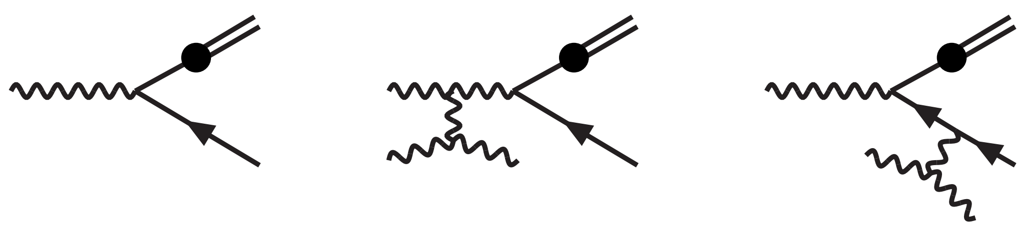
Indirect 2↔2 processes



$$\Gamma_W^{\rm soft} = \frac{g_2^2 T}{4\pi} \int_0^\infty dq_\perp \, q_\perp \left[\frac{1}{q_\perp^2 + m_W^2} - \frac{1}{q_\perp^2 + m_W^2 + m_{E2}^2} \right]$$

- At low T these approximations are inaccurate, they don't go into the Fermi limit
- We replace them with the Fermi limit results from Asaka Laine Shaposhnikov **JHEP01** (2007) (in a more compact form, as the masses of all scatterers are negligible for T>5 GeV)

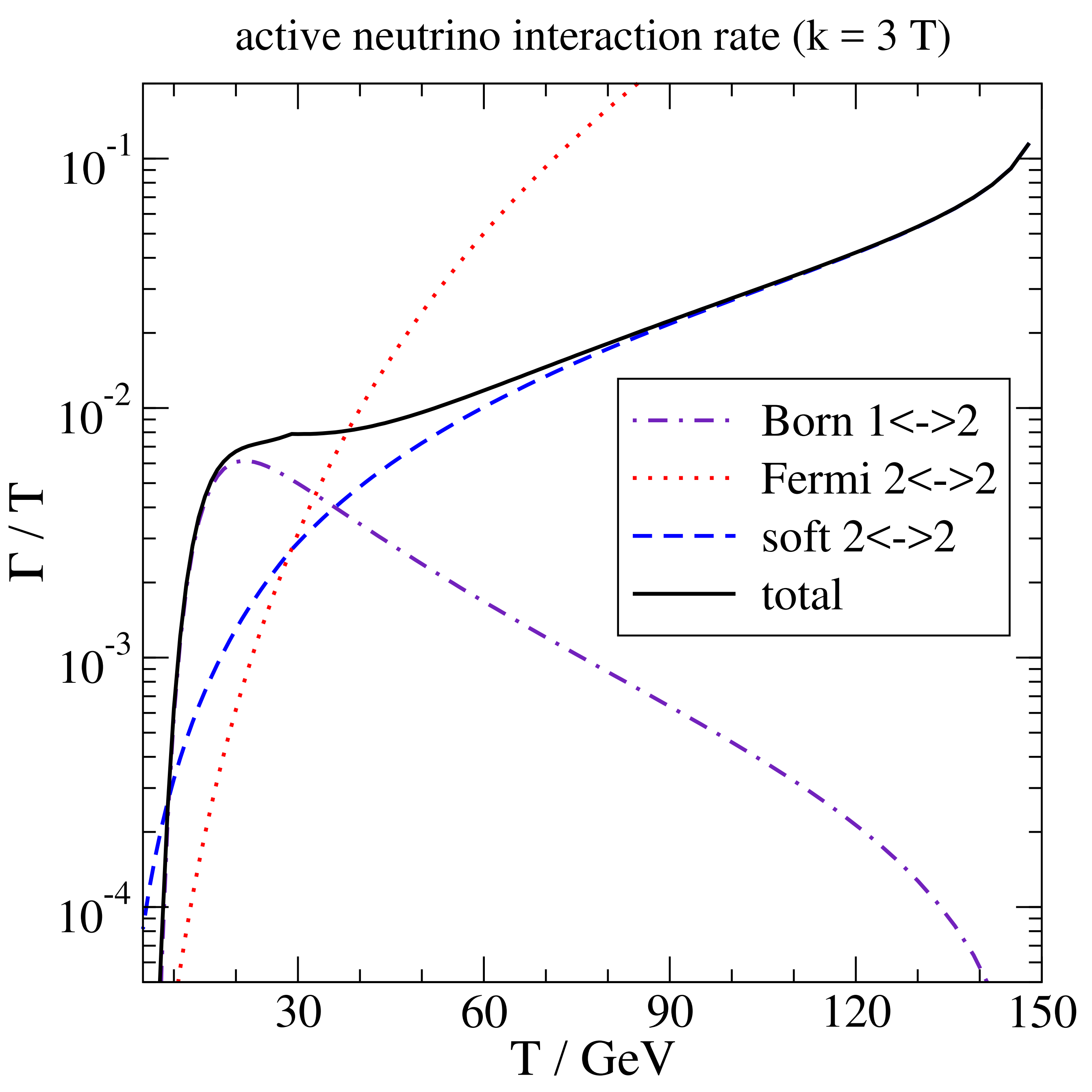
Indirect 1↔2 processes



- At **high** ***T*** they are very similar to the direct 1↔2 processes, with the scalar replaced by a gauge boson and the coupling $h \rightarrow g$. Hence $k_0\Gamma \sim g^2 m^2 \sim g^4 T^2$ and thus **negligible** w.r.t. the indirect 2↔2 processes
- At **low** ***T*** the LPM effect becomes negligible. The Born-level decays of gauge bosons into leptons $k_0\Gamma \sim g^2 m^2$ become the **leading contribution**, also w.r.t the indirect 2↔2 processes

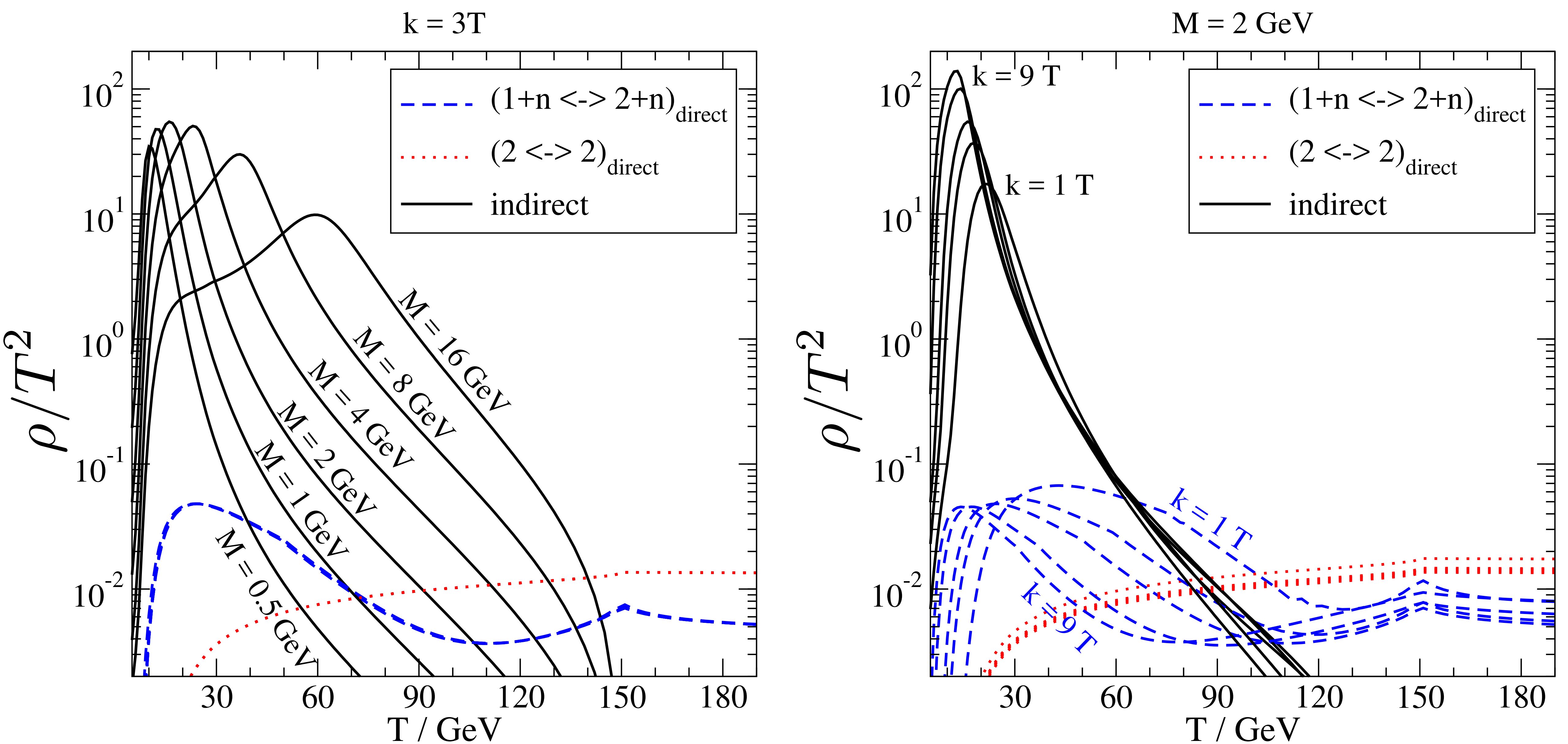
Indirect processes

- Soft 2↔2 scatterings, leading at high *T*
- 2↔2 scatterings in the Fermi limit, accurate but subleading at low *T*
- Born 1↔2 rate, leading at low *T*, inaccurate but negligible at high *T*
- **Total:** 1↔2 + the appropriate (smallest) 2↔2



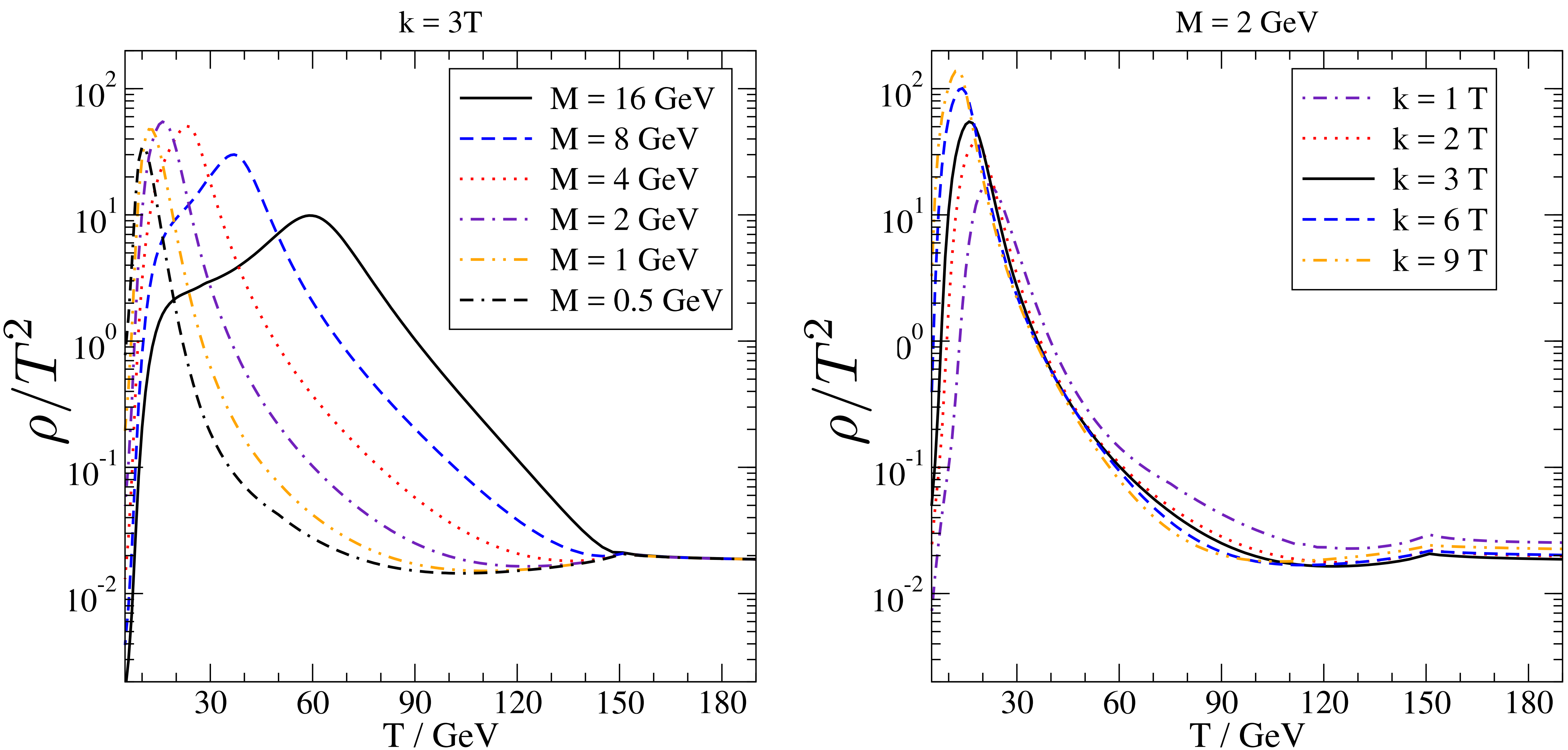
Results

- Indirect processes rapidly dominate and peak at low T (in our 1-loop parameter fixing $T_{EW} \approx 150$ GeV)



Results

- Spectra available for download at http://www.laine.itp.unibe.ch/production-midT/



Cosmological implications

- Compare the equilibration and washout rates to the Hubble rate

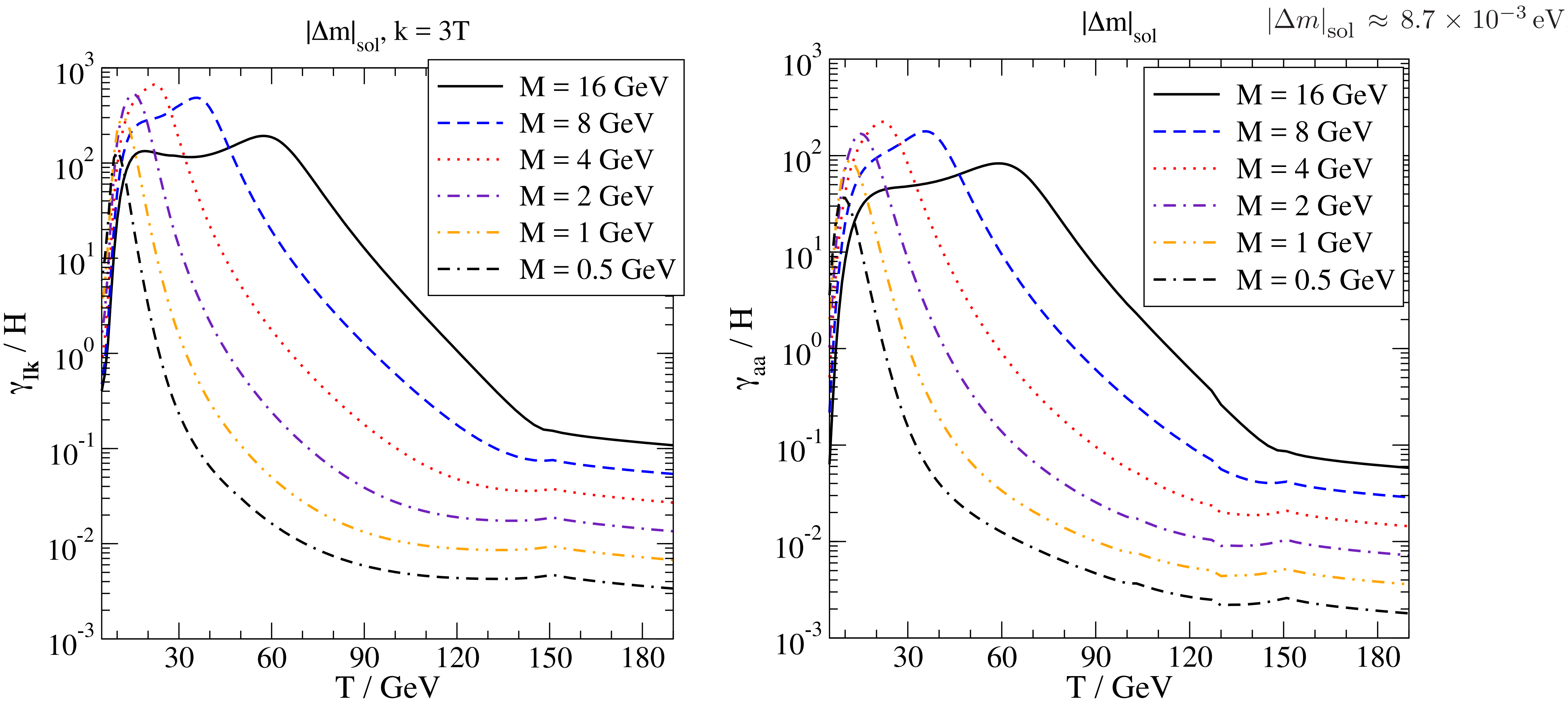
$$\gamma_{I\mathbf{k}} = \sum_a \frac{|h_{Ia}|^2 \rho(K)}{E_I}$$

$$H = \sqrt{\frac{8\pi e}{3 m_{\rm Pl}^2}}$$

$$\gamma_{ab} = -\sum_I \int \frac{d^3k}{(2\pi)^3} \frac{2 n_{\rm F}'(E_I) |h_{Ia}|^2 \rho(K)}{E_I} \Xi^{-1}_{ab}$$

- Fix the RHNs Yukawa couplings in a seesaw scenario with hierarchical neutrinos, with only one Yukawa coupling contributing to a given mass difference $|\Delta m| = |h_{Ia}|^2 v^2/(2M)$

Cosmological implications



- Leptogenesis possible because no equilibrium at T≳130 GeV
- Resonant generation of keV scale RHNs hindered by washout at T≲30 GeV. Fine-tuned windows still possible. Helicity could also play a role Eijima Shaposhnikov **PLB771** (2017)

Resonant sterile neutrino production

- If somehow a significant lepton asymmetry survive, how large does it need to be to account for DM abundance? And how large should the mixing angles be?

- To answer these questions, derive and solve the coupled equations Laine Shaposhnikov (2008), JG Laine (2015)

$$\dot{f}_k = \frac{1}{2}\sum_a \Big\{ \big[n_{\rm F}(E_1+\mu_a) - f_k\big] R_a^-(k) + \big[n_{\rm F}(E_1-\mu_a) - f_k\big] R_a^+(k) \Big\} \,,$$

$$\dot{n}_a = \int_{\mathbf{k}} \Big\{ \big[n_{\rm F}(E_1+\mu_a) - f_k\big] R_a^-(k) - \big[n_{\rm F}(E_1-\mu_a) - f_k\big] R_a^+(k) \Big\}$$

Mixing rates for interactions with leptons and antileptons in the presence of asymmetry

$$R_a^\pm(k) \equiv \frac{|h_{1a}|^2 \mathrm{Tr}\,[\,\not{\mathcal{K}}\,\rho_{aa}(\pm\mathcal{K})a_{\rm R}]}{E_1}$$

Resonant sterile neutrino production

- Mixing rates for interactions with leptons and antileptons in the presence of asymmetry

$$R_a^-(k) \approx \frac{|M_D|^2_{1a}\, M_1^2\, \Gamma}{[M_1^2 + 2E_1(b+c) + (b+c)^2]^2 + E_1^2\Gamma^2}\,, \qquad R_a^+(k) = R_a^-(k)\big|_{c\to -c}\,.$$

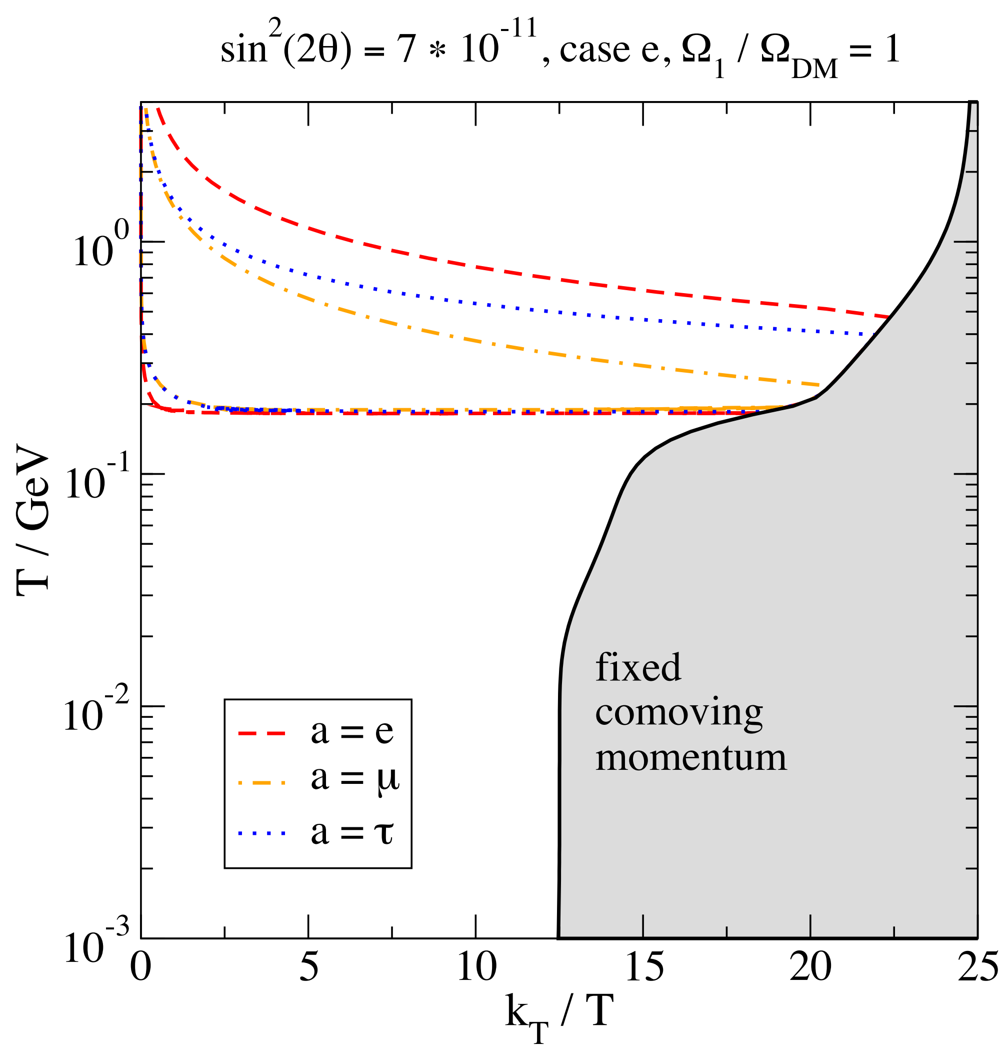
- Active neutrino width from Fermi-type processes. For small asymmetry μ-independent
- For small asymmetry matter potential b is μ-independent
- For small asymmetry matter potential c is linear in μ and causes resonance

$$c = \sqrt{2}G_F \Big[2n_{\nu_a} + \sum_{b\neq a} n_{\nu_b} + \Big(\frac{1}{2} + 2\sin^2\theta_w\Big)n_{e_a} - \Big(\frac{1}{2} - 2\sin^2\theta_w\Big)\sum_{b\neq a} n_{e_b}$$
$$+\Big(\frac{1}{2} - \frac{4}{3}\sin^2\theta_w\Big)\sum_{i=u,c} n_i - \Big(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w\Big)\sum_{i=d,s,b} n_i\Big]\,,$$

Resonant sterile neutrino production

- Solve numerically the coupled equations (with Hubble expansion and QCD transition) from T=5 GeV down to T_*=1 MeV. Very narrow resonance, numerically tricky.
  JG Laine **JHEP1511** (2015)

$$k_T \equiv k_* \left[\frac{s(T)}{s(T_*)} \right]^{1/3}$$



- Two resonances in most of the range. The second one, approximately at the QCD transition, is the strongest

only $n_{\nu_e} \neq 0$ at $T = T_{\max}$; only $h_{1e} \neq 0$; non-equilibrated active flavours.

Resonant sterile neutrino production

- Solve numerically the coupled equations (with Hubble expansion and QCD transition) from T=5 GeV down to T_*=1 MeV. Very narrow resonance, numerically tricky.
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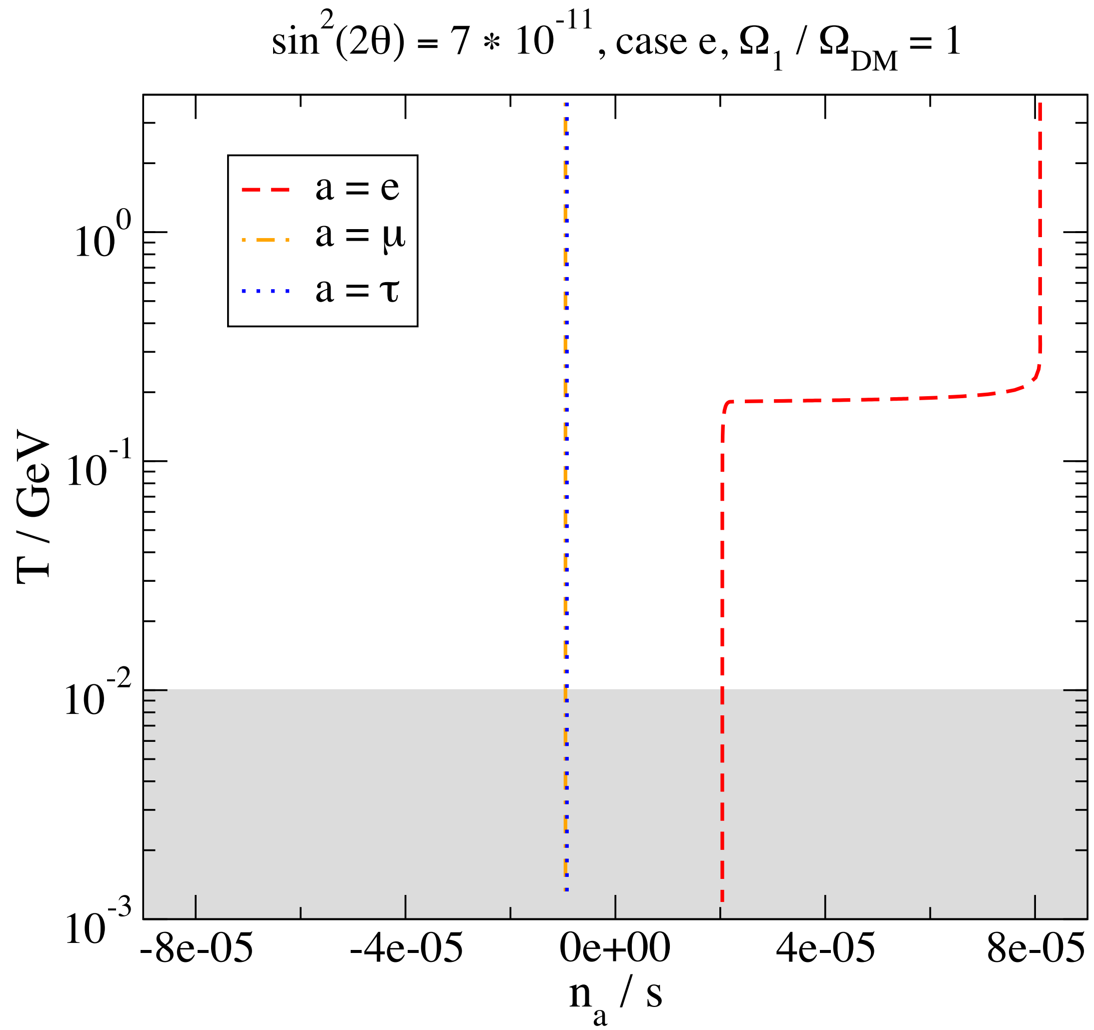
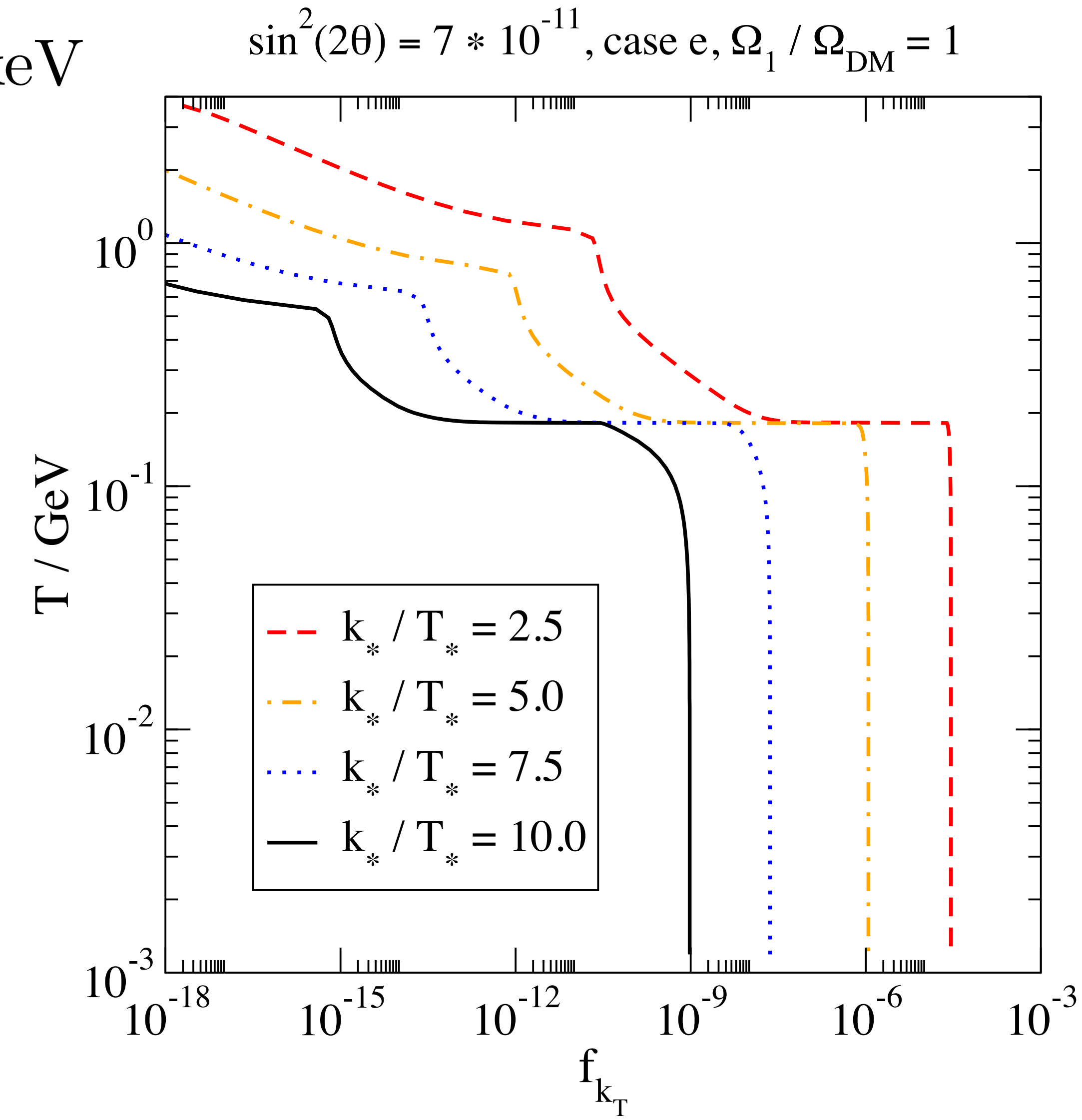
$m_N = 7.1$ keV





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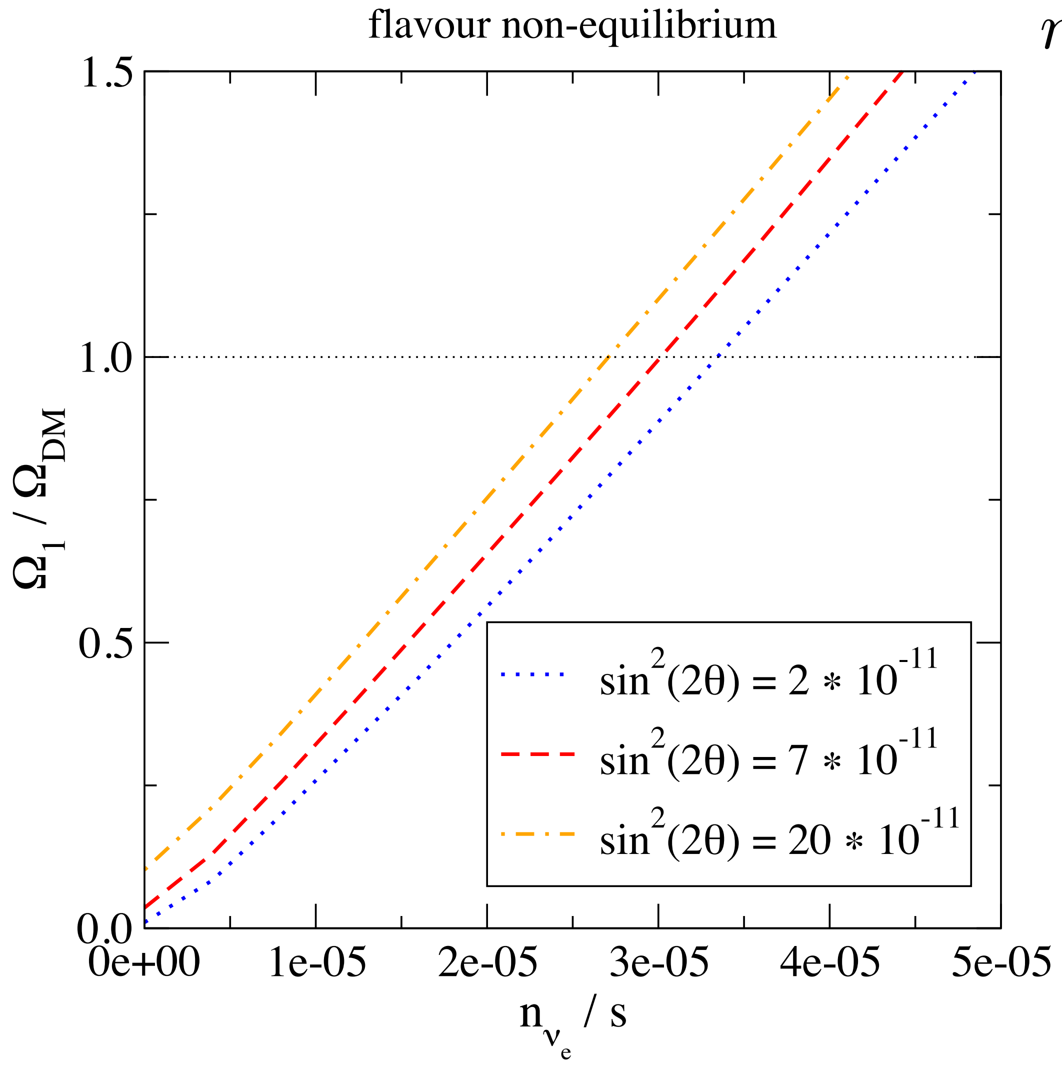
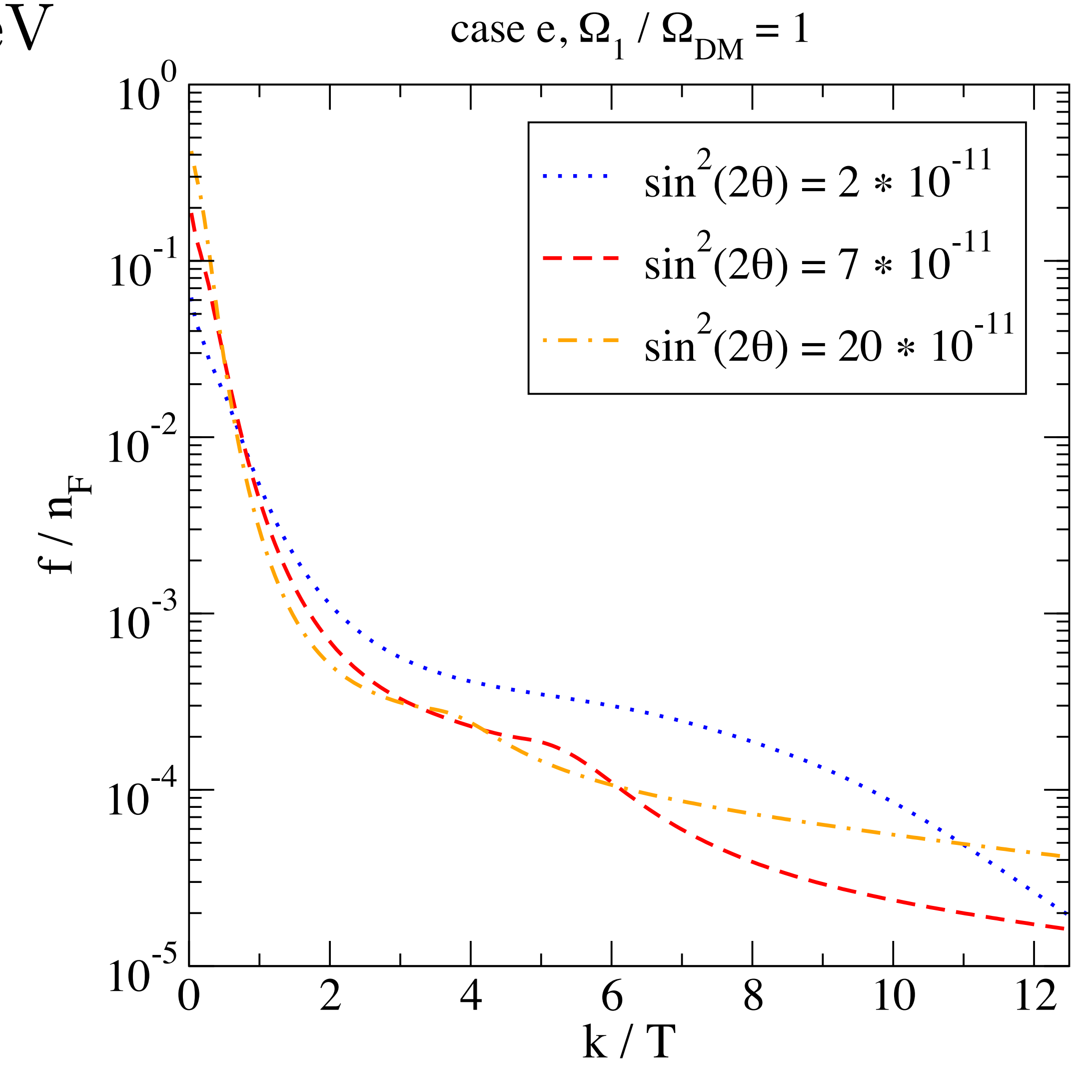
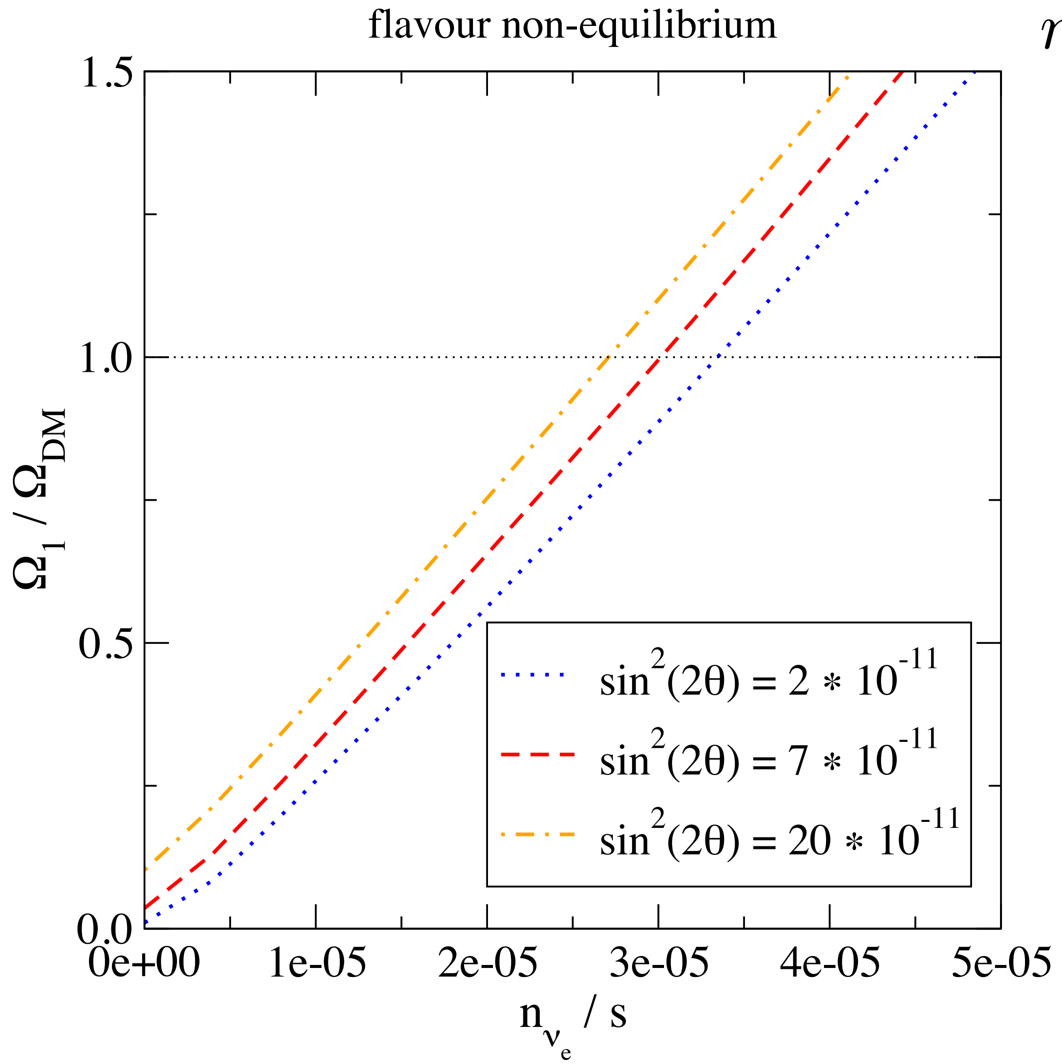


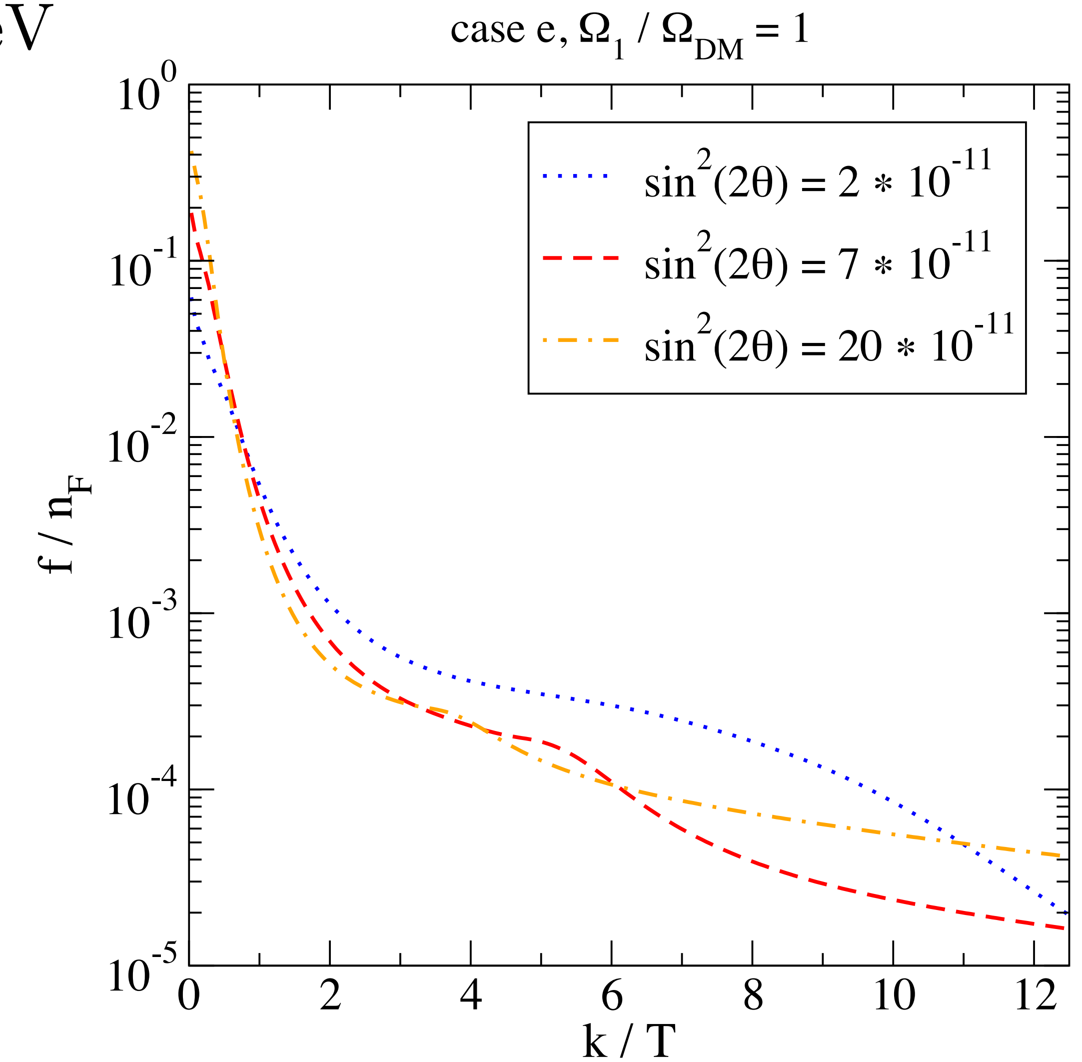


Resonant sterile neutrino production

- All results and code available at http://www.laine.itp.unibe.ch/dmpheno/

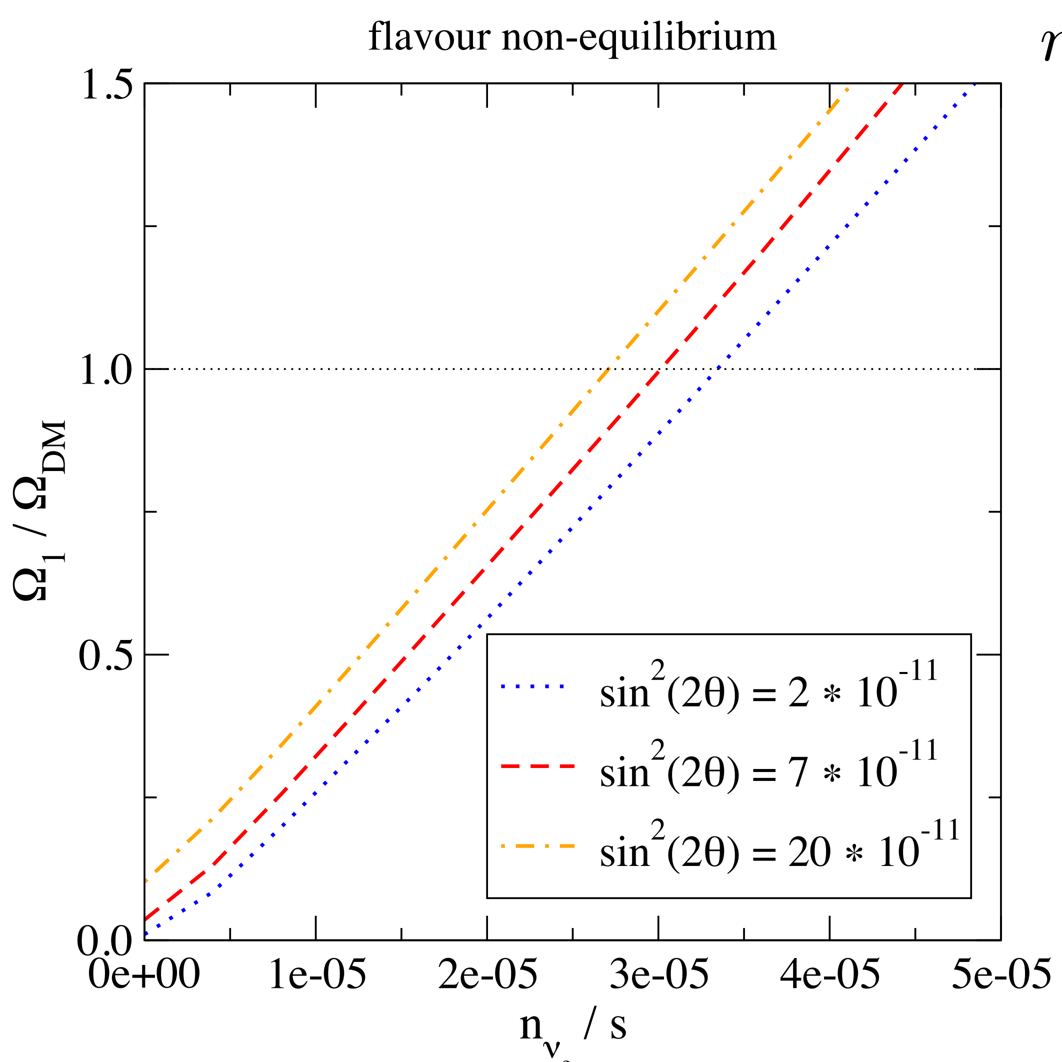
$m_N = 7.1$ keV



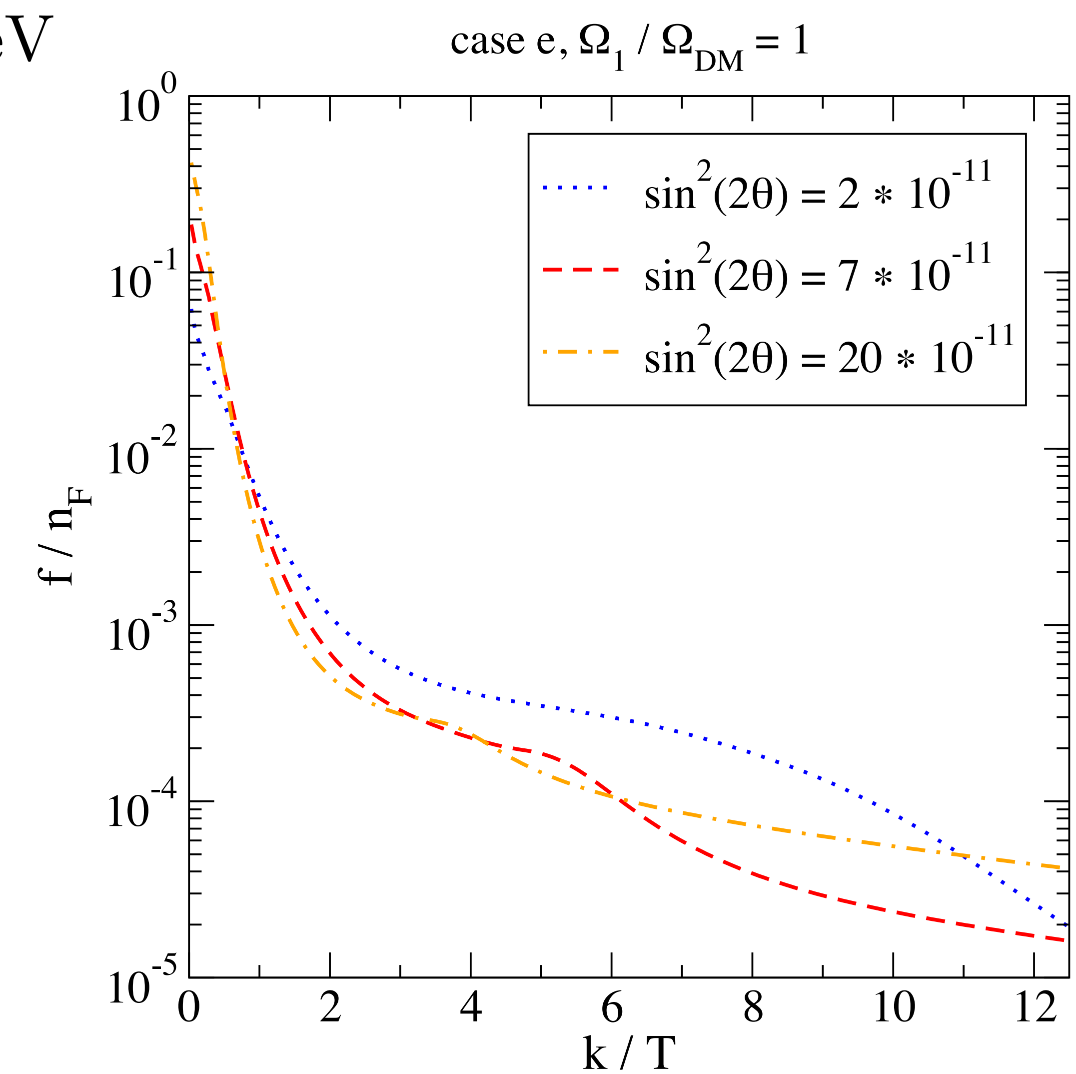


Resonant sterile neutrino production

- Some tension with recent astrophysical constraints Baur *et al* **1706.03118**



$m_N = 7.1 \text{ keV}$



Summary

- Right-handed neutrinos are an economical extension of the SM potentially capable of accounting for three shortcomings
- We have determined the **equilibration** and **washout rates** for **GeV-scale RHNs** at **leading order** for 5 GeV<T<160 GeV
- In the broken phase **these rates peak at T~10-30 GeV**, due to the **efficient, resonance-like indirect processes**, with consequences for leptogenesis and keV scale dark matter
- Illustration of the **resonant production** mechanism for **keV scale dark matter**