Thermal photon rate at next-to-leading order

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Outline

- Introduction and motivation
- The leading-order calculation
- NLO regions and divergences
- Light-cone magic
- Results

JG Hong Kurkela Lu Moore Teaney **1302.5970**

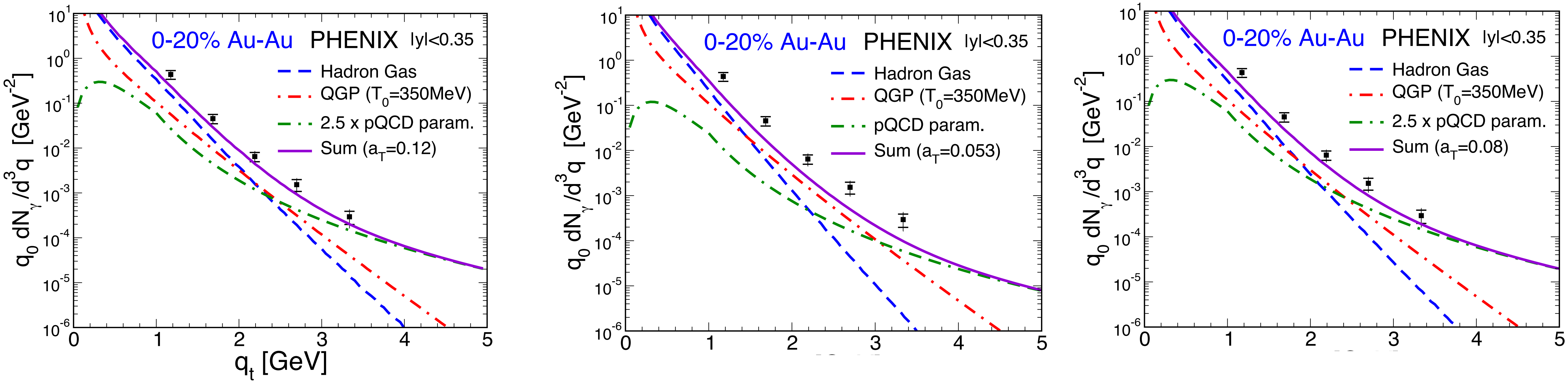
Photons from heavy ion collisions

- The hard partonic processes in the heavy ion collision produce quarks, gluons and *primary photons*
- At a later stage, quarks and gluons form a plasma.
  - A jet traveling through the QGP can radiate *jet-thermal photons*
  - Scatterings of thermal partons can produce *thermal photons*
- Later on, partons hadronize. Interactions between charged hadrons produce *hadron gas thermal photons*
- Hadrons may decay into *decay photons*

Thermal photons

- $\alpha_{\mathrm{EM}} \ll 1 \Rightarrow$ re-interactions negligible
  "Photons escape from the plasma"
- Thermal photons might then be a good hard probe of QGP properties
- The resulting spectrum is not a blackbody spectrum with some T_{QGP}

Thermal photons



van Hees, Gale, Rapp PRC84 (2011)

Thermal photon production

- Single-photon production probability

$$2k^0(2\pi)^3\frac{d\mathrm{Prob}}{d^3k} = \sum_X \mathrm{Tr}\,\rho\, U^\dagger(t)|X,\gamma\rangle\langle X,\gamma|U(t)$$

- Expand the time evolution operator

$$U(t) = 1 - i\int^t dt' \int d^3x e A^\mu(x,t') J_\mu(x,t') + \mathcal{O}(e^2)$$

- Assume translation invariance

$$\frac{d\mathrm{Prob}}{d^3k} = \frac{Vte^2}{(2\pi)^3 2k^0}\int d^4Y e^{-iK\cdot Y}\sum_X \mathrm{Tr}\rho J^\mu(Y)|X\rangle\langle X|J_\mu(0)$$

- Vt is the pos. space volume ⇒ get the rate

$$\frac{d\Gamma}{d^3k} = \frac{e^2}{(2\pi)^3 2k^0}\int d^4Y e^{-iK\cdot Y}\langle J^\mu(Y)J_\mu(0)\rangle$$

Wightman correlator of the e.m. current-current thermal expectation value (with k^0=k). Our operative definition, with k~T, always **hard**

Motivation

- The production rate is known at leading order $\alpha_{\mathrm{EM}}\,\alpha_{\mathrm{s}}$ (more later)
- An NLO ($\alpha_{\mathrm{EM}}\,g^3$) determination
  - Improve the phenomenological analyses, if not by giving reliable theory error bands
  - On the theory side, show if the pattern of large NLO corrections in transport coefficients is reproduced
  - A posteriori: pattern for other NLO transport calculations

NLO transport coefficients

- The only transport coefficient known so far at NLO is the *heavy quark momentum diffusion coefficient*, which is defined through the noise-noise correlator in a Langevin formalism. In field theory it can be written as

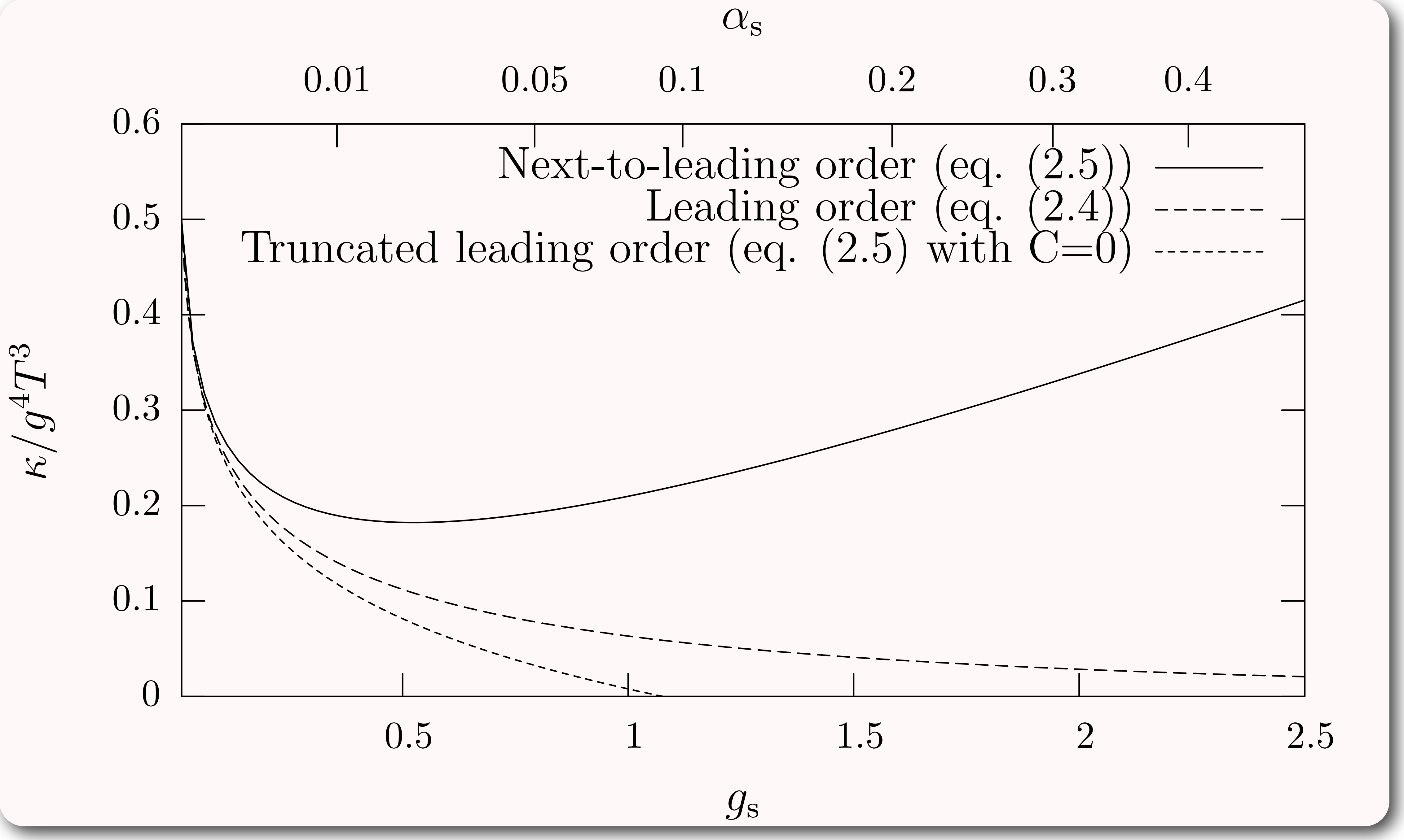
$$\kappa = \frac{g^2}{3N_c}\int_{-\infty}^{+\infty} dt \mathrm{Tr}\langle U(t,-\infty)^\dagger E_i(t) U(t,0) E_i(0) U(0,-\infty)\rangle$$

- The NLO computation factors in the coefficient C, which turns out to be sizeable

$$\kappa = \frac{C_H g^4 T^3}{18\pi}\left(\left[N_c + \frac{N_f}{2}\right]\left[\ln\frac{2T}{m_D} + \xi\right] + \frac{N_f \ln 2}{2} + \frac{N_c m_D}{T} C + \mathcal{O}(g^2)\right) \qquad \xi = \frac{1}{2} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)}$$

Caron-Huot Moore **PRL100, JHEP0802** (2008)

NLO transport coefficients



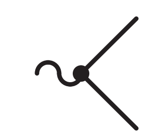
Caron-Huot Moore **PRL100, JHEP0802** (2008)

The LO calculation

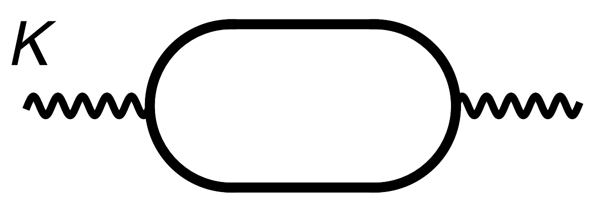
Looking at the diagrams

- Our starting point is the Wightman current-current correlator (with $k^0 = k$)

$$\frac{d\Gamma}{d^3k} = \frac{e^2}{(2\pi)^3 2k^0}\int d^4Y e^{-iK\cdot Y}\langle J^\mu(Y)J_\mu(0)\rangle \qquad J^\mu = \sum_{q=uds} e_q \bar{q}\gamma^\mu q :$$

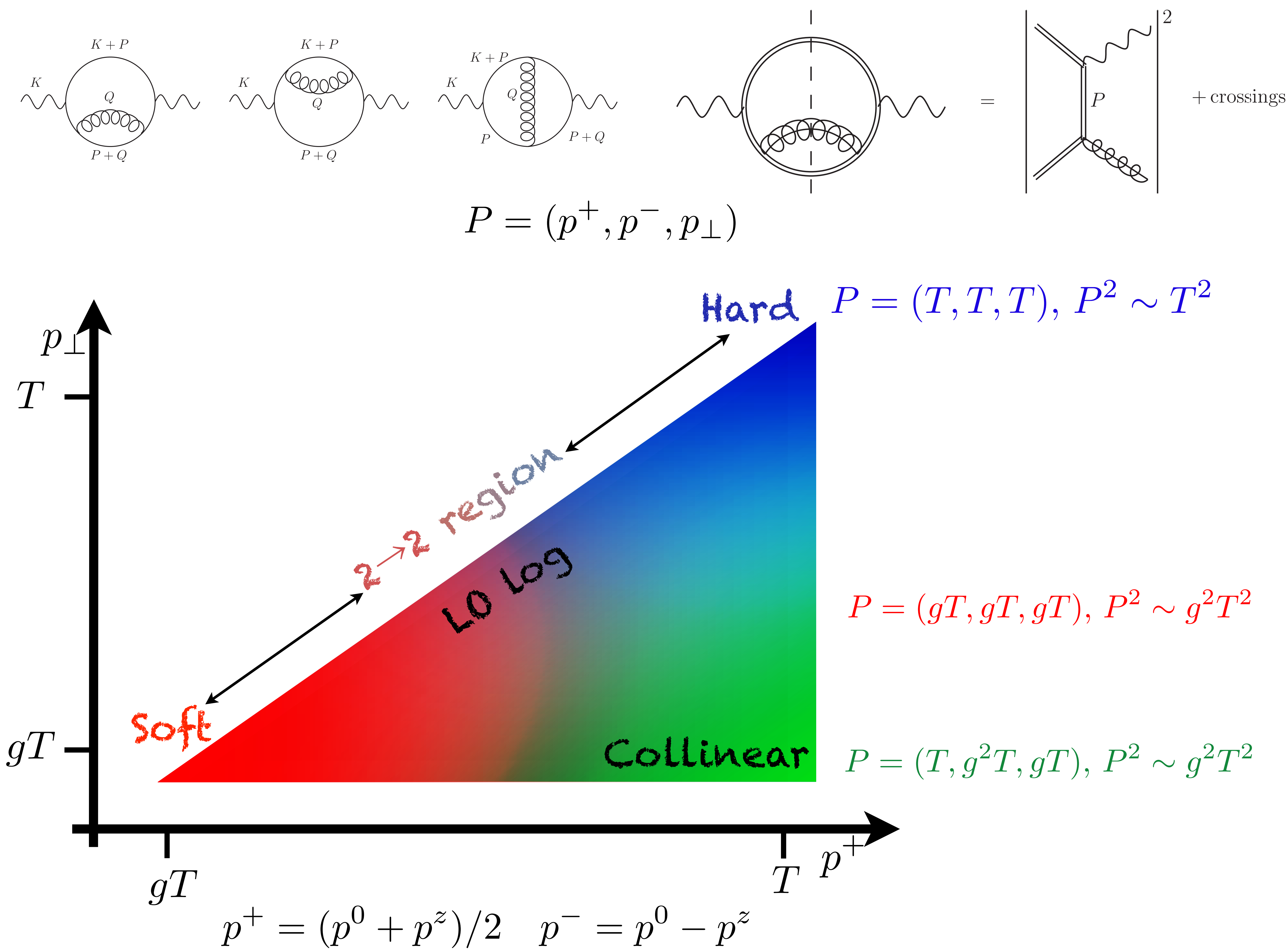


- At one loop ($\alpha_{\mathrm{EM}}\, g^0$):



Kinematically impossible to radiate an on-shell photon from on-shell quarks. Need something to kick one of the quarks (slightly) off-shell

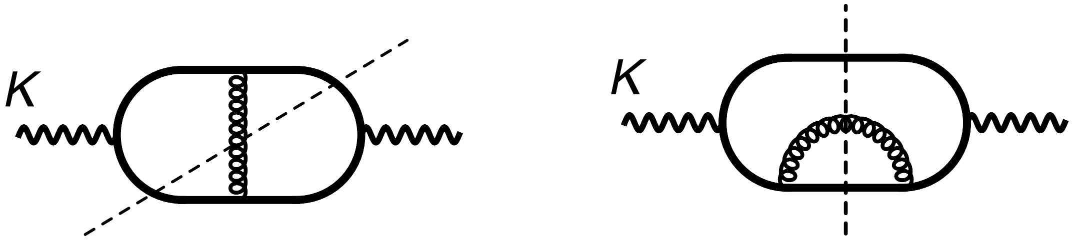
- Two separated phase space regions, 2↔2 and collinear



K
K + P
Q
P + Q
P
2
+ crossings
$P = (p^+, p^-, p_\perp)$
Hard
$P = (T, T, T),\ P^2 \sim T^2$
2→2 region
LO Log
$P = (gT, gT, gT),\ P^2 \sim g^2T^2$
Soft
Collinear
$P = (T, g^2T, gT),\ P^2 \sim g^2T^2$
$p_\perp$
T
gT
p^+
$p^+ = (p^0 + p^z)/2 \quad p^- = p^0 - p^z$

The 2↔2 region

- Two loop diagrams ($\alpha_{\text{EM}}\, g^2$)



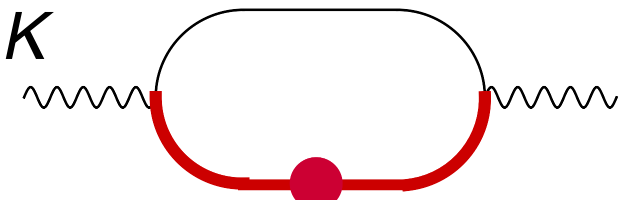
where the cuts correspond to the so-called 2↔2 processes (with their crossings and interferences):

- IR divergence (Compton) when t goes to zero

Introducing the soft scale

- The IR divergence is the signal of missing IR physics and is cured by a proper resummation in the soft sector through the Hard Thermal Loop effective theory

- The Landau cut of the HTL propagator opens up the phase space in this (apparently one-loop) diagram



- In the end one obtains the result

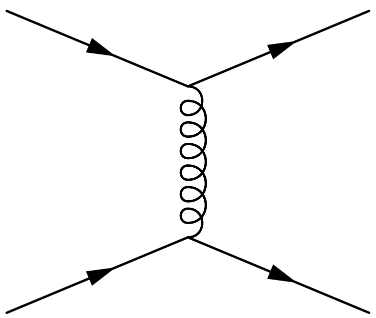
$$\left.\frac{d\Gamma_\gamma}{d^3k}\right|_{2\leftrightarrow 2} \propto e^2 g^2 \left[\log\frac{T}{m_\infty} + C_{2\leftrightarrow 2}\left(\frac{k}{T}\right)\right]$$

The dependence on the cutoff cancels out

Kapusta Lichard Siebert **PRD44** (1991) Baier Nakkagawa Niegawa Redlich **ZPC53** (1992)

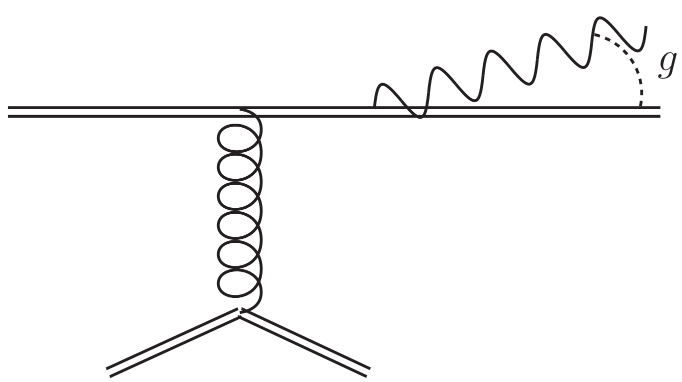
The collinear region

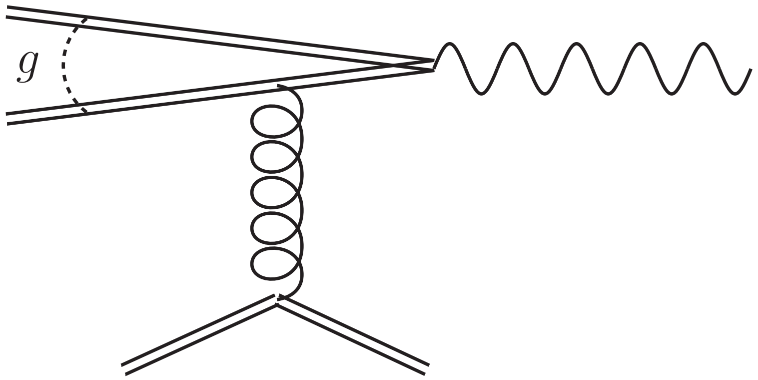
- Consider this simple power-counting argument:



$$\propto \alpha_s^2 \int d^2 q_\perp \frac{s}{(q_\perp^2 + m^2)^2} \sim \alpha_s \qquad \begin{array}{c} s \sim T \\ m \sim gT \end{array}$$

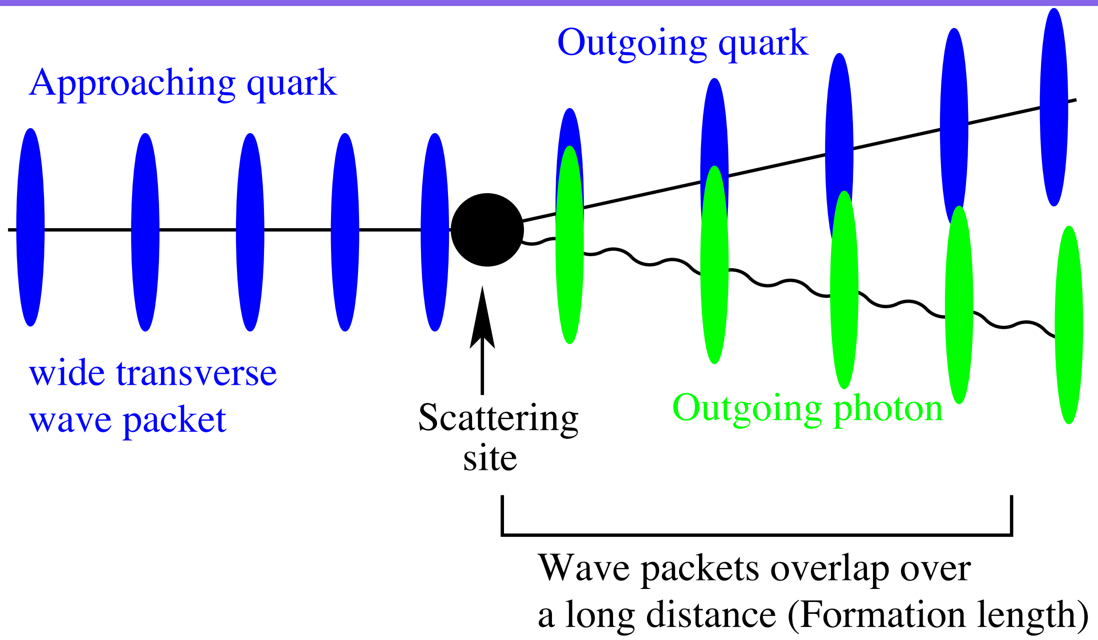
- There is then an $\alpha_{\rm EM}$ probability of radiating a photon





- The collinear enhancement brings these bremsstrahlung and pair annihilation diagrams to LO
Aurenche Gelis Kobes Petitgirard Zaraket 1998-2000

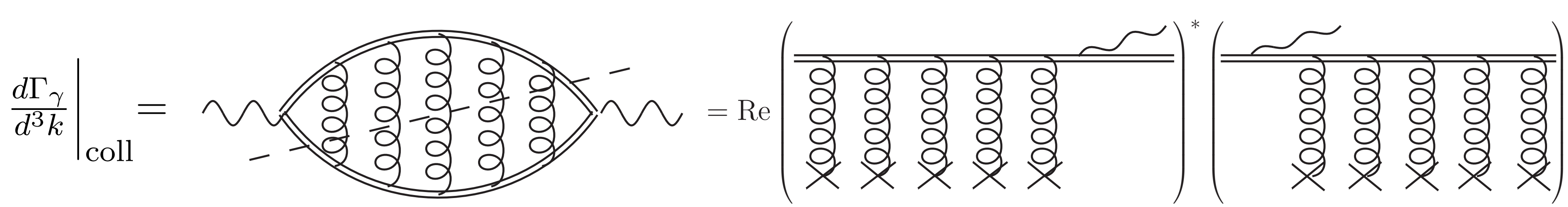
The collinear region: LPM effect



- Photon is collinear, $\theta \sim g \quad p_\perp \sim gT$
  spatial transverse size large $\Delta x_\perp \sim p_\perp^{-1}$
  long separation (formation) time $t \sim \Delta x_\perp / \theta \sim 1/(g^2 T)$
- The interference with other scattering events cannot be neglected (scattering rate ~ g^2T)
- This multiple scattering interference gives a suppression called the Landau-Pomeranchuk-Migdal (LPM) effect

The LPM effect

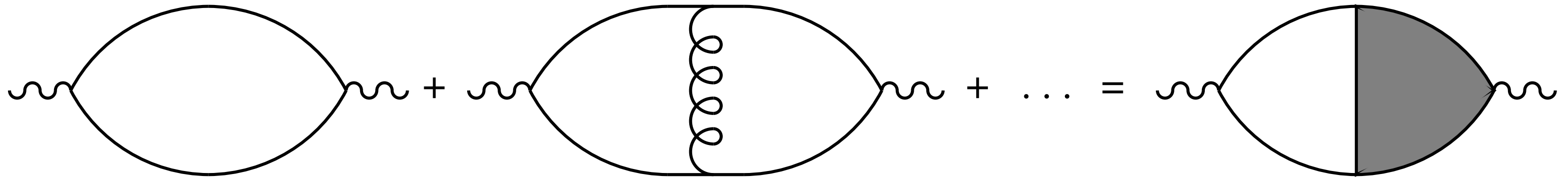
- Introduced by Landau and Pomeranchuk (then Migdal) for QED in the 50’s
- Extended to photons in QCD in Baier Dokshitzer Mueller Peigne Schiff **NPB478** (1996)
- Rigorous treatment and diagrammatics in AMY (Arnold Moore Yaffe) **JHEP** 0111, 0112, 0226 (2001-02)
- In the JJ correlator diagrams like

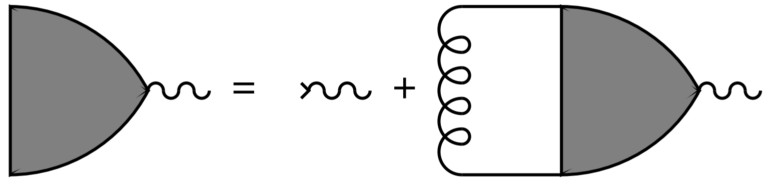


have to be resummed consistently

AMY resummation

- Define a dressed vertex determined by an integral equation





- The emission rate in the collinear region becomes

$$\left.\frac{d\Gamma_\gamma}{d^3k}\right|_{\rm coll} = \frac{\mathcal{A}(k)}{(2\pi)^3}\int_{-\infty}^{\infty} dp^+ \left[\frac{(p^+)^2+(p^++k)^2}{(p^+)^2(p^++k)^2}\right] \frac{n_F(k+p^+)[1-n_F(p^+)]}{n_F(k)} \times\frac{1}{g^2C_RT^2}\int\frac{d^2p_\perp}{(2\pi)^2}\,{\rm Re}\,2\mathbf{p}_\perp\cdot\mathbf{f}(\mathbf{p}_\perp,p^+,k)\,,$$

where $\mathcal{A}(k) = \alpha_{\rm EM}\frac{g^2C_fT^2}{2k}n_f(k)\sum_s d_f q_s^2$

and **f** is implicitly defined by

$$2\mathbf{p}_\perp = i\delta E\mathbf{f}(\mathbf{p}_\perp;p,k)+\int\frac{d^2q_\perp}{(2\pi)^2}\mathcal{C}(q_\perp)\Big[\mathbf{f}(\mathbf{p}_\perp)-\mathbf{f}(\mathbf{p}_\perp+\mathbf{q}_\perp)\Big]$$

AMY resumation

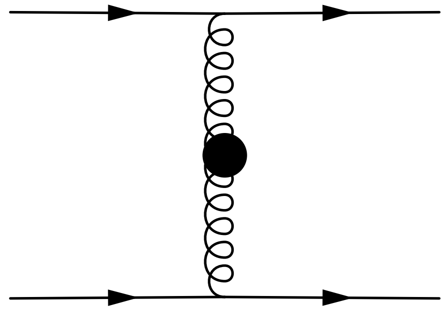
$$2\mathbf{p}_\perp = i\delta E \mathbf{f}(\mathbf{p}_\perp; p, k) + \int \frac{d^2q_\perp}{(2\pi)^2}\mathcal{C}(q_\perp)\Big[\mathbf{f}(\mathbf{p}_\perp) - \mathbf{f}(\mathbf{p}_\perp + \mathbf{q}_\perp)\Big]$$

- Two inputs
  - Difference in energy after and before radiation

$$\delta E = k^0 + E_{\mathbf{p}} - E_{\mathbf{p}+\mathbf{k}} \simeq \frac{k}{p(k+p)}\frac{\mathbf{p}_\perp^2 + m_\infty^2}{2}$$

  - Rate of soft collisions through the collision kernel

$$\mathcal{C}(q_\perp) = \frac{d\Gamma}{dq_\perp^2} \sim g^2 T \frac{m_D^2}{q_\perp^2(q_\perp^2 + m_D^2)}$$



relevance for jet quenching $\hat{q} \equiv \int_0^{q_{\max}} \frac{d^2q_\perp}{(2\pi)^2} q_\perp^2 C(q_\perp)$

Full LO results

- Numerically solving the implicit equation for the collinear region yields the full LO results for the thermal photon production rate

$$(2\pi)^3 \frac{d\Gamma}{d^3k}\bigg|_{\rm LO} = \mathcal{A}(k)\left[\log\frac{T}{m_\infty} + C_{2\to 2}(k) + C_{\rm coll}(k)\right]$$

$$\mathcal{A}(k) \;\; = \;\; \alpha_{\rm EM}\frac{g^2 C_f T^2}{2k} n_f(k) \sum_s d_f q_s^2$$

Arnold Moore Yaffe **JHEP0112** (2001)

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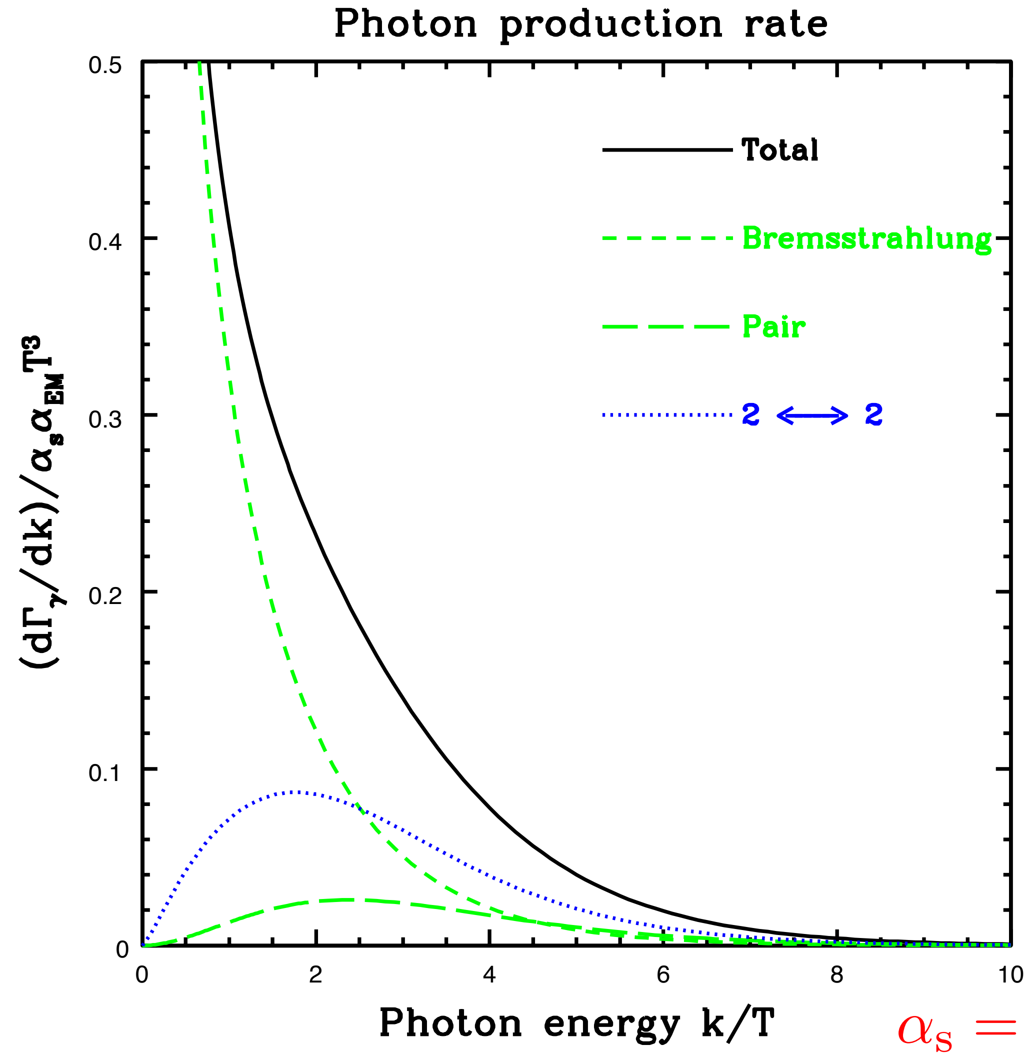
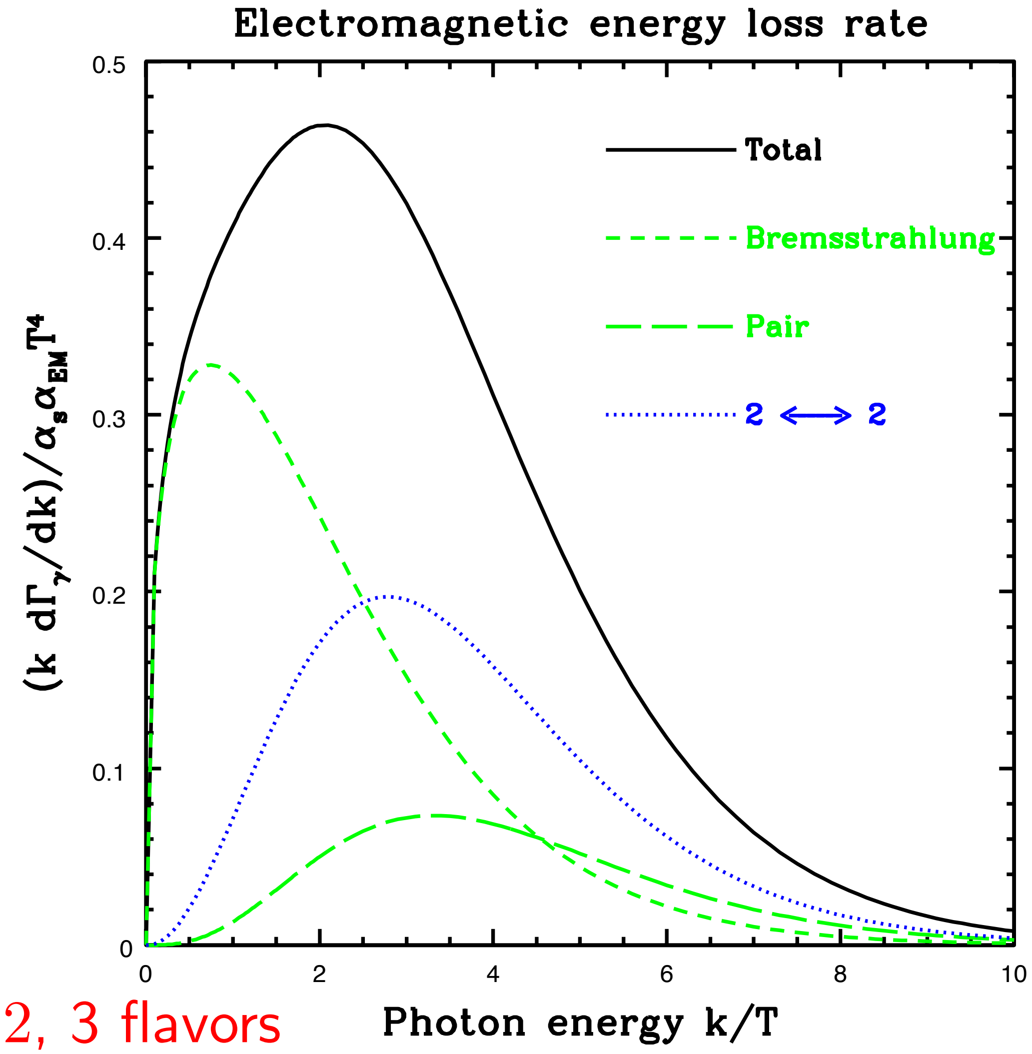




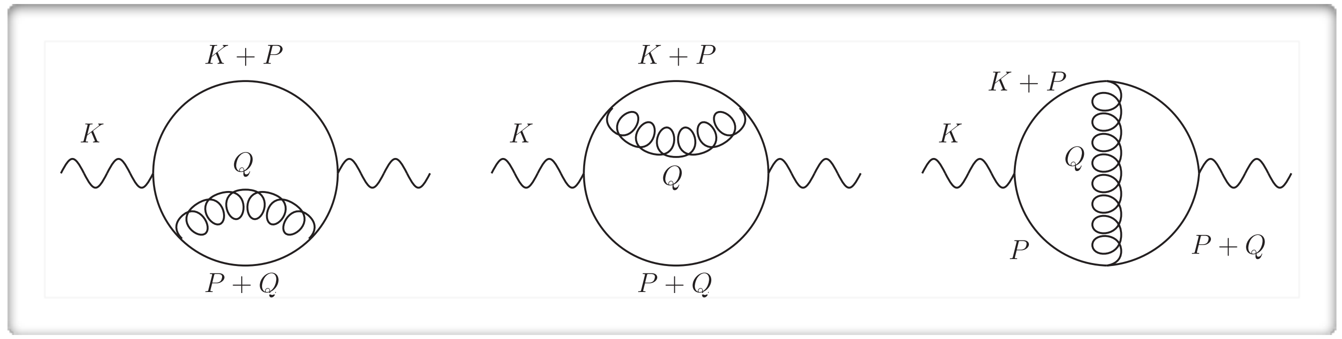
$\alpha_s = 0.2$, 3 flavors

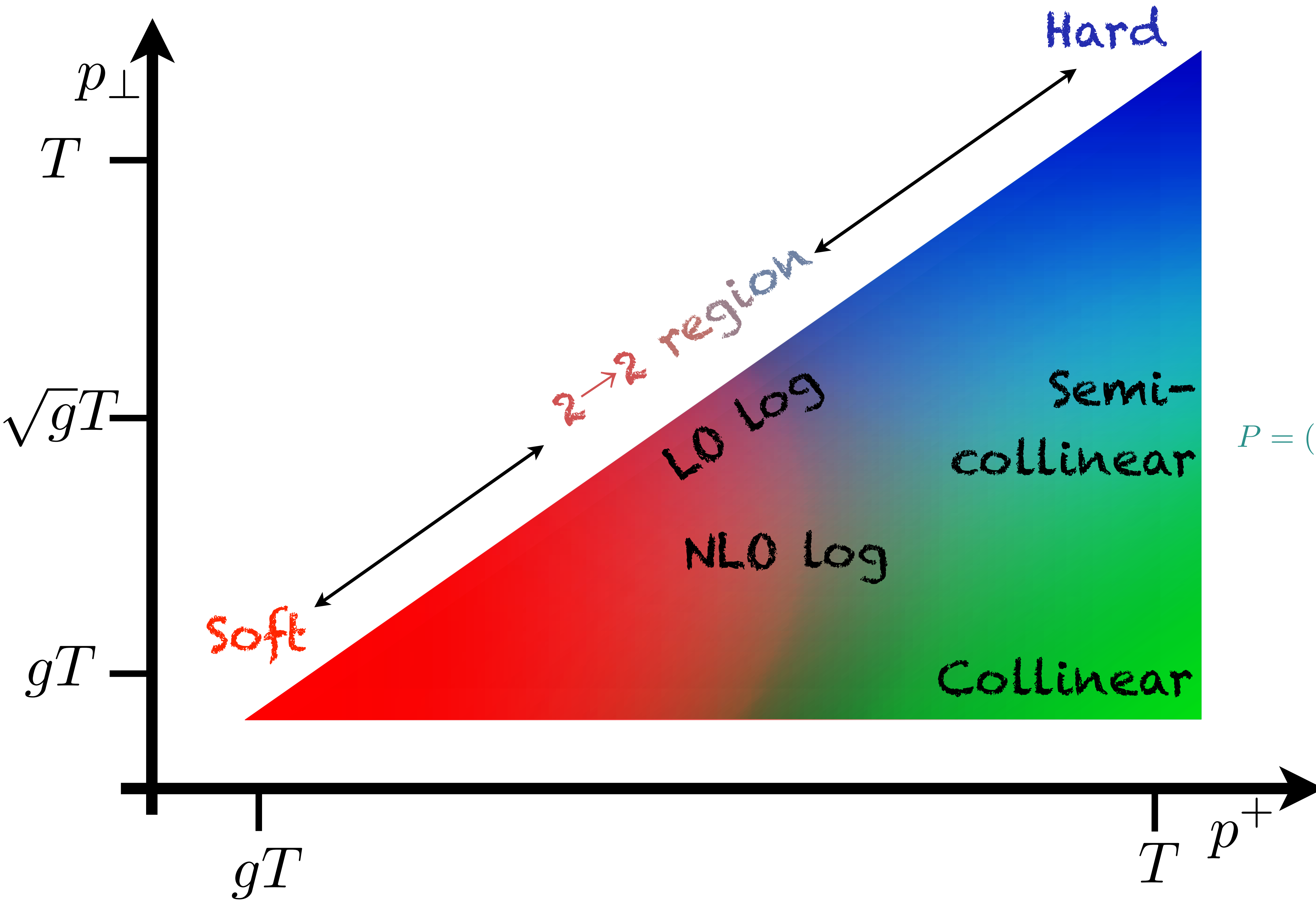
Arnold Moore Yaffe **JHEP0112** (2001)

Going to NLO

Sources of NLO corrections

- As usual in thermal field theory, the soft scale gT introduces NLO $O(g)$ corrections
- The soft region and the collinear region both receive $O(g)$ corrections
- There is a new semi-collinear region
- The NLO calculation is still not sensitive to the magnetic scale g^2T. Ideas for NNLO?





$P = (T, gT, \sqrt{g}T),\; P^2 \sim gT^2$

Euclideanization of light-cone soft physics

- For $v=x_z/t=\infty$ correlators (such as propagators) are the equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n, p)e^{i\mathbf{p}\cdot\mathbf{x}}$$

- Boost invariance: true for $v>1$. For soft fields the $v\rightarrow 1^+$ limit is smooth (feeling the medium in uncorrelated, eikonalized way)

$$G_{rr}(t=x_z,\mathbf{x}_\perp) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n, p_\perp, p_z + i\omega_n)e^{i(\mathbf{p}_\perp\cdot\mathbf{x}_\perp + p_z x_z)}$$

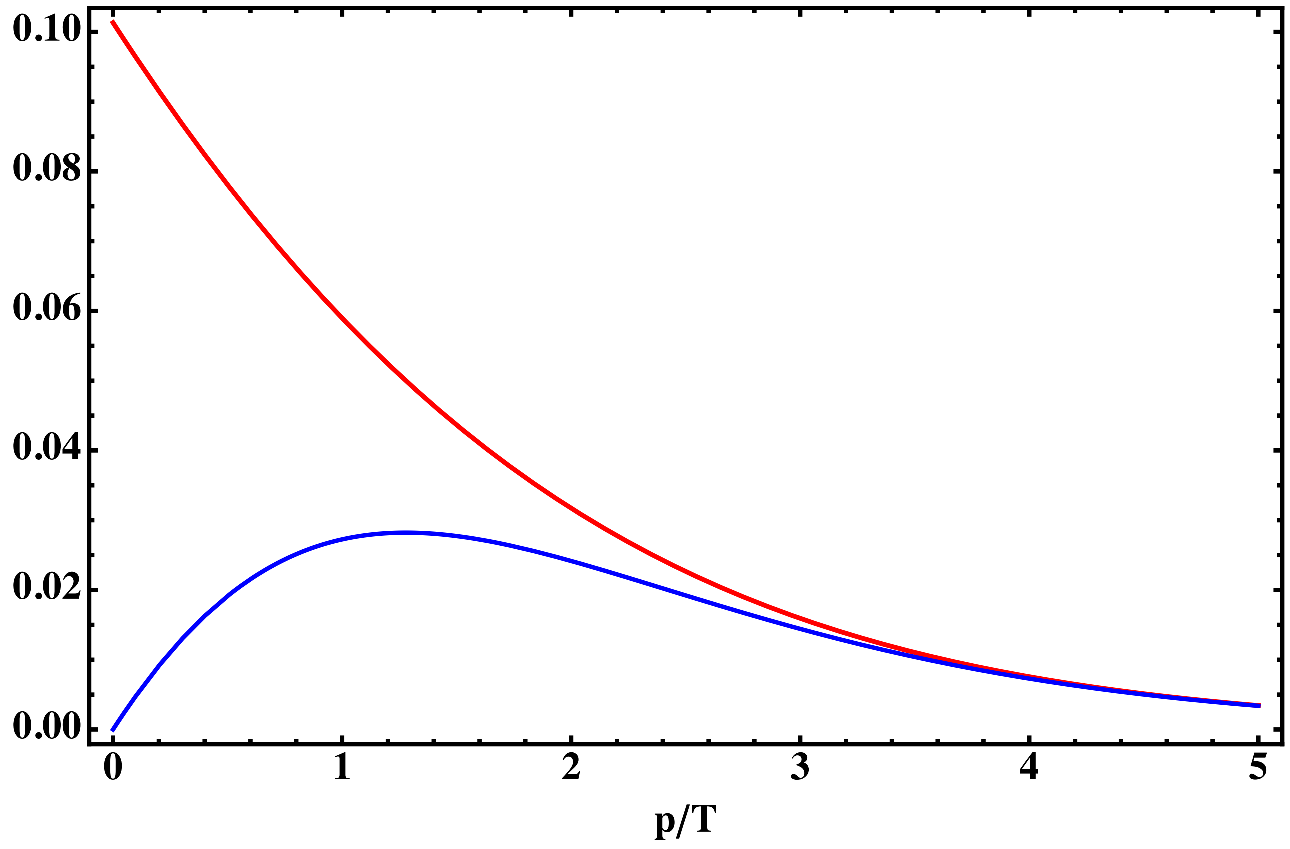
- The sums are dominated by the zero mode for soft physics=>EQCD!

- Equivalent to sum rules   Caron-Huot **PRD79** (2009)

Soft sensitivity and subtractions

- Consider the asymptotic mass for a fermion (a not-so-randomly chosen example). The dispersion relation approaches $p_0^2 = p^2 + m_\infty^2$ for $p^0 \approx p \gg gT$
- At leading order $m_\infty^2 = 2g^2 C_R \left(\int \frac{d^3p}{(2\pi)^3} \frac{n_\mathrm{B}(p)}{p} + \int \frac{d^3p}{(2\pi)^3} \frac{n_\mathrm{F}(p)}{p} \right)$

$$\frac{p\, n(p)}{\pi^2}$$



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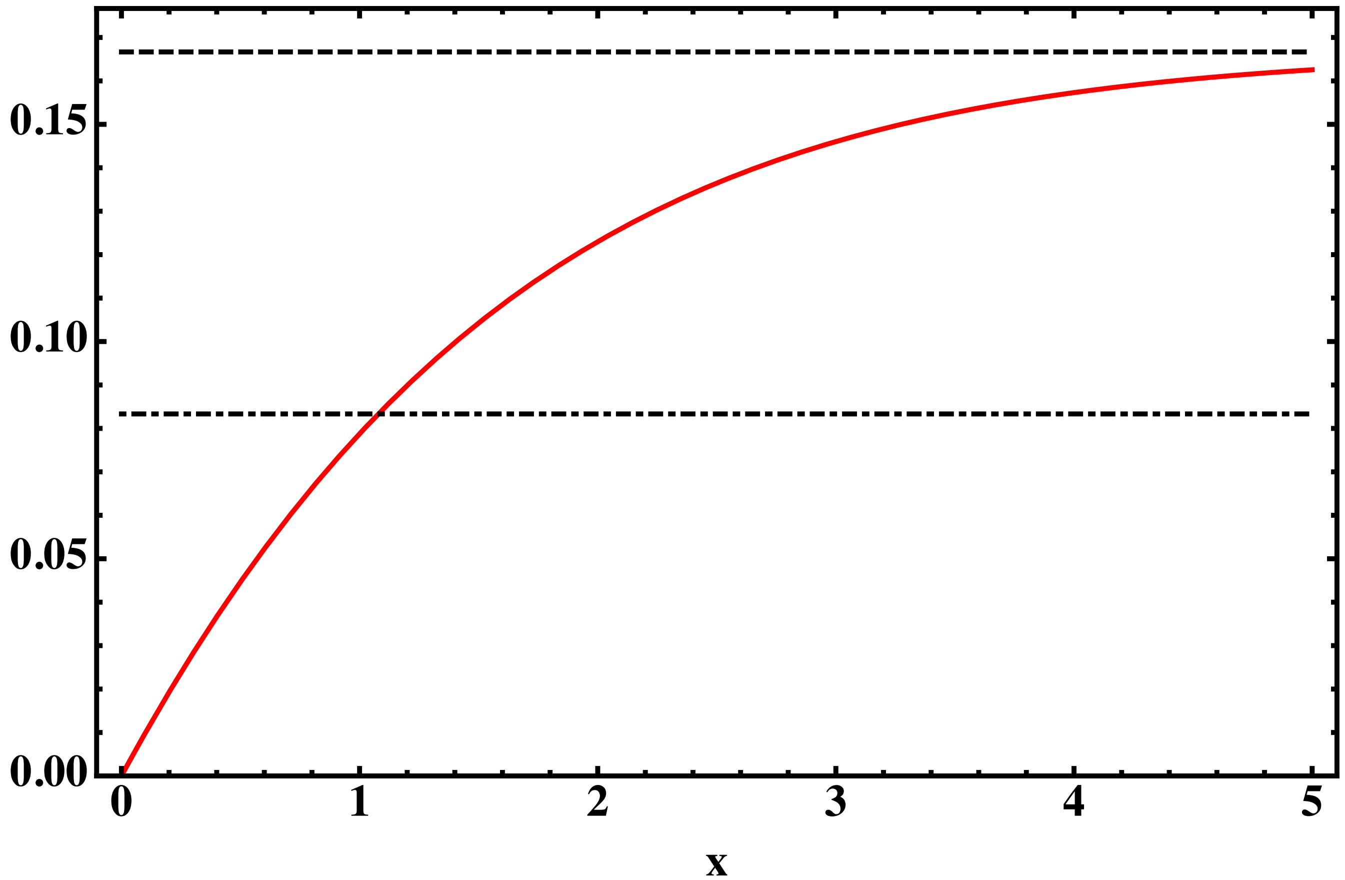
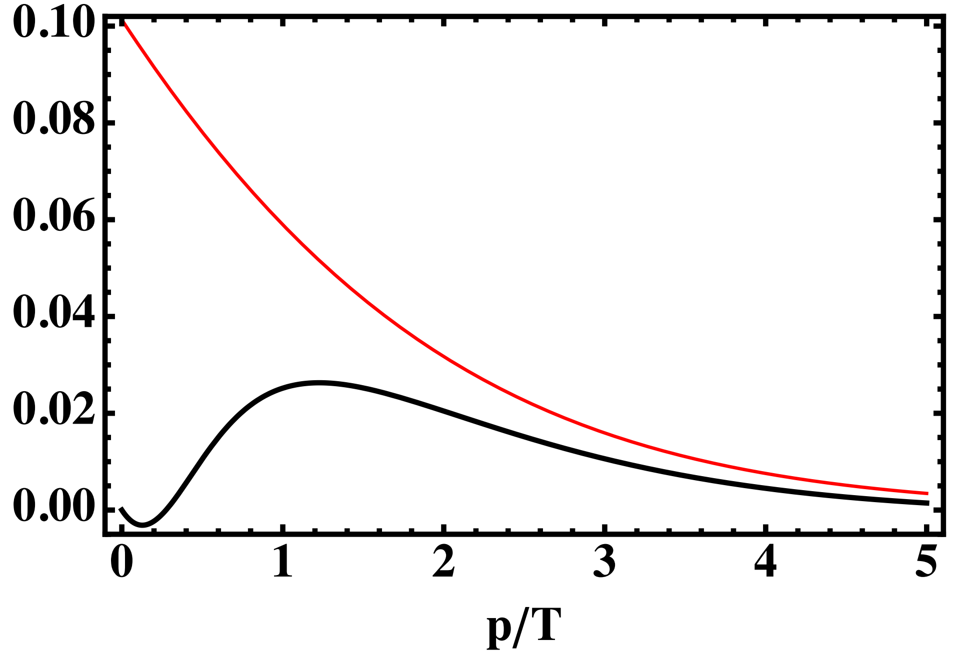
$$\int_0^{x} dy \frac{y\, n_B(y\,T)}{\pi^2}$$



NLO asymptotic mass

- The soft contribution is large and handled incorrectly. This part of the integrand needs to be subtracted and replaced by a proper evaluation with HTL
- NLO correction computed in Caron-Huot **PRD79** (2009) with Euclidean techniques

$$\delta m_\infty^2 = 2g^2 C_R T \int \frac{d^3q}{(2\pi)^3} \left(\frac{1}{q^2+m_D^2} - \frac{1}{q^2} \right) = -g^2 C_R \frac{T m_D}{2\pi}$$



Light-cone condensates

- Asymptotic mass Caron-Huot **PRD79** (2009)

$$m_\infty^2 = g^2 C_R (Z_g + Z_f)$$

$$Z_g \equiv \frac{1}{d_A}\left\langle v_\mu F^{\mu\rho}\frac{-1}{(v\cdot D)^2}v_\nu F^\nu_{\ \rho}\right\rangle \qquad v_k = (1,0,0,1)$$

$$= \frac{-1}{d_A}\int_0^\infty dx^+\, x^+ \langle v_{k\,\mu} F_a^{\mu\nu}(x^+,0,0_\perp) U_A^{ab}(x^+,0,0_\perp;0,0,0_\perp) v_{k\,\rho} F_{b\ \nu}^{\rho}(0)\rangle$$

$$Z_f \equiv \frac{1}{2d_R}\left\langle \overline{\psi}\frac{\not{v}}{v\cdot D}\psi\right\rangle$$

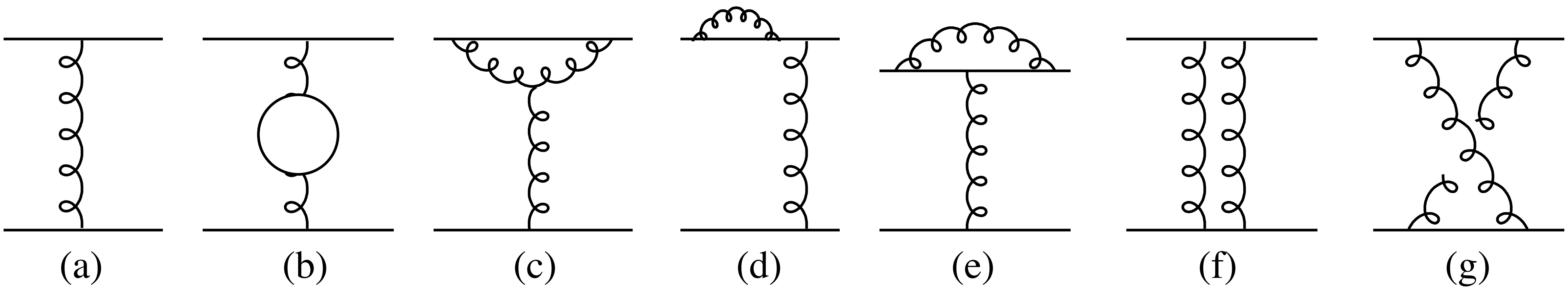
The collinear sector

- The AMY resummation equation is

$$\frac{d\Gamma_\gamma}{d^3k}\bigg|_{\rm coll} = \frac{\mathcal{A}(k)}{(2\pi)^3}\int_{-\infty}^{\infty} dp^+ \left[\frac{(p^+)^2+(p^++k)^2}{(p^+)^2(p^++k)^2}\right] \frac{n_F(k+p^+)[1-n_F(p^+)]}{n_F(k)} \times \frac{1}{g^2 C_R T^2}\int \frac{d^2p_\perp}{(2\pi)^2}\,\mathrm{Re}\,2\mathbf{p}_\perp\cdot\mathbf{f}(\mathbf{p}_\perp,p^+,k)\,,$$

- Four sources of $O(g)$ corrections
- $p^+ \sim gT$ or $p^+ + k \sim gT$. Mistreated soft limit
- $p_\perp \sim \sqrt{g}T, p^- \sim gT$. Mistreated semi-collinear limit
- The two inputs in the integral equation, m_∞^2 and $\mathcal{C}(q_\perp)$ receive $O(g)$ corrections. The former we know about.

The NLO collision kernel



- At the LO only (a) has been used as a rung in the AMY ladder resummation. At the NLO all these diagrams have to be evaluated at the soft scale (remember that the quark lines are on the light cone)
- This calculation has been carried out in Caron-Huot **PRD79** (2009) using Euclidean technology

Subtraction regions

$$\left.\frac{d\Gamma_\gamma}{d^3k}\right|_{\rm coll} = \frac{\mathcal{A}(k)}{(2\pi)^3}\int_{-\infty}^{\infty} dp^+ \left[\frac{(p^+)^2+(p^+ + k)^2}{(p^+)^2(p^+ + k)^2}\right] \frac{n_F(k+p^+)[1-n_F(p^+)]}{n_F(k)}$$

$$\times\frac{1}{g^2 C_R T^2}\int\frac{d^2p_\perp}{(2\pi)^2}\,{\rm Re}\, 2\mathbf{p}_\perp\cdot\mathbf{f}(\mathbf{p}_\perp,p^+,k)\,,$$

$$2\mathbf{p}_\perp = i\delta E\,\mathbf{f}(\mathbf{p}_\perp;p,k)+\int\frac{d^2q_\perp}{(2\pi)^2}\mathcal{C}(q_\perp)\Big[\mathbf{f}(\mathbf{p}_\perp)-\mathbf{f}(\mathbf{p}_\perp+\mathbf{q}_\perp)\Big]$$

- For small p $\quad \delta E = \frac{k}{p(k+p)}\frac{p_\perp^2+m_\infty^2}{2} \to \frac{p_\perp^2+m_\infty^2}{2p} \sim gT$
- This then implies

$$\delta E \sim \frac{T}{p}\int\frac{d^2q_\perp}{(2\pi)^2}\mathcal{C}(q_\perp)^{\rm LO} \qquad C(q_\perp)^{\rm (LO)} = \frac{g^2TC_s m_D^2}{q_\perp^2(q_\perp^2+m_D^2)}$$

- The AMY equation can be solved analytically by substitution (single-scattering regime), yielding

$$\left.\frac{d\Gamma_\gamma}{d^3k}\right|_{\rm soft}^{\rm subtr.} = \frac{\mathcal{A}(k)}{(2\pi)^3}\int_{-\mu^+}^{+\mu^+} dp^+\,\frac{8}{T}\int\frac{d^2p_\perp d^2q_\perp}{(2\pi)^4}\frac{m_D^2}{q_\perp^2(q_\perp^2+m_D^2)}\left(\frac{\mathbf{p}_\perp}{p_\perp^2+m_\infty^2}-\frac{\mathbf{p}_\perp+\mathbf{q}_\perp}{(\mathbf{p}_\perp+\mathbf{q}_\perp)^2+m_\infty^2}\right)^2$$

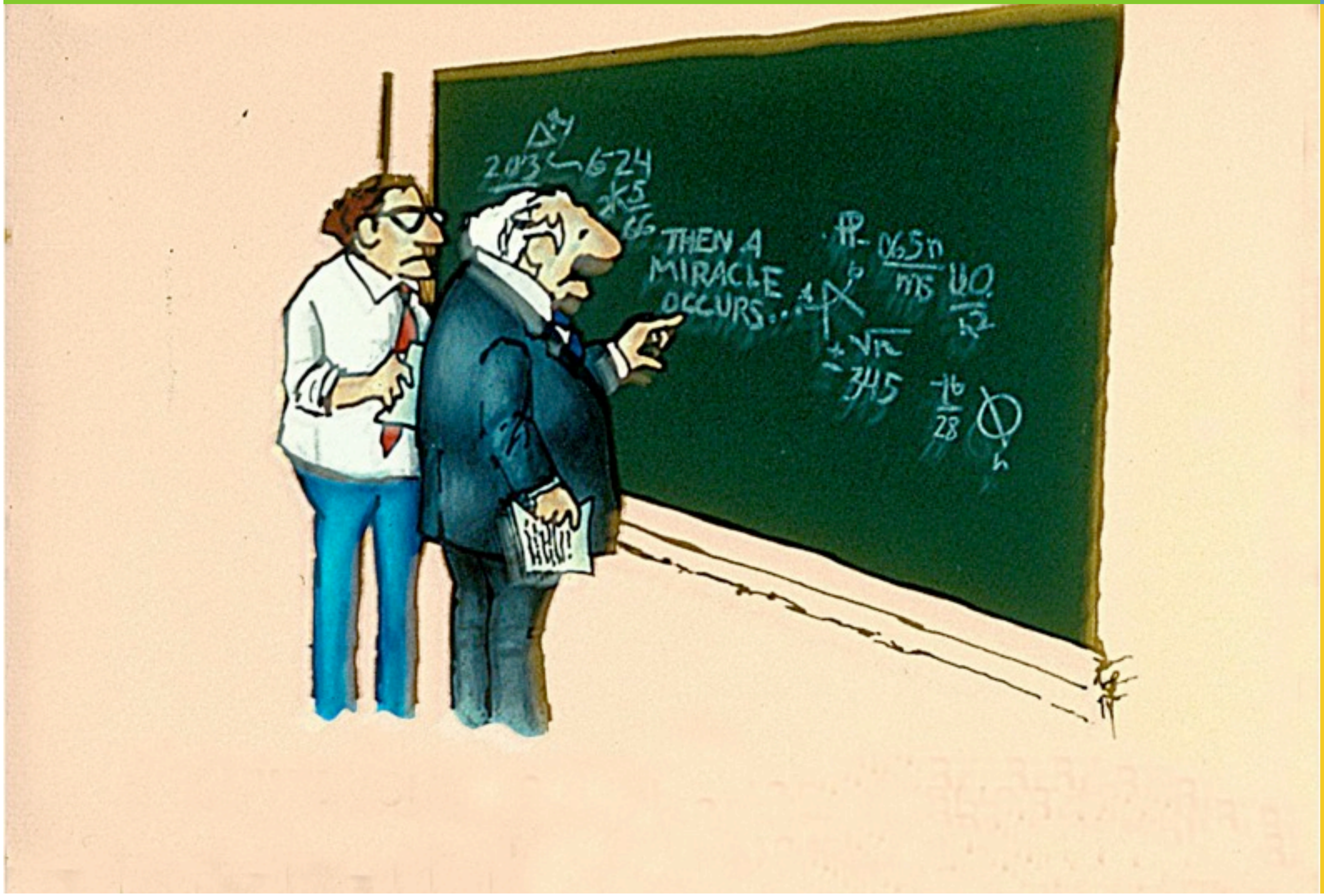
Numerical solution

- The contribution from the NLO asymptotic mass and scattering kernel is then to be solved for numerically.
- Going into impact parameter space is useful: integral equation $\Rightarrow$ differential equation
  Aurenche Gelis Moore Zaraket **JHEP0212** (2002)
- The results for the numerical solution of the collinear region can be written in this form

$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\text{NLO coll}} = \mathcal{A}(k)\left[\frac{\delta m_\infty^2}{m_\infty^2} C_{\text{coll}}^{\delta m}(k) + \frac{g^2 C_A T}{m_D} C_{\text{coll}}^{\delta \mathcal{C}}(k)\right]$$

$$\frac{\delta m_\infty^2}{m_\infty^2} = -\frac{2m_D}{\pi T}$$

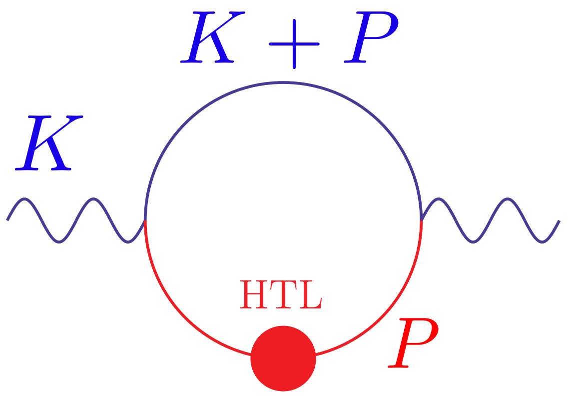
The soft sector



Fermionic sum rules

- We have found the fermionic analogue of the AGZ sum rule

- The leading-order soft contribution (P fully soft)



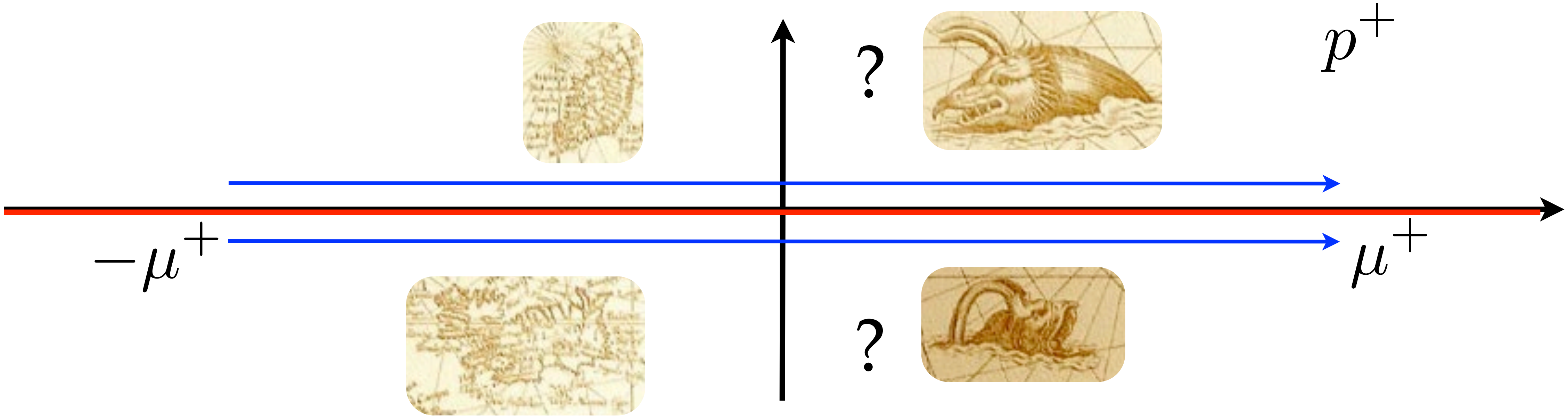
$$(2\pi)^3 \frac{d\Gamma_\gamma}{d^3k}_{\text{soft}} \propto \int \frac{dp^+\, d^2p_\perp}{(2\pi)^3} \text{Tr}\left[\gamma^-(S_R(P) - S_A(P))\right]_{p^-=0}$$

where $$S(P) = \frac{1}{2}\left[(\gamma^0 - \vec{\gamma}\cdot\hat{p})S^+(P) + (\gamma^0 + \vec{\gamma}\cdot\hat{p})S^-(P)\right]$$

$$S^{\pm}_R(P) = \frac{i}{p^0 \mp \left[p + \frac{\omega_0^2}{p}\left(1 - \frac{p^0 \mp p}{2p}\ln\left(\frac{p^0+p}{p^0-p}\right)\right)\right]}\Bigg|_{p^0=p^0+i\epsilon}$$

Fermionic sum rules

$$(2\pi)^3 \frac{d\Gamma_\gamma}{d^3k}_{\text{soft}} \propto \int \frac{dp^+ \, d^2p_\perp}{(2\pi)^3} \mathrm{Tr}\left[\gamma^-(S_R(P) - S_A(P))\right]_{p^-=0}$$



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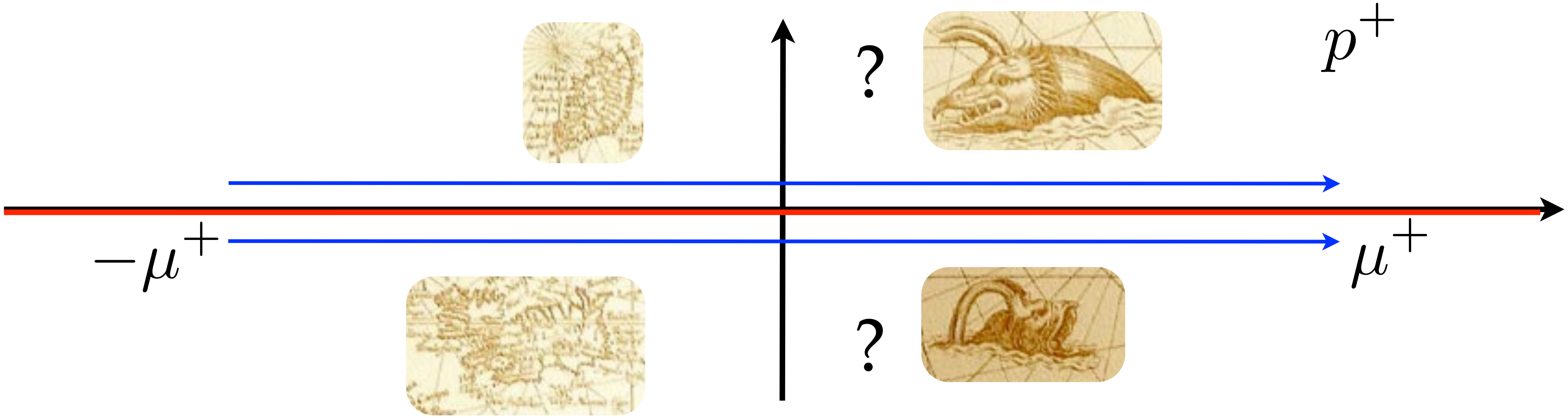

- A retarded propagator is an analytic function of Q in the upper half-plane not just in the frequency, but in any time-like or light-like variable

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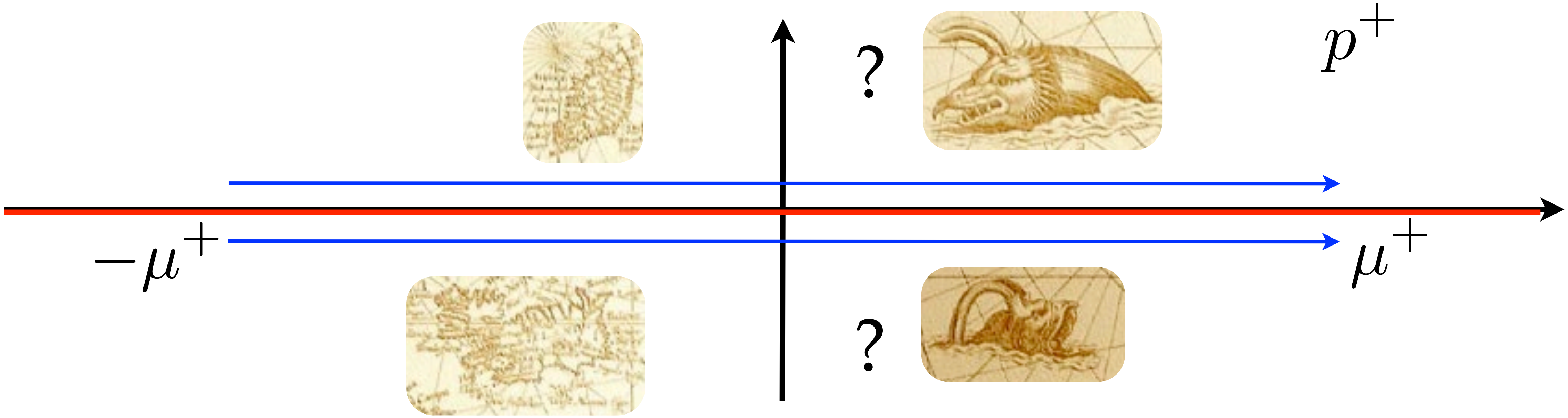

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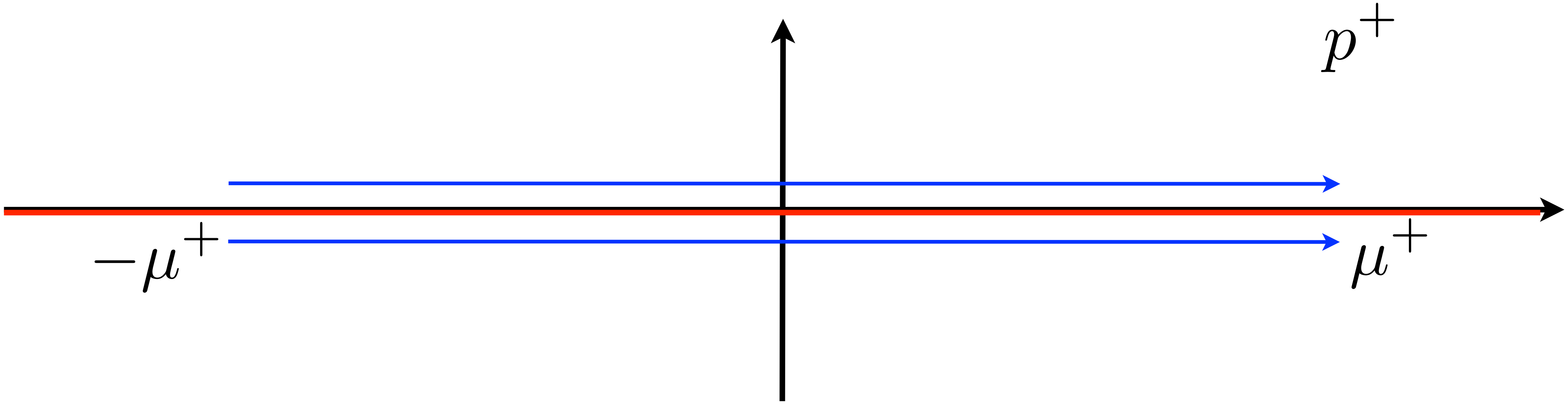

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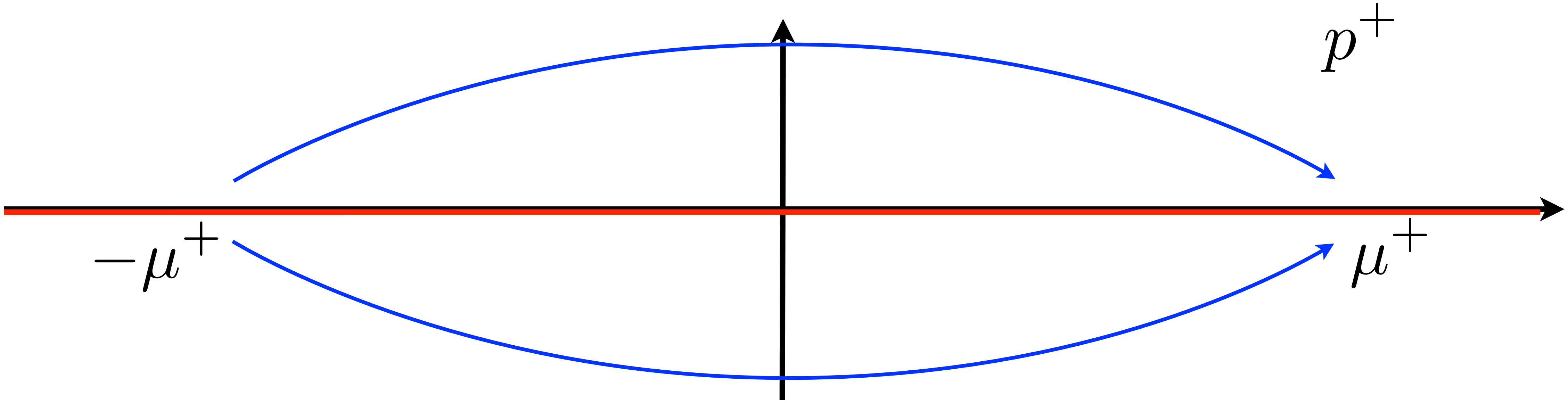

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- Along the arcs at large complex p^+ the integrand has a very simple behavior

$$\mathrm{Tr}\left[\gamma^-(S_R(P) - S_A(P)\right]_{p^-=0} = \frac{i}{p^+}\frac{m_\infty^2}{p_\perp^2 + m_\infty^2} + \mathcal{O}\left(\frac{1}{(p^+)^2}\right)$$

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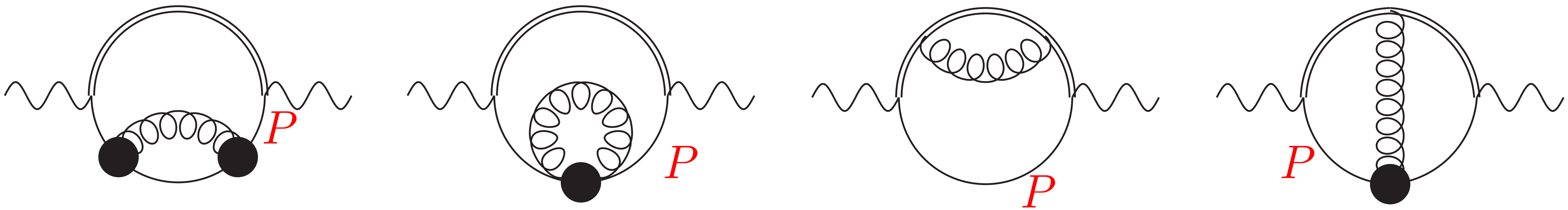
- The $p_\perp$ integral is UV-log divergent, giving the LO UV-divergence that cancels the IR divergence at the hard scale, now analytically

Independently obtained by Besak Bödeker **JCAP1203** (2012)

The NLO soft region



- At NLO one can use the KMS relations and the *ra* basis to write the diagrams in terms of fully retarded and fully advanced functions of P. The hard only depend on p^-.
- The contour deformations are then again possible and the diagrams can be expanded for large complex p^+. On general grounds we expect

$$(2\pi)^3 \frac{d\delta\Gamma_\gamma}{d^3k}\bigg|_{\rm soft} \propto \int \frac{dp^+ d^2p_\perp}{(2\pi)^3}\left[C_0\left(\frac{1}{p^+}\right)^0 + C_1\left(\frac{1}{p^+}\right)^1 + \ldots\right]$$

The soft region

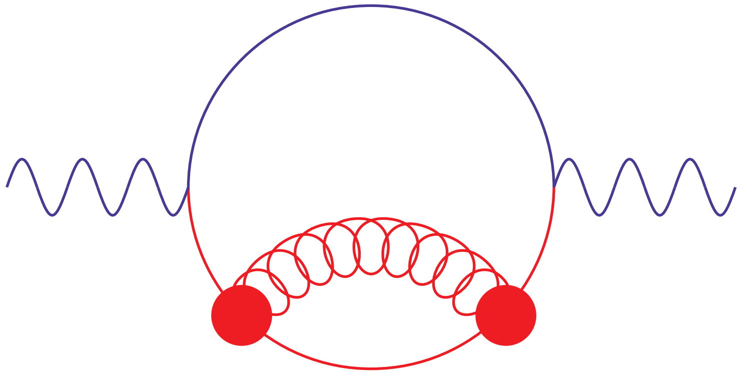
- The $(1/p^+)^0$ term has to be *exactly* the subtraction term we have seen before in the collinear region, to cancel the cutoff dependence. Confirmed by explicit calculation

- At order $1/p^+$ we had the LO result. We can expect

$$\frac{m_\infty^2}{p_\perp^2 + m_\infty^2} \to \frac{m_\infty^2 + \delta m_\infty^2}{p_\perp^2 + m_\infty^2 + \delta m_\infty^2} = \left(\frac{m_\infty^2}{p_\perp^2 + m_\infty^2} + \frac{\delta m_\infty^2 \, p_\perp^2}{(p_\perp^2 + m_\infty^2)^2} + \mathcal{O}(g^2) \right)$$

The explicit calculation finds just this contribution.

- The contribution from HTL vertices goes like $(1/p^+)^2$ or smaller on the arcs.



$$\sim \frac{1}{(p^+)^2}$$

The soft region

- Once the divergent part is subtracted the soft contribution is

$$(2\pi)^3 \frac{d\delta\Gamma_\gamma}{d^3k}\bigg|_{\mathrm{soft}} = \mathcal{A}(k)\frac{\delta m_\infty^2}{m_\infty^2}\int \frac{d^2p_\perp}{(2\pi)^2}\frac{p_\perp^2}{(p_\perp^2+m_\infty^2)^2}$$

- UV log-divergence has to cancel with the semi-collinear region, where $p_\perp \sim \sqrt{g}T$

Light-cone condensates

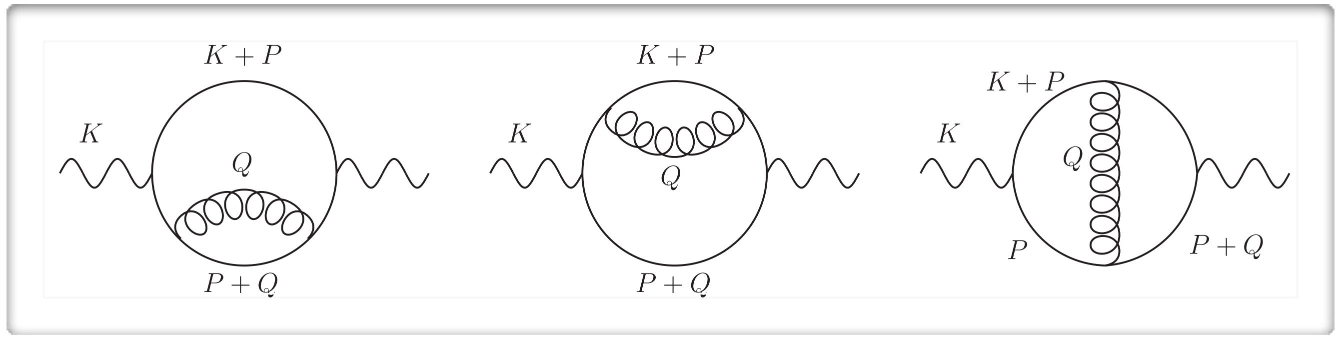
- Asymptotic mass Caron-Huot **PRD79** (2009)

$$m_\infty^2 = g^2 C_R (Z_g + Z_f)$$

$$Z_g \equiv \frac{1}{d_A} \left\langle v_\mu F^{\mu\rho} \frac{-1}{(v\cdot D)^2} v_\nu F^\nu_{\ \rho} \right\rangle \qquad v_k = (1,0,0,1)$$

$$= \frac{-1}{d_A} \int_0^\infty dx^+\, x^+ \langle v_{k\,\mu} F_a^{\mu\nu}(x^+,0,0_\perp) U_A^{ab}(x^+,0,0_\perp;0,0,0_\perp) v_{k\,\rho} F_{b\ \nu}^{\rho}(0)\rangle$$

$$Z_f \equiv \frac{1}{2d_R} \left\langle \overline{\psi} \frac{\not v}{v\cdot D} \psi \right\rangle$$


$K+P$
K
Q
$P+Q$
$K+P$
K
Q
$P+Q$
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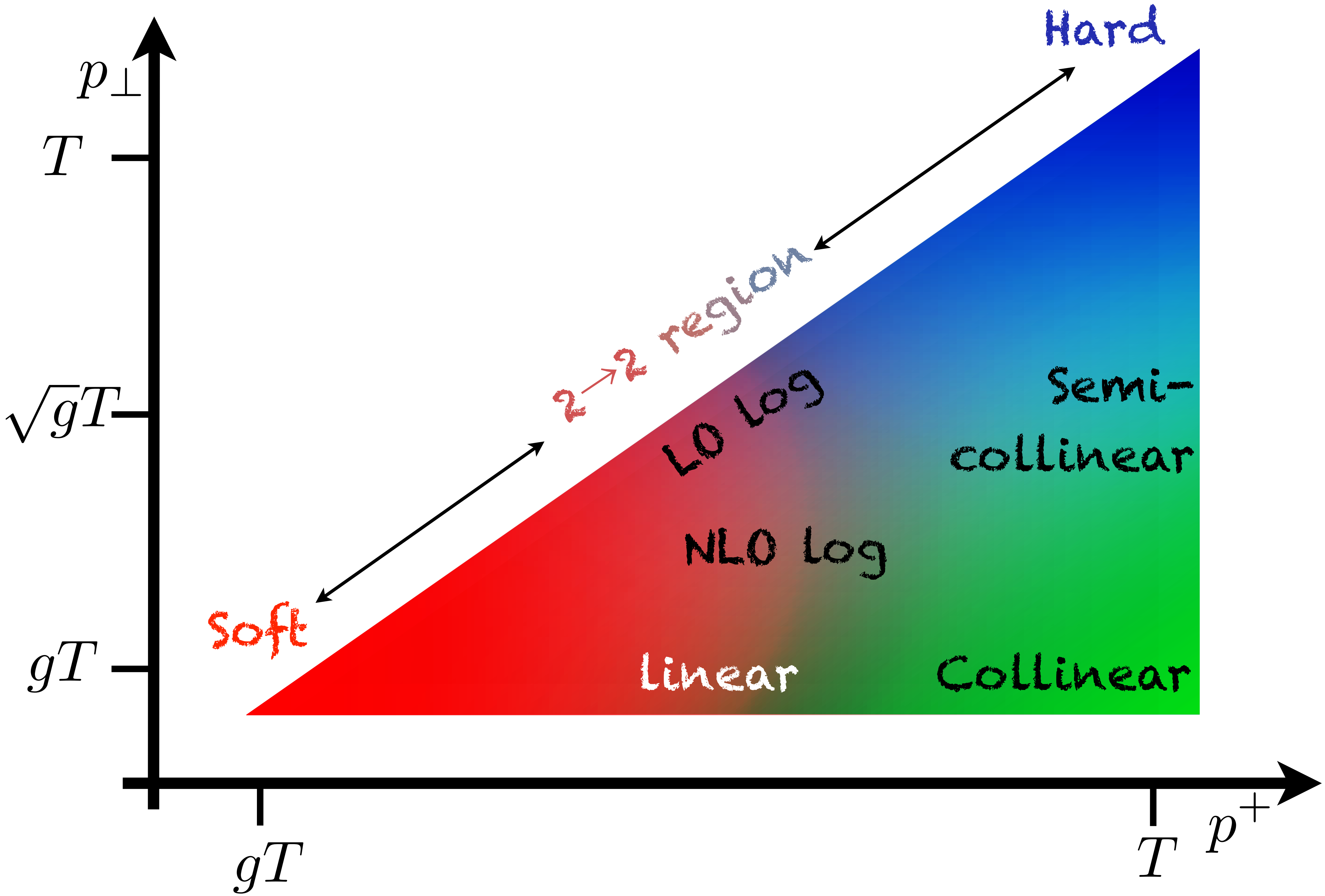
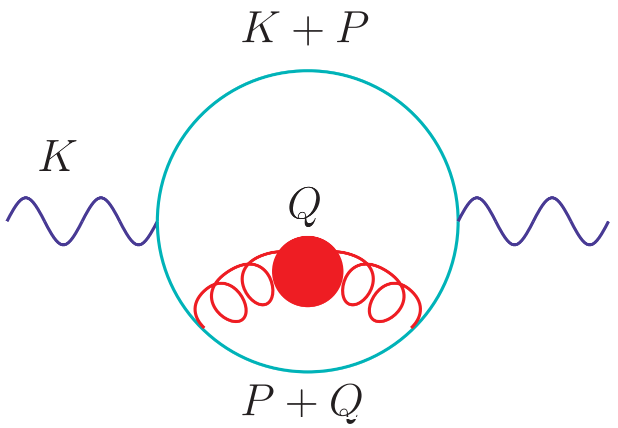
K
Q
P
$P+Q$


Hard
$p_\perp$
T
$\sqrt{g}T$
gT
2→2 region
LO log
Semi-
collinear
NLO log
Soft
linear
Collinear
gT
T
p^+

The semi-collinear region



P semi-collinear
Q soft

- Kinematical regions ⇒ different processes
  - Q timelike ⇒ 2↔2 processes with massive (plasmon) gluon
  - Q spacelike ⇒ 2↔3 processes: wider-angle bremsstrahlung and pair annihilation, no LPM interference

The semi-collinear region

- Subtraction term from the collinear region

$$\frac{d\delta\Gamma_\gamma}{d^3k}\bigg|^{\text{coll subtr.}}_{\text{semi-coll}} = 2\frac{\mathcal{A}(k)}{(2\pi)^3}\int dp^+ \left[\frac{(p^+)^2+(p^++k)^2}{(p^+)^2(p^++k)^2}\right]\frac{n_F(k+p^+)[1-n_F(p^+)]}{n_F(k)}$$
$$\times\frac{1}{g^2C_RT^2}\int\frac{d^2p_\perp}{(2\pi)^2}\frac{4(p^+)^2(p^++k)^2}{k^2p_\perp^4}\int\frac{d^2q_\perp}{(2\pi)^2}\,q_\perp^2\,\mathcal{C}(q_\perp)\,.$$

- Proper evaluation: replace

$$\frac{\hat{q}}{g^2C_R}\equiv\frac{1}{g^2C_R}\int\frac{d^2q_\perp}{(2\pi)^2}q_\perp^2\mathcal{C}(q_\perp)=\int\frac{d^4Q}{(2\pi)^3}\delta(q^-)q_\perp^2G_{rr}^{++}(Q)$$

with

$$\frac{\hat{q}(\delta E)}{g^2C_R}\equiv\int\frac{d^4Q}{(2\pi)^3}\delta(q^- - \delta E)\left[q_\perp^2G_{rr}^{++}(Q)+G_T^{rr}(Q)\left(\left[1+\frac{q_z^2}{q^2}\right]\delta E^2-2q_z\delta E\left[1-\frac{q_z^2}{q^2}\right]\right)\right]$$

because $\delta E \sim gT$ is no longer negligible

- The latter object too can be evaluated in Euclidean spacetime

Light-cone condensates

- Asymptotic mass Caron-Huot **PRD79** (2009)

$$m_\infty^2 = g^2 C_R (Z_g + Z_f)$$

$$Z_g \equiv \frac{1}{d_A} \left\langle v_\mu F^{\mu\rho} \frac{-1}{(v\cdot D)^2} v_\nu F^\nu_{\ \rho} \right\rangle \qquad v_k = (1,0,0,1)$$

$$= \frac{-1}{d_A} \int_0^\infty dx^+ \, x^+ \langle v_{k\,\mu} F_a^{\mu\nu}(x^+,0,0_\perp) U_A^{ab}(x^+,0,0_\perp;0,0,0_\perp) v_{k\,\rho} F_{b\ \nu}^{\rho}(0)\rangle$$

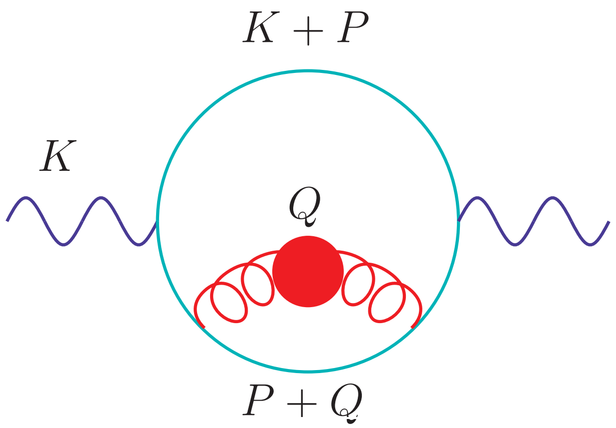
$$Z_f \equiv \frac{1}{2d_R} \left\langle \bar\psi \frac{\not v}{v\cdot D} \psi \right\rangle$$

- δE-dependent qhat

$$\frac{\hat q(\delta E)}{g^2 C_R} \equiv \int_{-\infty}^{\infty} dx^+ \, e^{ix^+\delta E} \, \frac{1}{d_A} \langle v_k^\mu F_\mu^{\ \nu}(x^+,0,0_\perp) U_A(x^+,0,0_\perp;0,0,0_\perp) v_k^\rho F_{\rho\nu}(0)\rangle,$$

For δE→0 the definition by audience members is recovered

The semi-collinear region



P semi-collinear
Q soft

- Limits and divergences

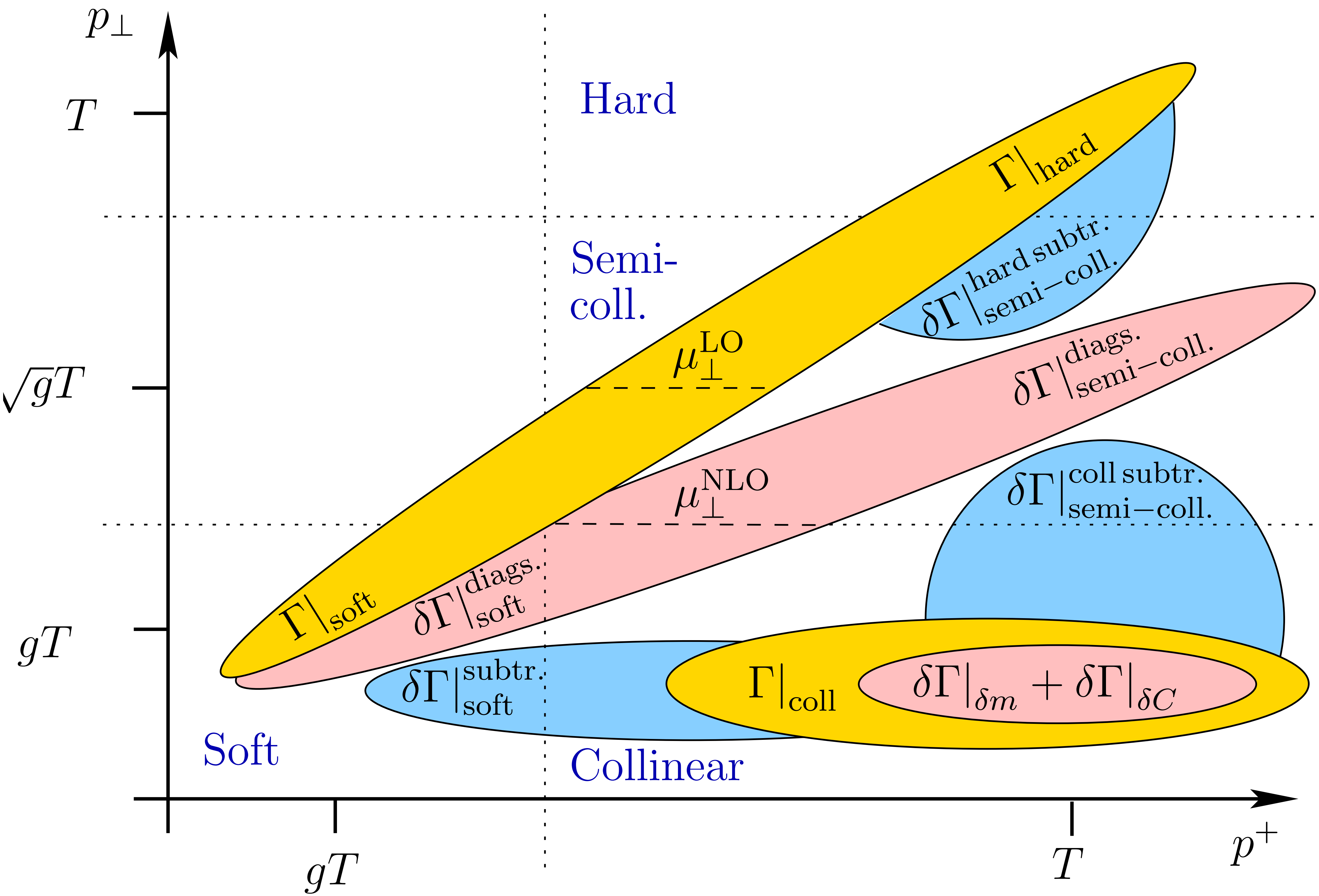
↑ $p_\perp \to \infty \, (\delta E \to \infty)$ subtract the hard limit

↓ $\mathrm{p}_\perp \to 0$ subtract the collinear limit $(p_\perp \gg q_\perp)$

↙ $\mathrm{p}_\perp \to 0 \wedge p^+ \to 0$ IR log, combines with UV soft log (NLO log)

- Aside from the IR-log, the general behaviour of the P integration can only be obtained numerically.

Kandinsky, Klee & Kurkela



Results

Summary

- LO rate

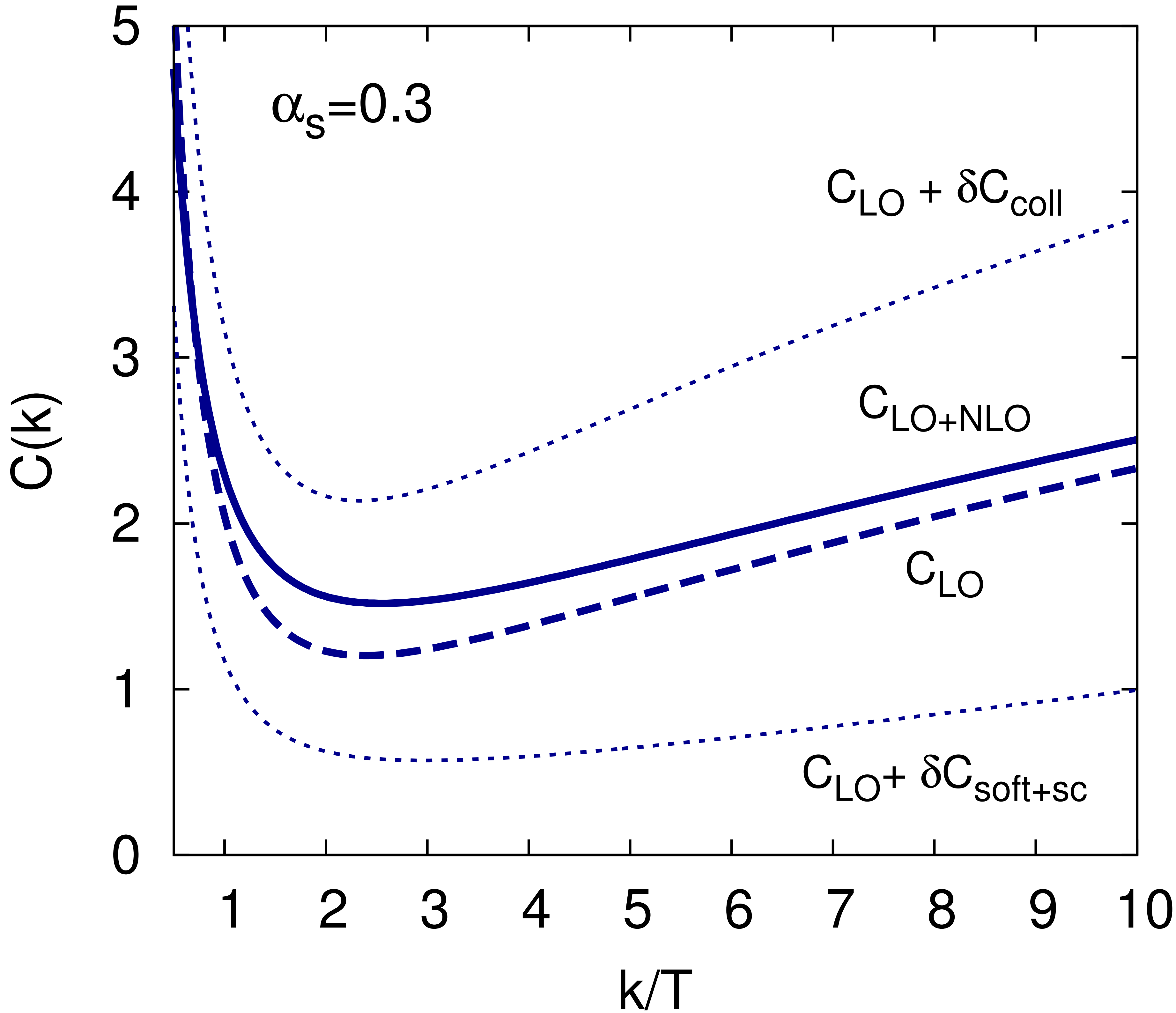
$$(2\pi)^3 \frac{d\Gamma}{d^3k}\bigg|_{\rm LO} = \mathcal{A}(k) \overbrace{\left[\log\frac{T}{m_\infty} + C_{2\to2}(k) + C_{\rm coll}(k)\right]}^{C_{\rm LO}(k)}$$

$$\mathcal{A}(k) = \alpha_{\rm EM}\, g^2\; C_F T^2 \frac{n_{\rm F}(k)}{2k} \sum_f Q_f^2 d_f$$

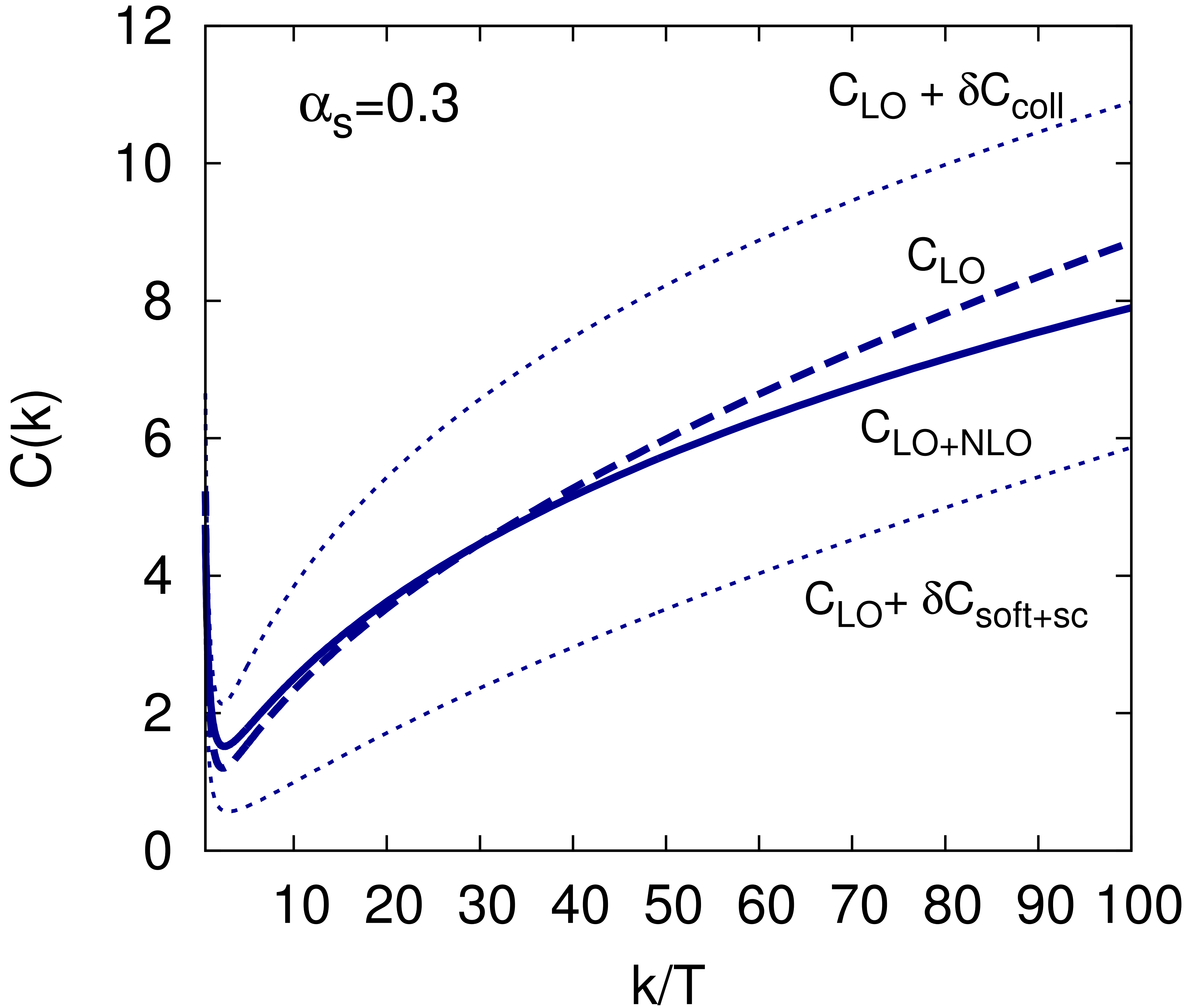
- NLO correction

$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\rm NLO} = \mathcal{A}(k) \overbrace{\Bigg[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2}C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2}C_{\rm coll}^{\delta m}(k) + \frac{g^2 C_A T}{m_D}C_{\rm coll}^{\delta \mathcal{C}}(k)}_{\delta C_{\rm coll}(k)}\Bigg]}^{\delta C_{\rm NLO}(k)}$$

$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\mathrm{NLO}} = \mathcal{A}(k)\left[\overbrace{\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2}C_{\mathrm{soft+sc}}(k)}_{\delta C_{\mathrm{soft+sc}}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2}C_{\mathrm{coll}}^{\delta m}(k) + \frac{g^2 C_A T}{m_D}C_{\mathrm{coll}}^{\delta \mathcal{C}}(k)}_{\delta C_{\mathrm{coll}}(k)}}^{\delta C_{\mathrm{NLO}}(k)}\right]$$



$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\rm NLO} = \mathcal{A}(k)\overbrace{\Bigg[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2}C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2}C_{\rm coll}^{\delta m}(k) + \frac{g^2C_AT}{m_D}C_{\rm coll}^{\delta\mathcal{C}}(k)}_{\delta C_{\rm coll}(k)}\Bigg]}^{\delta C_{\rm NLO}(k)}$$



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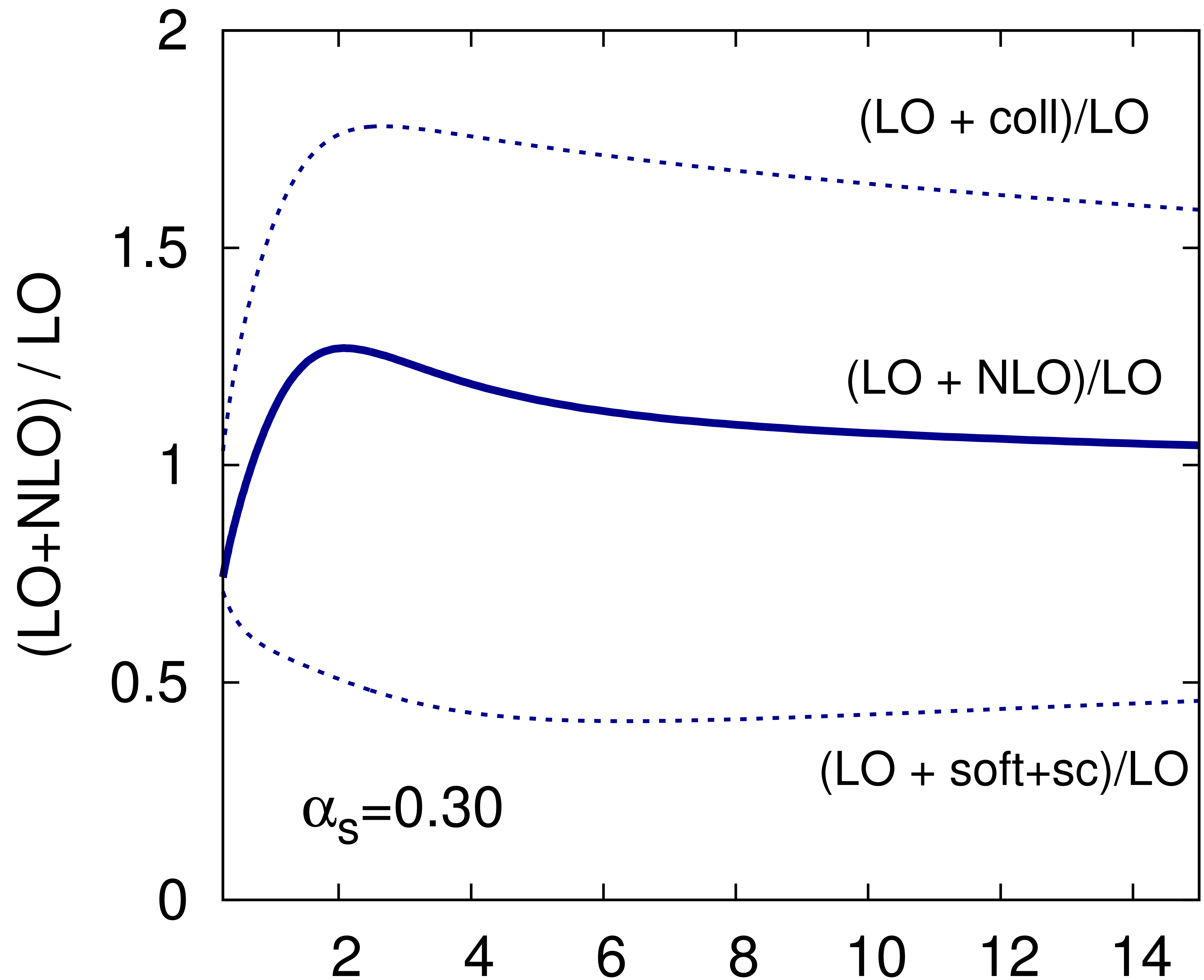

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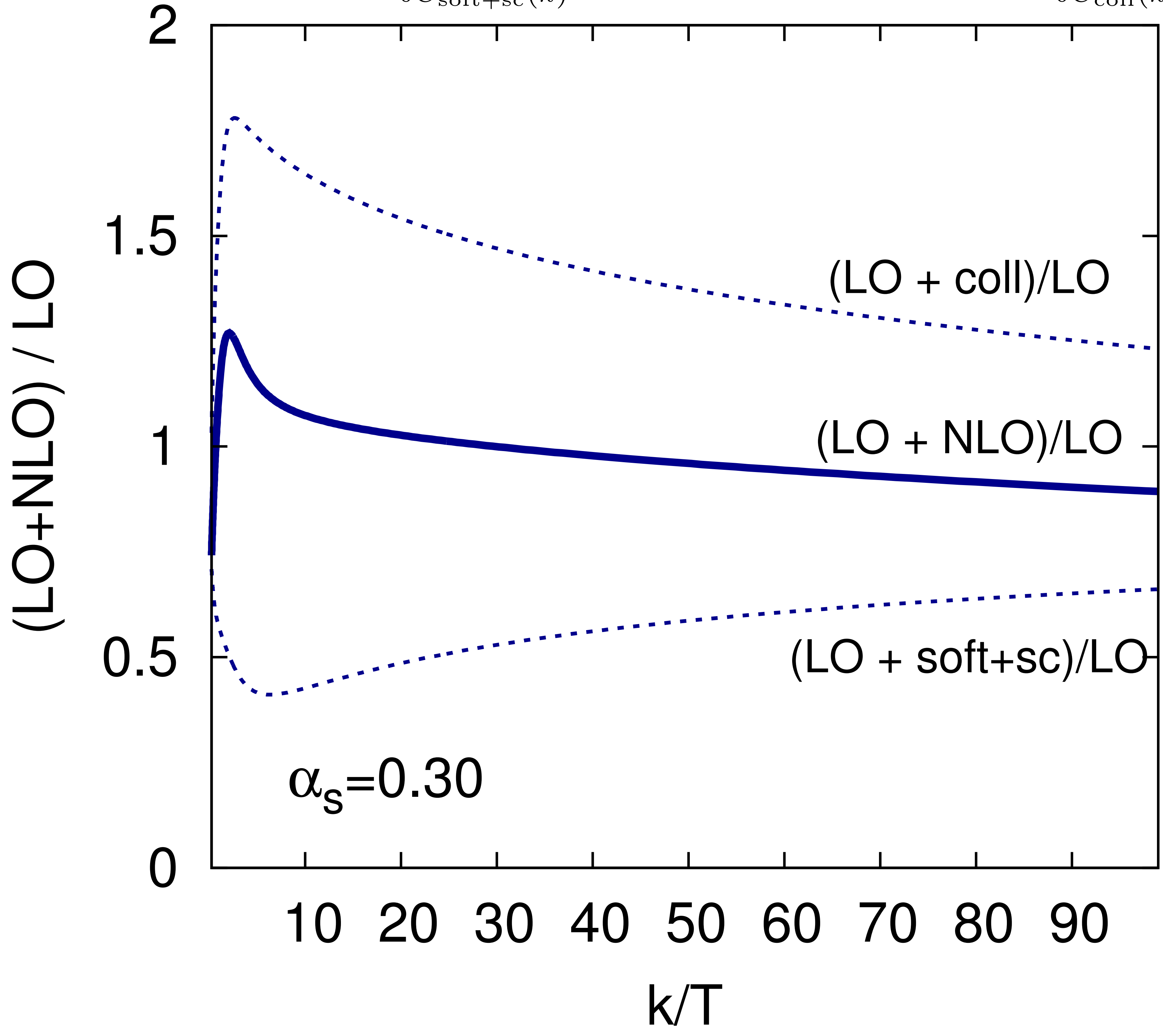
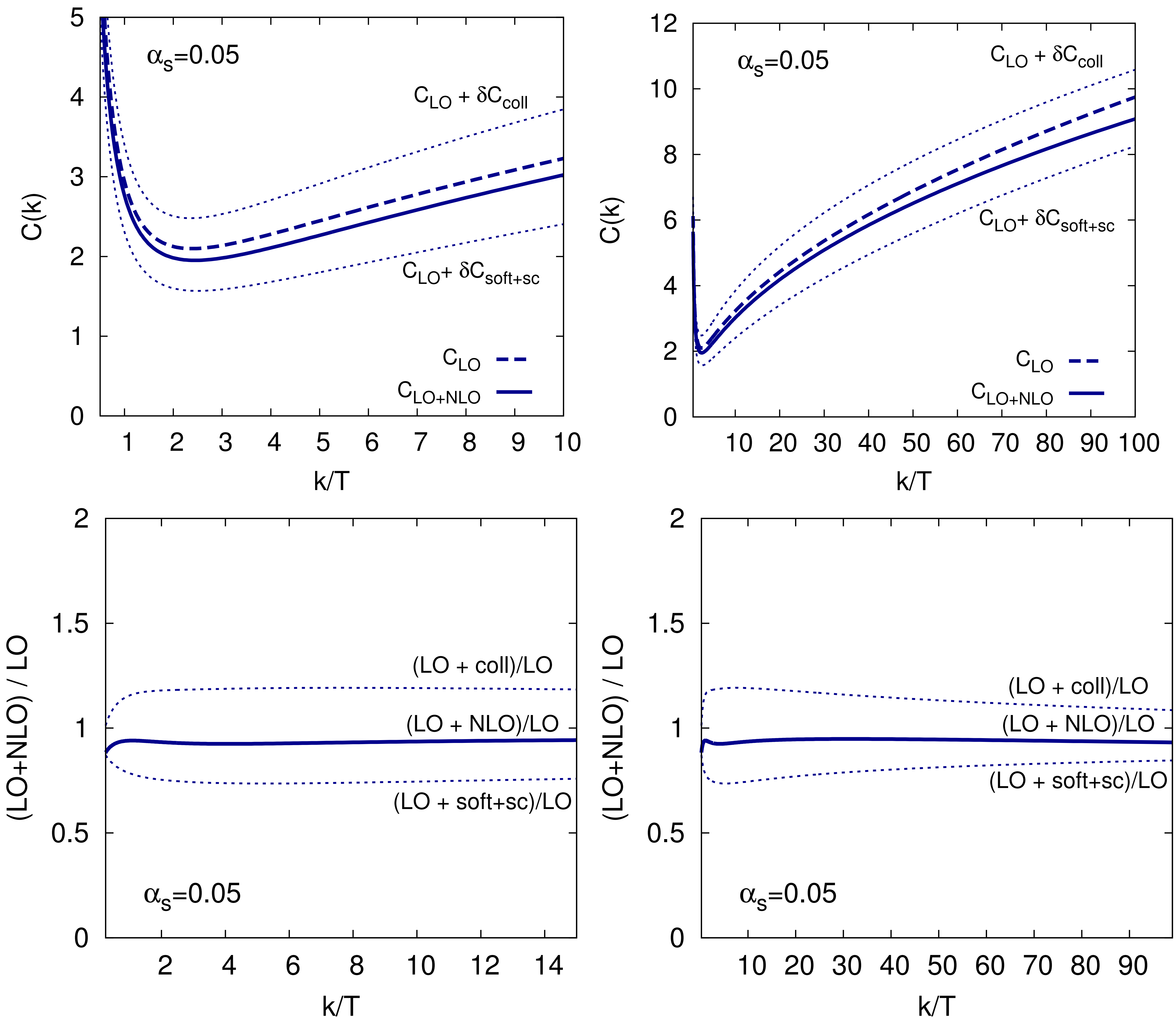
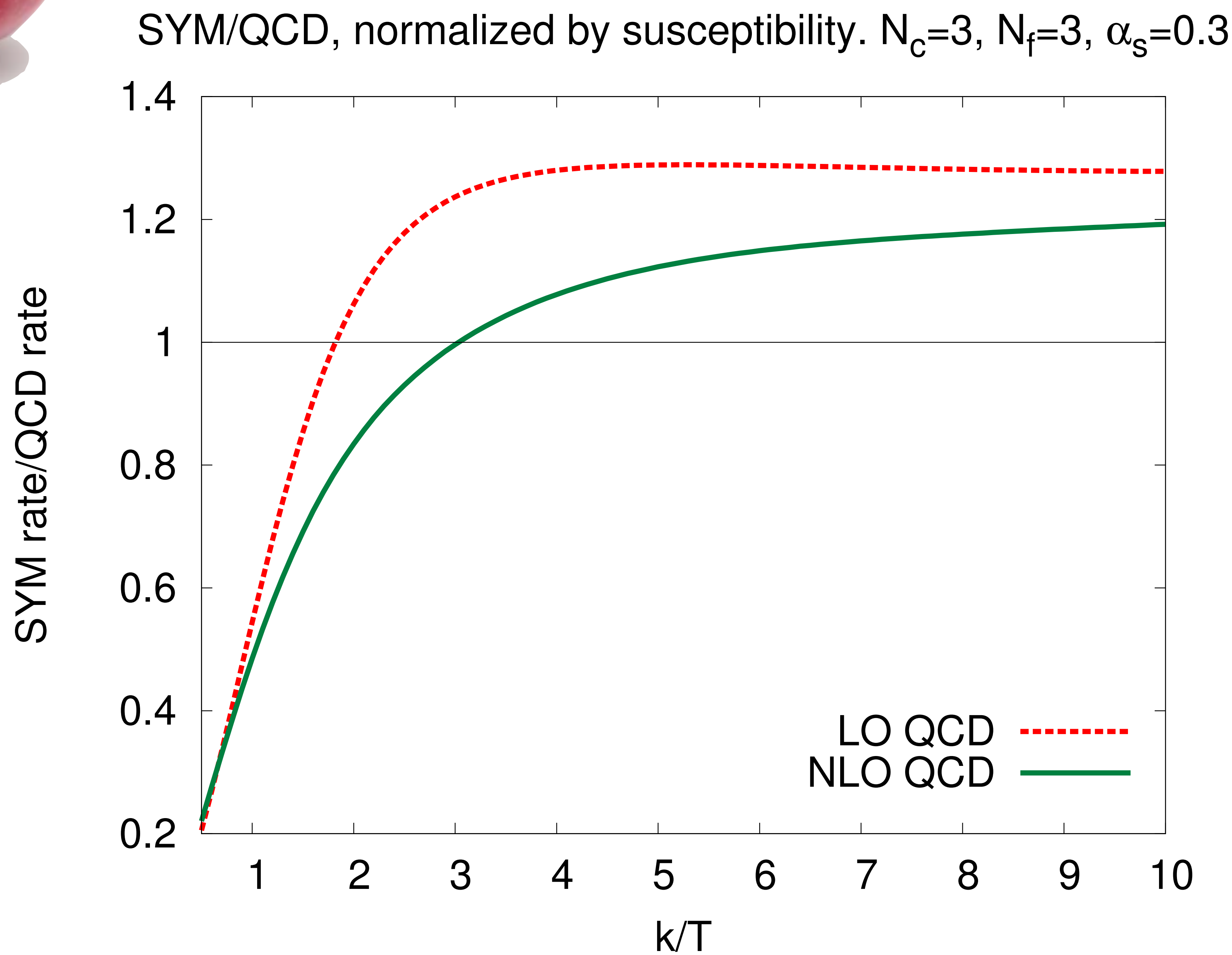




α_s=0.05
C_{LO} + δC_{coll}
C_{LO}+ $\delta C_{soft+sc}$
C_{LO}
C_{LO+NLO}
C(k)
k/T
(LO + coll)/LO
(LO + NLO)/LO
(LO + soft+sc)/LO
(LO+NLO) / LO

QCD and SYM



- Strongly-coupled $\mathcal{N}$=4: Caron-Huot Kovtun Moore Starinets Yaffe **JHEP0612** (2006)

Conclusions

$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\rm NLO} = \mathcal{A}(k) \Bigg[\overbrace{\underbrace{\frac{\delta m_\infty^2}{m_\infty^2} \log \frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2} C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2} C^{\delta m}_{\rm coll}(k) + \frac{g^2 C_A T}{m_D} C^{\delta C}_{\rm coll}(k)}_{\delta C_{\rm coll}(k)}}^{\delta C_{\rm NLO}(k)} \Bigg]$$

- The NLO contribution is made of four terms, with a semicollinear / soft log ($\sim g^{1/2}$)
- These four terms combine in two large and opposite contributions that largely cancel giving a relatively small NLO correction. Is the cancellation accidental? At α_s=0.3 the NLO is initially positive, then turns negative and keeps growing at large k/T . At small α_s (α_s=0.05) the correction is always negative
- In the phenomenologically interesting window up to the NLO correction is 10%-20% for αs=0.3

Conclusions

- Contrary to the heavy-quark diffusion case, here we probe soft fields at light-like separations. After a few headaches, it turns out this is computationally easier and better convergent
- Light-cone sum rules are a powerful instrument. Is there a Euclidean picture for fermions too?
- Finding out that there is a bridge is as important as being able to go to the other side. Other applications for it? Tackling new NLO calculations?